Final Exam in Data Structures

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Location: PB
Time: 9 – 14
No books or calculator allowed

Directions:
1. Answer only one problem on each sheet of paper
2. Do not write on the back of the paper
3. Write your name on each sheet of paper

Good Luck!

Problem 1 (20p)
Consider the recurrence
\[ T(n) = \begin{cases} 
1 & \text{if } n = 1 \\
2T\left(\left\lfloor \frac{n}{2} \right\rfloor \right) + \frac{n^2}{3} & \text{if } n > 1 
\end{cases} \]
Is it the case that \( T(n) = \Omega(n \log n) \)? If your answer is yes, provide the relevant constant and threshold, and then use induction to prove your claim. Otherwise, explain why this is not the case.

Problem 2 (8p)
Show how \textsc{Quicksort} sorts the following string:
\textsc{EasyQuestion}
Assume that letters have the standard alphabetical ordering.

Problem 3 (12p)
What is the maximum number of times during the execution of \textsc{Quicksort} that the largest element can be moved, for an array of size \( n \).
Problem 4 (5p)
Is an array which is sorted in descending order a heap?

Problem 5 (15p)
The largest element in a heap must appear in position 1, and the next largest element in position 2 or position 3. Give the list of positions in a heap of size 15 where the $k$th largest element (i) can appear, and (ii) cannot appear, for $k = 3, 4, 5$. Assume that the values of all elements are distinct.

Problem 6 (10p)
Draw the binary search tree you get by inserting the following sequence into an initially empty tree:

5 1 20 30 15 25 21 8 17 9

Assume that letters have the standard alphabetical ordering.

Problem 7 (10p)
Consider a binary search tree $T$. Suppose that we first insert a new element $z$ into $T$, and then we remove the same element from $T$. In other words, we first perform TREE-INSERT($T, z$) followed by TREE-DELETE($T, z$). Call the resulting tree $T_1$. Are $T_1$ and $T$ identical? Consider two cases:

- When all elements in $T$ have distinct values.
- When two elements in $T$ may have identical values.

Motivate your answers.
Problem 8 (10p)
Consider the following version of INSERTION-SORT.

\begin{verbatim}
INSERTION-SORT(A)
1 for j ← 2 to length[A]
2 do key ← A[j]
3 i ← j − 1
4 while i > 0 and A[i] ≥ key
5 do A[i + 1] ← A[i]
6 i ← i − 1
7 A[i + 1] ← key
\end{verbatim}

Is the algorithm stable? If yes motivate why; otherwise describe how to modify the algorithm in order to obtain a stable version.

Problem 9 (10p)
Consider inserting the following keys into a hash table of length $m = 13$:

152 44 39 22 134 53 144 131 0 135

The auxiliary hash function is given by $(k \mod m)$. Draw the resulting hash table if we use

- linear probing; or
- quadratic probing with $c_1 = 2$ and $c_2 = 3$. 