Constraint Technology

A Programming Paradigm on the Rise

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Invited talks at UU / IT / TDB (2008-03-12)
and at UU / IT / TAPVES (2008-03-13)

Outline

1. **What is Constraint Technology?**
   - Constraint Problems
   - Constraint Technology
   - Constraint Modelling
   - Constraint Solving by Global Search
   - Constraint Solving by Local Search
   - History & Success Stories
2. **Enabling Basic Research and Tools**
   - Abstract Constraint Modelling
   - Symmetry
   - Constraint-Based Local Search
3. **Technology Transfer**
   - Air-Traffic Management
   - Computational Biology
   - Computational Finance

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Constraint Problems

Example (The Tourist Site Competition (TSC) problem)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka							
Falun							
Lund							
Mora							
Sigtuna							
Uppsala							
Ystad							

1. Every tourist site is visited by $r = 3$ judges.
2. Every judge visits $c = 3$ tourist sites.
3. Every pair of sites is visited by $\lambda = 1$ common judge.

Constraint Problems

Example (The Tourist Site Competition (TSC) problem)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓				
Falun	✓			✓	✓		
Lund	✓					✓	✓
Mora		✓		✓		✓	
Sigtuna		✓			✓		✓
Uppsala			✓	✓			✓
Ystad			✓		✓	✓	

1. Every tourist site is visited by $r = 3$ judges.
2. Every judge visits $c = 3$ tourist sites.
3. Every pair of sites is visited by $\lambda = 1$ common judge.

Constraint Problems

Many increasingly ubiquitous and important real-life problems *must* be solved by intelligent search:

- Rostering, scheduling, time-tabling
- Planning
- Configuration, design
- RNA structure prediction, alignment, sequencing, . . .
- Financial investment instrument design
- VLSI circuit layout
- Hardware / software specification verification
- . . .

In a constraint problem, decisions have to be made so that:

- Some constraints are satisfied.
- Optionally: Some cost/benefit is minimised/maximised.

Outline

Constraint Technology

Constraint Technology (CT) offers methods and tools for:

- Effectively modelling constraint problems.
- Efficiently solving constraint problems,
  by global search (in Sudoku fashion)
  or by local search (see below).

Slogan of CT:

Constraint Program = Model + Search

This is orthogonal and complementary to

- Operations Research (OR):
  linear programming (LP), integer LP (ILP),
  mixed integer linear programming (MILP), . . .
- Boolean satisfiability (SAT)
- . . .

leading to hybridised optimisation technologies!

Scope of Constraint Technology

Constraint Technology addresses:

- satisfaction problems and optimisation problems
- discrete variables and continuous variables
- linear constraints and non-linear constraints

in any combination thereof,
by:

- global search, if optimality more important than speed
- local search, if speed is more important than optimality

Built-in support for:

- explanations
- soft constraints
- . . .

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Constraint Modelling

Example (TSC integer model)

set *Sites*, *Judges*
cst r, c, λ : **int**
var *TSC* : **array**[*Sites*, *Judges*] **of** 0..1
solve
  forall s **in** *Sites* : $r = \textbf{sum}(j \textbf{ in } Judges)\ TSC[s, j]$
  forall j **in** *Judges* : $c = \textbf{sum}(s \textbf{ in } Sites)\ TSC[s, j]$
  forall $s_1 \neq s_2$ **in** *Sites* :
    $\lambda = \textbf{count}(j \textbf{ in } Judges)\ TSC[s_1, j] = 1 = TSC[s_2, j]$

Example (Instance data for the Sweden TSC instance)

$Sites = \{Birka, Falun, Lund, Mora, Sigtuna, Uppsala, Ystad\}$
$Judges = \{Ali, Dan, Eva, Jim, Leo, Mia, Ulla\}$
$\langle r, c, \lambda \rangle = \langle 3, 3, 1 \rangle$

