Abduction

No, we’re not kidnappping anyone....
Abduction

- Deduction - finding the effect, given the cause and the rule
- Induction - finding the rule, given the cause and the effect
- Abduction - finding the cause, given the rule and the effect
Abduction

- Deduction: “All men are mortal. Socrates is a man. Therefore, Socrates is mortal”
- Induction: “All swans we see are white. Therefore all swans are white.”
- Abduction: “Drunk people do not walk straight. Jack is not walking straight. Therefore, Jack is drunk”
Abduction

- If a then b
- b
  ---------------
- a
Abduction

- You have a fever, a sore throat and a headache. It is the middle of December. You go to the doctor. After examining you, the doctor says “You have the flu”.

- It appears that the doctor is reasoning as follows: Symptoms --> Disease
Abduction

• Let $D$= disease $S$=symptom

• $P(\ S \mid \ D \ ) = \frac{P(\ S \cap \ D)}{P(\ D)}$

• $P(S \cap D) = P(D \cap S) = P(S|D)P(D)$

• $P(D|S) = \frac{P(D \cap S)}{P(S)} = \frac{P(S|D)P(D)}{P(S)}$

• We can express the probability of a disease given a symptom in terms we already have!
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Diseases
- $P(\text{pickled liver}) = 2^{-17}$
- $P(\text{iron-poor blood}) = 2^{-13}$

Symptoms
- $P(\text{yellow skin}) = 2^{-10}$
- $P(\text{bloodshot eyes}) = 2^{-6}$

Conditionals
- $P(\text{yellow skin} \mid \text{iron-poor blood})=2^{-1}$
- $P(\text{yellow skin} \mid \text{pickled liver})=2^{-3}$
- $P(\text{bloodshot eyes} \mid \text{iron-poor blood})=2^{-6}$
- $P(\text{bloodshot eyes} \mid \text{pickled liver})=2^{-1}$
A patient has yellow skin....

\[
P(\text{pickled liver} \mid \text{yellow skin}) = \frac{P(\text{yellow skin} \mid \text{pickled liver}) \cdot P(\text{pickled liver})}{P(\text{yellow skin})} = 2^{-3} \cdot 2^{-17} / 2^{-10} = 2^{-10}
\]

\[
P(\text{iron-poor blood} \mid \text{yellow skin}) = \frac{P(\text{yellow skin} \mid \text{iron-poor blood}) \cdot P(\text{iron-poor blood})}{P(\text{yellow skin})} = 2^{-1} \cdot 2^{-13} / 2^{-10} = 2^{-4}
\]

Conclusion: patient has iron-poor blood
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We select the disease that maximizes the following

\[ D_{\text{MAP}} = \arg\max_{D_i} P(D_i|S) = \arg\max_{D_i} P(S|D_i)P(D_i)/P(S) \]

Note 1: This is sometimes called the MAP (Maximum a Posteriori) estimate. We’ll discuss further later......

Note 2: one can disregard \( P(S) \) in the denominator, as it plays no role in selecting the estimator
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The plot thickens.......the patient has TWO symptoms ($S_1$ and $S_2$)....yellow skin AND blood-shot eyes.

$$P(D|S_1 \text{ and } S_2) = \frac{P(D)P(S_1 \text{ and } S_2|D)}{P(S_1 \text{ and } S_2)}$$
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For single disease with single symptoms:
Assume m diseases and n symptoms, then
\[ P(D|S) = \frac{P(S|D)P(D)}{P(S)} \]

- \( P(S|D) \) requires \( mn \) numbers
- \( P(D) \) requires \( m \) numbers
- \( P(S) \) requires \( n \) numbers

Total storage is \( mn + m + n \sim mn \)
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\[ P(D|S_1 \text{ and } S_2) = \frac{P(D)P(S_1 \text{ and } S_2|D)}{P(S_1 \text{ and } S_2)} \]

- \( P(S_1 \text{ and } S_2|D) \) requires \( mn^2 \)
- \( P(S_1 \text{ and } S_2) \) requires \( n^2 \)

Total storage is \( mn^2 + m + n^2 \sim mn^2 \)
Suppose there are more symptoms??
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We make the Naive assumption:

a) the symptoms are independent among people at large

b) the symptoms are independent within the subset of people with disease D
Abduction

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a) the symptoms are independent among people at large

\[ P(S_i \text{ and } S_j) = P(S_i)P(S_j) \]

b) the symptoms are independent within the subset of people with disease D

\[ P(S_i|S_j \text{ and } D) = P(S_i|D) \]
Abduction

We make the *Naive* assumption:

a) the symptoms are independent among people at large

\[ P(S_i \text{ and } S_j) = P(S_i)P(S_j) \]

b) the symptoms are independent within the subset of people with disease D

\[ P(S_i|S_j \text{ and } D) = P(S_i|D) \]

This permits:

\[ P(S_i \text{ and } S_j|D) = P(S_i|d)P(S_j|D)^* \]

\[ P(D|S_i \text{ and } S_j) = P(D)P(S_i|D)P(S_j|D)/P(S_i)P(S_j) \]

* Problem: prove this follows from b)
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\[ P(D|S_i \text{ and } S_j) = \frac{P(D)P(S_i|D)P(S_j|D)}{P(S_i)P(S_j)} \]

We define \( I(D|S) = \frac{P(S|D)}{P(S)} \)

\[ P(D|S_1 \text{ and } S_2 \text{ and } \ldots S_n) = P(D)I(D|S_1)I(D|S_2)\ldots I(D|S_n) \]