Data Structures, Tutorial 4

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Graphs - what are they good for?
Provide a good representation of...

- Data structures (liked lists, tables, trees)
- The internet
- DNA
- A road net
- A sewer system
- The circulation of the blood in the body
- ...

Google uses graphs!
And then?
With the representation, we can isolate interesting properties of the graph, and thus discover interesting properties of the underlying system.

**Example:** A road system is abstracted as a directed graph in the natural way. What information would computing the strongly connected components of this graph give us?

- Whether there is a path from every junction to all other junctions.
Fundamentals: DFS and BFS

- We look a vertex at a time, examining it and discovering its neighbours (moving the frontier).
- The difference lies in how when the neighbours are examined!

DFS

- When examining a node, just tag it as discovered and examine all not yet discovered neighbours first.
- So we examine the nodes using a LIFO policy.

BFS

- When examining a node, put all not yet discovered neighbours on hold and let them “wait for their turn”.
- So we examine the nodes using a FIFO policy.
The essence

- DFS uses LIFO, BFS uses FIFO.
- This means they are essentially the same, except that DFS uses a stack and BFS uses a queue.

Note that this glosses over a lot of important points, read the book for more detailed explanation.
Exercise on bipartite graphs: A graph $G = (V, E)$ is bipartite iff its vertices $V$ can be divided into two subsets $A$ and $B$ such that:

- $A \cup B = V$ and $A \cap B = \emptyset$
- For every edge $(v_1, v_2) \in E$, either $v_1 \in A$ and $v_2 \in B$, or $v_2 \in A$ or $v_1 \in B$.

A pair $(A, B)$ of such subsets is called a bipartition.

1. Which of the following graphs are bipartite?
   - $G = (\{1\}, \emptyset)$
   - $G = (\{1\}, \{(1, 1)\})$
   - $G = (\{1, 2, 3\}, \{(1, 2), (2, 3)\})$
   - $G = (\{1, 2, 3\}, \{(1, 2), (2, 3), (3, 1)\})$

2. How we can modify BFS to check whether a graph is bipartite, and if so, find its bipartition?
Implementation of graphs

- Implement some representation of a graph (i.e. adjacency list or incidence matrix)
- Implement a BFS search on your graph.

...and also:
- Implement the possibility of bipartitioning the graph.

Deadline: 15/10