Constraint Modelling

Example (TSC set model)

$$
\begin{array}{l}
\textbf{set } \mathit{Sites}, \mathit{Judges} \\
\textbf{cst } r, c, \lambda : \textbf{int} \\
\textbf{var } \mathit{TSC} : \textbf{array}[\mathit{Sites}] \textbf{ of setof}(\mathit{Judges}) \\
\textbf{solve} \\
\quad \textbf{forall } s \textbf{ in } \mathit{Sites} : r = \textbf{size}(\mathit{TSC}[s]) \\
\quad \textbf{forall } j \textbf{ in } \mathit{Judges} : c = \textbf{count}(s \textbf{ in } \mathit{Sites})\ j \in \mathit{TSC}[s] \\
\quad \textbf{forall } s_1 \neq s_2 \textbf{ in } \mathit{Sites} : \\
\qquad \lambda = \textbf{size}(\mathit{TSC}[s_1] \cap \mathit{TSC}[s_2])
\end{array}
$$

Example (Instance data for the Sweden TSC instance)

$$\mathit{Sites} = \{\mathit{Birka}, \mathit{Falun}, \mathit{Lund}, \mathit{Mora}, \mathit{Sigtuna}, \mathit{Uppsala}, \mathit{Ystad}\}$$
$$\mathit{Judges} = \{\mathit{Ali}, \mathit{Dan}, \mathit{Eva}, \mathit{Jim}, \mathit{Leo}, \mathit{Mia}, \mathit{Ulla}\}$$
$$\langle r, c, \lambda \rangle = \langle 3, 3, 1 \rangle$$

Constraint Modelling

Example (Sudoku)

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

```
range N = 1..9
var s : array[N, N] of N
solve
    ... // load clues
    forall r in N : AllDifferent(s[r, *])
    forall c in N : AllDifferent(s[*, c])
    forall r, c in 1..9 by 3 :
        AllDifferent(s[r + 0..2, c + 0..2])
```

Fill in the grid so that every row, every column, and every highlighted 3 × 3 box contains the digits 1 through 9.

Constraint Modelling

Example (Sudoku)

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

```
range $N = 1..9$
var $s$ : array$[N, N]$ of $N$
solve
   ... // load clues
   forall $r$ in $N$ : $AllDifferent(s[r, *])$
   forall $c$ in $N$ : $AllDifferent(s[*, c])$
   forall $r, c$ in $1..9$ by $3$ :
      $AllDifferent(s[r + 0..2, c + 0..2])$
```

Fill in the grid so that every row, every column, and every highlighted 3 × 3 box contains the digits 1 through 9.

Constraint Modelling: Global Constraints

Example (*AllDifferent*)

Consider the n-ary constraint *AllDifferent*, with $n = 4$ here:

$$AllDifferent([a, b, c, d]) \tag{1}$$

Declaratively, (1) is equivalent to the $\Theta(n^2)$ 2-ary constraints

$$a \neq b,\ a \neq c,\ a \neq d,\ b \neq c,\ b \neq d,\ c \neq d \tag{2}$$

but it provides convenient genericity in constraint models.
Operationally, (1) prunes much stronger than (2). Example:

$$a \in \{2, 3\},\ \ b \in \{2, 3\},\ \ c \in \{1, 3\},\ \ d \in \{1, 2, 3, 4\}$$

No pruning by (2). But perfect pruning by (1).

Constraint Modelling: Global Constraints

Example (*AllDifferent*)

Consider the n-ary constraint *AllDifferent*, with $n = 4$ here:

$$AllDifferent([a, b, c, d]) \qquad (1)$$

Declaratively, (1) is equivalent to the $\Theta(n^2)$ 2-ary constraints

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but it provides convenient genericity in constraint models.
Operationally, (1) prunes much stronger than (2). Example:

$$a \in \{2, 3\},\ b \in \{2, 3\},\ c \in \{1, \cancel{3}\},\ d \in \{\cancel{1}, \cancel{2}, \cancel{3}, 4\}$$

No pruning by (2). But perfect pruning by (1).

