Often heuristic power can be gained at the expense of giving up admissibility by using for $\hat{h}$ some function that is not a lower bound on h. This added heuristic power then allows us to solve much harder problems. In the 8-puzzle, the function $\hat{h}(n) = W(n)$ (where $W(n)$ is the number of tiles in the wrong place) is a lower bound on $h(n)$, but it does not provide a very good estimate of the difficulty (in terms of number of steps to the goal) of a tile configuration. A better estimate is the function $\hat{h}(n) = P(n)$, where $P(n)$ is the sum of the distances that each tile is from "home" (ignoring intervening pieces). Even this estimate is too gross, however, in that it does not accurately appraise the difficulty of exchanging the positions of two adjacent tiles.

An estimate that works quite well for the 8-puzzle is

$$\hat{h}(n) = P(n) + 3\,S(n)$$

The quantity $S(n)$ is a *sequence score* obtained by checking around the noncentral squares in turn, allotting 2 for every tile not followed by its proper successor and 0 for every other tile, except that a piece in the center scores 1. We note that this $\hat{h}$ function does not provide a lower bound for h.

With this $\hat{h}$ used in the evaluation function $\hat{f}(n) = \hat{g}(n) + \hat{h}(n)$, we can easily solve much more difficult 8-puzzles than the one we solved earlier. In Fig. 3-8 we show the tree resulting from applying the ordered-search algorithm with this evaluation function to the problem of transforming

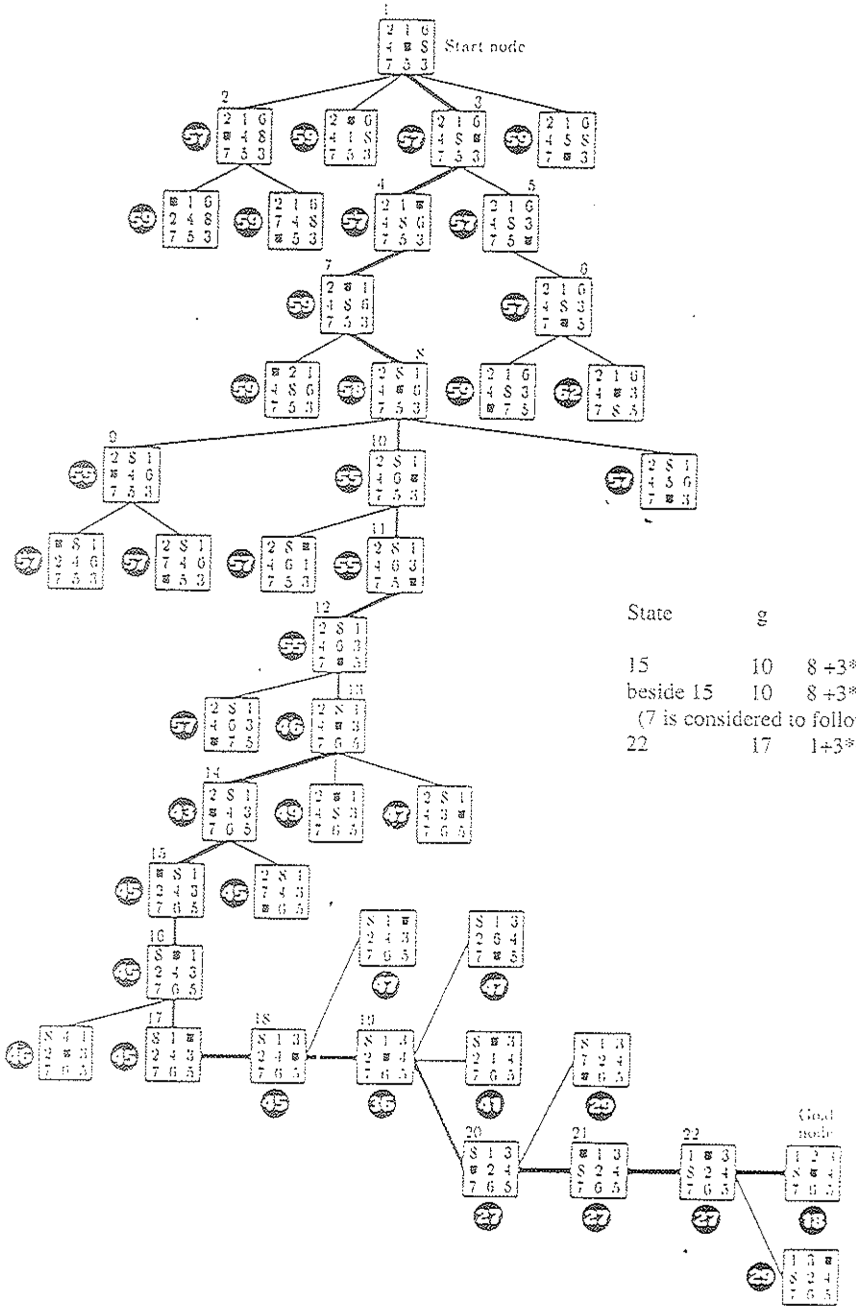
2	1	6
4	▨	8
7	5	3

into

1	2	3
8	▨	4
7	6	5

Again the $\hat{f}$ value of each node is circled, and the uncircled numbers show the order in which nodes are expanded.

The solution path found happens to be of minimal length (18 steps), although since the $\hat{h}$ function is not a lower bound for h, we were not guaranteed finding an optimal path. Note that this $\hat{h}$ function results in a focusing of search rather directly toward the goal; only a very limited amount of spreading occurred, and that was near the start.



State	g	h	f
15	10	8 +3*(2*4+1)	45
beside 15	10	8 +3*(2*4+1)	45
(7 is considered to follow 6)			
22	17	1+3*(2*1+1)	27

FIG. 3-8 *A tree produced by an ordered search.*