

Algoritmer och datastrukturer DV2,

Exam October 2007

The grades on the exam was distributed as follows

points	Num of students	Grade UU	Grace ETCS	comment
0-7.5	0	U	F	
8-12.5	11	U+	FX	Need oral exam to pass, you will get an email from Arne A
13-14.5	5	3	E	
15.5-17.5	6	3	D	
18-19.5	7	4	C	
20-22.5	4	5	B	
23-27	2	5	A	

Schedules for oral exam, see course home page

1. Correct answer (note that the first function is decreasing, the denominator is $\log n \log n$, not $\log \log n$):

$$\frac{\sqrt{\log n}}{\log n \log n}, \log \log n, \left\{ \log n \right\}, n^{\log n}, 2^{2n}, 5^n$$

2. Always pick the next possible concert with the earliest ending time, not the shortest one! Since the complexity analysis depends on whether the number of halls is regarded as a constant or not, I decided to be a bit generous when judging complexity analysis.

3. (a) is solved with the Master Theorem.

(b) is solved by thinking: the number of recursive levels is $O(\log \log n)$ and each level costs a total of $O(n)$.

7. (a,b)

Let k be the number of coins.

Let $A(i)$ be the optimal number of coins we need to create the sum i , we store $A[1..W]$ in an array.

The proper solution is

For $i = 1$ to W

For each coin value c

If we use c to create the sum i , the cost is $1+A(i-c)$ ($i-c$ must be positive)

Select the smallest of these costs, and store as $A(i)$

When we are done with this, we have the value of $A(W)$.

The loop "For $i = 1$ to W " runs W times. The inner loop "For each coin value c " runs k times.

All in all, the cost is $O(kW)$.

7c) The answer is trivial, if the largest coin value is C_{\max} , the solution is calculated in $O(1)$ time as $\left\lceil \frac{C_{\max}}{W} \right\rceil$.

(One exception, if $W=0$, the answer is 0).

Many of you replied that one should repeat taking the largest coin as long as needed, but that is of course too slow!

8. The solution:

1. Construct a line segment containing the query point p and a point above all points in the polygon.

2. For each segment in the polygon, check if it intersects the constructed line segment.

If we are really into details, we should consider the cases when the constructed line goes through an endpoint (or both endpoints) of any of the polygon's segments. One way handle this is to do as follows: If p lies exactly on the same x -coordinate as one or more endpoints, then pretend as if the endpoint is to the left of p . that is

- if the segment has its right endpoint above p , we say that the constructed line crosses the segment
- if the segment has its left endpoint above p , we say that the constructed line does not cross the segment
- if the segment has both its endpoints above p , the constructed line does not cross the segment.

A common mistake on this question was: instead of just say that we check if the constructed line crosses a segment, we test if both y -coordinates of the segment's endpoints are above p and the x -coordinates are on different sides of p . However, such a test does not work, it is easy to find counterexamples.

1. Arrange the functions below in increasing order according to asymptotic growth.
 5^n , $\frac{\sqrt{\log n}}{\log n \log n}$, $\log \log n$, 2^{2^n} , $3 \log \sqrt{n}$, $n^{\log n}$, $\log n$

2p

2. Alice is attending a music festival. There are several concert halls with concerts given in parallel. The different concerts are of varying length and partially overlapping, like in the example schedule below.

a) Describe a greedy algorithm that allows Alice to attend the largest possible number of concerts, provided that she can not attend a partial concert; each concert has to be visited from start to end. 2p

b) Assuming that the schedule is given as a set of sorted lists, one per hall, what is the complexity of the greedy algorithm? 2p

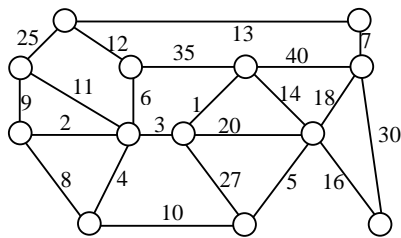
Hall A	Hall B	Hall C	Hall D
Band 1	Band 5	Band 9	Band 12
Band 2	Band 6		
Band 3	Band 7	Band 10	Band 13
		Band 11	
Band 4	Band 8		Band 14

3. a) Solve $T(n) = 4T(n/5) + 1$, $T(1) = 1$. Explain the solution! 2p

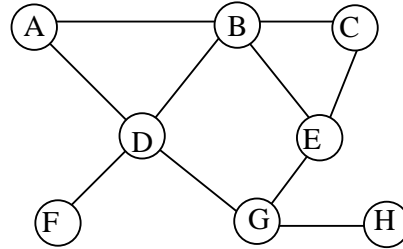
b) Solve $T(n) = \sqrt{n} \cdot T(\sqrt{n}) + n$, $T(2) = 1$. Explain the solution! 2p

4. Explain how modular arithmetic is used in the Rabin-Karp algorithm for finding a pattern in a string. 3p

5. a) Mark the minimum spanning tree in the graph below. 2p
 b) Assuming that the MST is calculated with Kruskal's algorithm, which is the last edge to be included? 1p



6. a) Show how the undirected graph below is represented as an adjacency matrix. 1p
 b) Show how the undirected graph below is represented as adjacency lists. 1p
 c) Give the nodes in the order they will be visited in a breadth-first traversal starting from D. 1p



7. Consider the following problem: Given a set of n coin values $v_1, v_2, v_3, \dots, v_n$, and a sum W , find the smallest number of coins that adds to exactly W . We assume that one of the values is 1.

Example: We have 4 values: 1, 2, 20, 50, and $W = 65$, the solution is 6 coins: $3 \times 20 + 2 \times 2 + 1$

- a) Describe an algorithm that solves the problem by dynamic programming. 2p
- b) Analyse the complexity of your algorithm above in terms of W . 2p
- c) If we instead of finding the smallest number summing to *exactly* W wish to find the smallest number of coins summing to *at least* W we get a trivial solution, how? 1p

8. Given a point and a simple polygon, we wish to find out if the point lies inside the polygon or not. One way to do this is to draw a straight line starting at the point and count how many times the line crosses the polygon; if the number of crossings is odd the point is inside, otherwise it is outside. See the example below, the line crosses the polygon 4 times, and since 4 is an even number the point is outside the polygon.

Describe an algorithm that solves this problem in $O(n)$ time, n being the number of segments in the polygon. Input: the query point and the points in the polygon in clockwise order, starting from an arbitrary point. (Hint, the solution is very simple, just think!)