Constraint Modelling: Global Constraints

Global constraints are a much admired feature of CT: they allow the preservation of combinatorial sub-structures of a problem, both while modelling it and while solving it.

About 300 global constraints have been implemented so far, declaratively encapsulating advanced algorithms from combinatorics, graph theory, flow theory, matching theory, geometry, automata theory, . . . :

- Rostering under balancing, counting, and coverage constraints
- Scheduling under resource constraints
- Geometrical constraints between points, segments, . . .
- . . .

Constraint Modelling

Pride:

*Constraint programming represents
one of the closest approaches computer science
has yet made to the Holy Grail of programming:
the user states the problem, the computer solves it.*
— Eugene Freuder

Prejudice:

*The contribution of the article should be the reduction
of an engineering problem to a known optimization format.
[. . .] showcases pseudo code [. . .] submit this
work to a journal interested in code semantics [. . .].*
— Reviewer of a paper of ours at a prestigious OR journal

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Constraint Solving by Local Search
History & Success Stories
Enabling Basic Research and Tools
Technology Transfer

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	?						
Uppsala	✗						
Ystad	✗						

Ali cannot be a judge of Sigtuna as that would violate the second constraint (every judge visits $c = 3$ tourist sites). Actually, Ali cannot be a judge of Mora, Uppsala, or Ystad either, for the same reason, and this was already inferred when trying the decision that Ali be a judge of Lund!

Constraint Solving by Global Search

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Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	?						
Uppsala	✗						
Ystad	✗						

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Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗						
Uppsala	✗						
Ystad	✗						

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Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗						
Uppsala	✗						
Ystad	✗						

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Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗						
Uppsala	✗						
Ystad	✗						

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Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	?	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Constraint Solving by Global Search

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Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	?	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Decision: Dan is a judge of Sigtuna. (✓ decisions first.)

Constraint Solving by Global Search

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	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Decision: Dan is a judge of Sigtuna. (✓ decisions first.)

Constraint Solving by Global Search

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	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Inference: Dan cannot be a judge of Uppsala and Ystad as otherwise the second constraint (every judge visits $c = 3$ tourist sites) would be violated for Dan.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Inference: Dan cannot be a judge of Uppsala and Ystad as otherwise the second constraint (every judge visits $c = 3$ tourist sites) would be violated for Dan.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Inference: Eva, Jim, and Mia cannot be judges of Sigtuna as otherwise the third constraint (every pair of sites is visited by $\lambda = 1$ common judge) would be violated.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Inference: Eva, Jim, and Mia cannot be judges of Sigtuna as otherwise the third constraint (every pair of sites is visited by $\lambda = 1$ common judge) would be violated.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Inference: Leo and Ulla must now be judges of Sigtuna as otherwise the first constraint (every tourist site is visited by $r = 3$ judges) would be violated for Sigtuna.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Inference: Leo and Ulla must now be judges of Sigtuna as otherwise the first constraint (every tourist site is visited by $r = 3$ judges) would be violated for Sigtuna.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Inference: Eva must now be a judge of Uppsala and Ystad as otherwise the second constraint (every judge visits $c = 3$ tourist sites) would be violated for Eva.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Inference: Eva must now be a judge of Uppsala and Ystad as otherwise the second constraint (every judge visits $c = 3$ tourist sites) would be violated for Eva.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Fixpoint reached: No more propagation possible.

Constraint Solving by Global Search

Example (TSC: Sweden partial assignment)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓	✗	✗	✗	✗
Falun	✓	✗	✗	✓	✓	✗	✗
Lund	✓	✗	✗	✗	✗	✓	✓
Mora	✗	✓	✗	✓	✗	✓	✗
Sigtuna	✗	✓	✗	✗	✓	✗	✓
Uppsala	✗	✗	✓				
Ystad	✗	✗	✓				

Decision: Jim is a judge of Uppsala. (✓ decisions first.)

Inference: etc.

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

a		1	2	3	4	5	6	7	8	9
b	0	1	2	3	4	5	6	7	8	

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

Posting $2 \cdot a + 4 \cdot b = 24$: pruning unwitnessed values of a:

a		1	2	3	4	5	6	7	8	9
b	0	1	2	3	4	5	6	7	8	

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Posting $2 \cdot a + 4 \cdot b = 24$: pruning unwitnessed values of b:

a			2		4		6		8	
b	0	1	2	3	4	5	6	7	8	

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Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

a			2		4		6		8	
b			2	3	4	5				

Suspending $2 \cdot a + 4 \cdot b = 24$, as it is not definitely true.

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

Posting $a + b = 9$: pruning unwitnessed values of a:

a			2		4		6		8	
b			2	3	4	5				

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \dots, 9\}$ and $b \in \{0, 1, \dots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

Posting $a + b = 9$: pruning unwitnessed values of a:

a			2		4		6		8	
b			2	3	4	5				

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

Posting $a + b = 9$: pruning unwitnessed values of b:

a					4		6			
b			2	3	4	5				

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

Posting $a + b = 9$: pruning unwitnessed values of b:

a					4		6			
b			2	3	4	5				

Constraint Solving by Global Search

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Find $a \in \{1, 2, \dots, 9\}$ and $b \in \{0, 1, \dots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

a					4		6			
b				3		5				

Suspending $a + b = 9$, as it is not definitely true.

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \dots, 9\}$ and $b \in \{0, 1, \dots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

Waking $2 \cdot a + 4 \cdot b = 24$: pruning unwitnessed values of a:

a					4		6			
b				3		5				

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

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Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

a							6			
b				3						

Deactivating $2 \cdot a + 4 \cdot b = 24$, as it is definitely true.

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

Waking $a + b = 9$: pruning unwitnessed values of a:

a							6			
b				3						

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Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

a							6			
b				3						

No suspended constraints: All solutions found. No search!

Constraint Solving by Global Search

Example (Local Propagation to Domain Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$\begin{aligned} 2 \cdot a + 4 \cdot b &= 24 \\ a + b &= 9 \end{aligned}$$

a							6			
b				3						

This general propagation method works for all systems of constraints (linear or not, equalities or inequalities, etc), no matter how many constraints and decision variables.

Constraint Solving by Global Search

Example (Local Propagation to Bounds Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$2 \cdot a + 4 \cdot b = 24$$

Posting $2 \cdot a + 4 \cdot b = 24$: pruning unwitnessed values of b:

a		1	2	3	4	5	6	7	8	9
b	0	1	2	3	4	5	6	7	8	

Suspending $2 \cdot a + 4 \cdot b = 24$, as it is not definitely true.

Suspended constraints: No solutions found yet. Search!

Constraint Solving by Global Search

Example (Local Propagation to Bounds Consistency)

Find $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 8\}$ such that

$$2 \cdot a + 4 \cdot b = 24$$

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Suspending $2 \cdot a + 4 \cdot b = 24$, as it is not definitely true.
Suspended constraints: No solutions found yet. Search!

Constraint Solving by Global Search

Local Propagation and Local Consistency

Use of polynomial-time propagators.
Use of incomplete propagators, when necessary/sufficient.

Example ($\mathit{AllDifferent}(x_1, \ldots, x_n)$)

Bounds consistency in $O(n \log n)$ time, but often $O(n)$ time.
Domain consistency in $O(n^{2.5})$ time.
Compare with the $\Theta(n^2)$ constraints $\forall 1 \leq i < j \leq n : x_i \neq x_j$.

Example (Arithmetic constraints over n variables)

Bounds consistency in $O(n)$ time.
Domain consistency in $O(2^n)$ time.

Constraint Solving by Global Search

Global Search Algorithm

```
post all given constraints (including propagation)
while there is at least one suspended constraint do
    pick a variable $x$ with $|dom(x)| \geq 2$
    pick some values $d_i \in dom(x)$
    branch on mutually exclusive constraints,
    say $x = d$ and $x \neq d$, or $x > d$ and $x \leq d$
end while
```

Heuristics

- Line 3: variable ordering heuristic: smallest domain, ...
- Line 4: value ordering heuristic: highest, middle, ...
- Tree exploration: depth-first search, ...
  with backtracking when a constraint is definitely false

Outline

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   - Constraint Problems
   - Constraint Technology
   - Constraint Modelling
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   - History & Success Stories
2. **Enabling Basic Research and Tools**
   - Abstract Constraint Modelling
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   - Computational Biology
   - Computational Finance

Constraint Solving by Local Search

Example (TSC: Sweden configuration after *i* moves)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓				
Falun	✓			✓		✓	
Lund	✓					✓	✓
Mora		✓		✓	✓	→	
Sigtuna		✓			✓		✓
Uppsala			✓	✓			✓
Ystad			✓		✓	✓	

1. Every tourist site is visited by $r = 3$ judges. (Invariant)
2. Every judge visits $c = 3$ tourist sites.
3. Ex: Mora and Sigtuna have $2 > \lambda = 1$ common judges: let Mia visit Mora instead of Leo.

Constraint Solving by Local Search

Example (TSC: Sweden configuration after $i+1$ moves)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓				
Falun	✓			✓	←	✓	
Lund	✓					✓	✓
Mora		✓		✓		✓	
Sigtuna		✓			✓		✓
Uppsala			✓	✓			✓
Ystad			✓		✓	✓	

1. Every tourist site is visited by $r = 3$ judges. (Invariant)
2. Leo visits $2 < c = 3$ sites; Mia visits $4 > c = 3$ sites.
3. Ex: Falun and Lund have $2 > \lambda = 1$ common judges: let Leo visit Falun instead of Mia.

Constraint Solving by Local Search

Example (TSC: Sweden configuration after $i+2$ moves)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓				
Falun	✓			✓	✓		
Lund	✓					✓	✓
Mora		✓		✓		✓	
Sigtuna		✓			✓		✓
Uppsala			✓	✓			✓
Ystad			✓		✓	✓	

1. Every tourist site is visited by $r = 3$ judges. (Invariant)
2. Every judge visits $c = 3$ tourist sites.
3. Every pair of sites is visited by $\lambda = 1$ common judge.

Constraint Solving by Local Search

Local Search Algorithm

```
1: let $k$ and $k^*$ be the same **computed** initial configuration
2: **for** $i := 1$ **to** *MaxIterations* **do**
3:     let $k$ be a **picked** neighbour configuration of $k$
4:     **if** $k$ is a solution and is better than $k^*$ **then** $k^* := k$
5: **end for**
6: **return** $k^*$
```

Heuristics: What Neighbour To Move To?

- Line 3: assign, flip, swap, add, drop, transfer, ...
- Line 3: best / first / random improvement, ...

Meta-Heuristics: How To Escape Local Optima?

- Lines 2 – 5: simulated annealing, tabu search, ...

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History of CT

Stand-alone languages:

- ALICE by Jean-Louis Laurière, France, 1976
- CHIP at ECRC, Germany, 1987 – 1990,
  then marketed by Cosytec, France, 1990 – 1992
- OPL, by P. Van Hentenryck, USA, and ILOG, France:
  front-end to both ILOG CP Optimizer and ILOG CPLEX
- Comet, by P. Van Hentenryck and L. Michel, USA

Libraries (the first few ones listed are open-source!):

- Prolog: . . . , ECLiPSe, GNU Prolog, SICStus Prolog, . . .
- C++: Gecode, ILOG CP Optimizer (ex Solver), . . .
- Java: Choco, Gecode/J, Koalog, . . .

The *Association for Computing Machinery* (ACM) identified CT as a strategic direction in computing research.

CT Success Stories I

CT is deployed in many products from industry leaders:

Alcatel·Lucent

Planning of satellite missions

CHRYSLER ENGINEERED BEAUTIFULLY

Vehicle production optimisation

CISCO

Routing

france telecom

Cabling

i2 The Supply Chain Results Company

Supply chain management

JEPPESEN A BOEING COMPANY

Crew rostering

MONSANTO

Production scheduling

CT Success Stories II

ORACLE	Scheduling
SAP	Logistics software
SIEMENS	Control software validation, circuit verification
SNCF	Resource allocation
Whirlpool	Manufacturing
XEROX	Copier component specification

⋮

CT has become the technology of choice in short-term scheduling, timetabling, and rostering.

CT Vision

> *Constraint programming is the ideal paradigm*
> *for encoding correct programs that must run efficiently*
> *on multi-processor hardware.*
> — Mark Wallace

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Outline

Abstract Constraint Modelling

Research Objectives

- Vision: Abstract and **solver-independent** constraint modelling will lead to simpler and leaner languages, to more intuitive and analysable models, as well as to more effective model formulation and maintenance.
- Specific Objectives: Development of intelligent **analysis** and **compilation** tools, so as to (help a modeller to) translate a high-level model into a lower-level program not unlike what an expert would have written.
- Benefit: Empowerment of a wider range of users to unleash the proven benefits of constraint solvers on a broader range of real-life constraint problems.

Outline

Symmetry

Example (Symmetry in the TSC problem)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓				
Falun	✓			✓	✓		
Lund	✓					✓	✓
Mora		✓		✓		✓	
Sigtuna		✓			✓		✓
Uppsala			✓	✓			✓
Ystad			✓		✓	✓	

- The tourist sites (row indices) are indistinguishable.
- ...

Symmetry

Example (Symmetry in the TSC problem)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓				
Falun			✓	✓			✓
Lund	✓					✓	✓
Mora		✓		✓		✓	
Sigtuna		✓			✓		✓
Uppsala	✓			✓	✓		
Ystad			✓		✓	✓	

- The tourist sites (row indices) are indistinguishable.
- . . .

Symmetry

Example (Symmetry in the TSC problem)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓	✓	✓				
Falun			✓	✓			✓
Lund	✓					✓	✓
Mora		✓		✓		✓	
Sigtuna		✓			✓		✓
Uppsala	✓			✓	✓		
Ystad			✓		✓	✓	

- …
- The judges (column indices) are indistinguishable.

Symmetry

Example (Symmetry in the TSC problem)

	Ali	Dan	Eva	Jim	Leo	Mia	Ulla
Birka	✓		✓			✓	
Falun			✓	✓			✓
Lund	✓	✓					✓
Mora		✓		✓		✓	
Sigtuna					✓	✓	✓
Uppsala	✓			✓	✓		
Ystad		✓	✓		✓		

- . . .
- The judges (column indices) are indistinguishable.

Symmetry

Problems with Unhandled Symmetry

- For every solution, there are at most as many symmetric solutions as symmetries.
- Worse: For every *non*-solution, there are at most as many symmetric *non*-solutions as symmetries.
- Similarly for *partial* solutions and *partial* non-solutions.

Hence: A problem solver may waste a lot of time exploring symmetric parts of the search space, when looking for a solution, or all solutions, or optimal solutions.

Symmetry

Symmetry Handling

We have shown that the $|Sites|! \cdot |Judges|!$ combinations of row and column symmetries (that is 25,401,600 symmetries for the Sweden instance) of the TSC problem can be:

- Detected automatically, in polynomial time, from an abstract model (in our ESRA language, say).
- Broken automatically and partially, in polynomial time and space, by requiring the rows and columns to be lexicographically ordered (another global constraint).

This sufficiently reduces the search space to solve problem instances that are one order of magnitude larger.

Symmetry

Research Objectives

- Vision: Real-life problem instances of unconsidered hardness can be solved via better symmetry handling.
- Specific Objectives: Automatable, effective, efficient techniques for symmetry **detection** and **breaking**.
- Benefit: Burden of symmetry handling shifted from the problem modeller to the developer of problem solvers.

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Constraint-Based Local Search

Research Objectives

- Vision: The design and maintenance of efficient local search algorithms can be simplified even more, with the user having to resort to even less low-level handcoding.
- Specific Objectives: Improve the modelling **versatility**, language **extensibility**, and solving **efficiency** of constraint-based local search.
- Benefit: Increased likelihood of experiments with local search. Hence: Increased chances of environmental, time, financial, or material savings.

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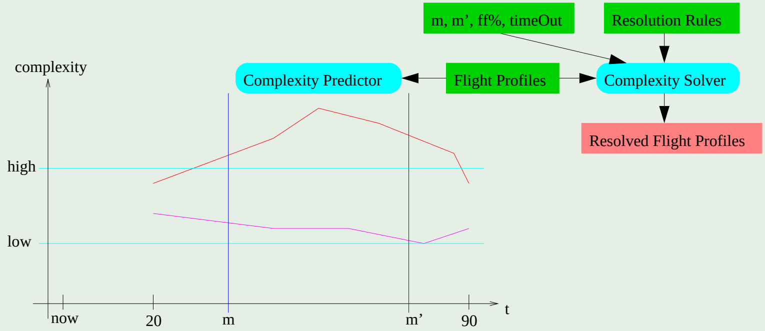
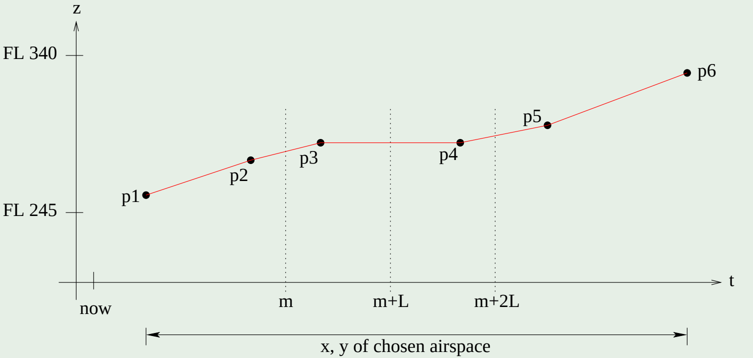
Air-Traffic Management

Example (Demonstration project to *EuroControl*, the *European Organisation for the Safety of Air Navigation*)



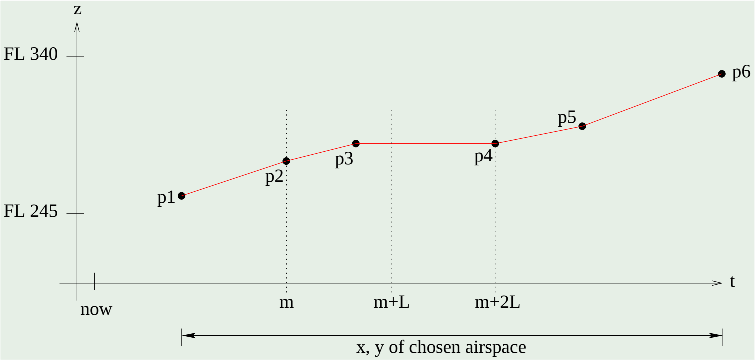
Air-Traffic Management

Example (Planned temporal profile)



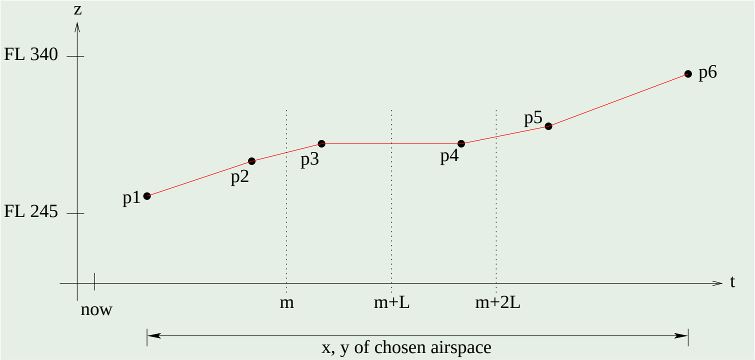
Air-Traffic Management

Example (Resolved temporal profile)



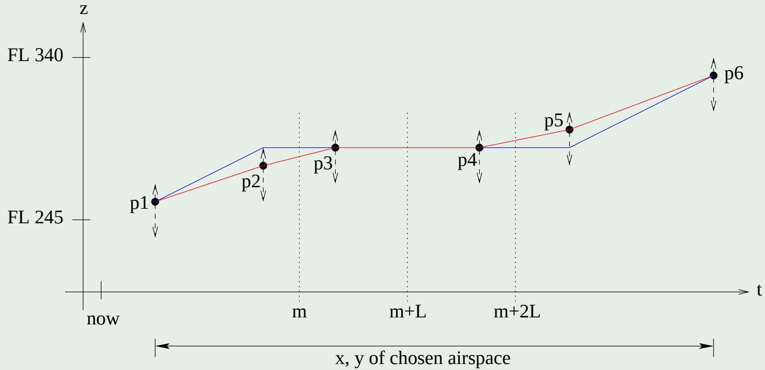
Air-Traffic Management

Example (Planned vertical profile)



Air-Traffic Management

Example (Planned and resolved vertical profiles)



Outline

Computational Biology

Cross-Fertilisation

Biology can benefit from constraint technology.
Constraint technology can benefit from biology:
Indeed, the current CT research on

- Symmetry handling
- Graph decision variables
- Our *tree* partitioning constraint (see below)
- …

was actually motivated by biological problems,
but has also found applications in routing, linguistics, etc.

Computational Biology

Existing Collaboration

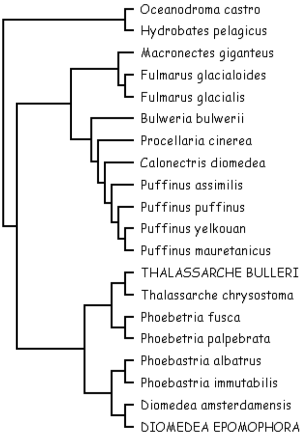
With:

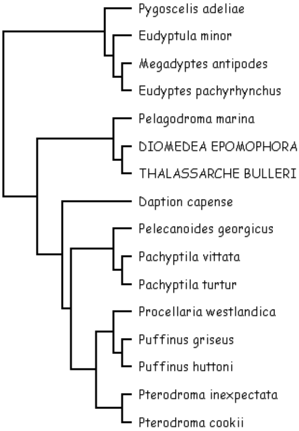
- École des Mines de Nantes, France
- University of East Anglia, Norwich, UK
- Sabancı University, İstanbul, Turkey

On:

- Phylogenetic supertree construction
- RNA secondary structure prediction
- Haplotype inference
- Motif discovery in promoter sequences of DNA

Computational Biology: Phylogenetic Trees





Objective: Construct a supertree that is (maximally) consistent with several given species trees.

Outline

1. **What is Constraint Technology?**
   - Constraint Problems
   - Constraint Technology
   - Constraint Modelling
   - Constraint Solving by Global Search
   - Constraint Solving by Local Search
   - History & Success Stories
2. **Enabling Basic Research and Tools**
   - Abstract Constraint Modelling
   - Symmetry
   - Constraint-Based Local Search
3. **Technology Transfer**
   - Air-Traffic Management
   - Computational Biology
   - Computational Finance

Computational Finance

Example (Financial investment instrument design with Merrill Lynch, New York, USA)

	Acer	Apple	Dell	HP	IBM	Siemens	Sony
B 1	✓	✓	✓				
B 2	✓			✓	✓		
B 3	✓					✓	✓
B 4		✓		✓		✓	
B 5		✓			✓		✓
B 6			✓	✓			✓
B 7			✓		✓	✓	

1. Every basket contains $r = 3$ credits.
2. (Every credit is in any amount of baskets.)
3. Minimise maximum overlap λ between any two baskets.