Uppsala University
Department of Information Technology
Division of Scientific Computing

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Emil Kieri

Analysis of numerical methods
Assignment 1

1. Discretise the advection equation with periodic boundary conditions,

$$\begin{aligned} u_t + u_x &= 0, && 0 < x < 2\pi, t > 0, \\ u(0,t) &= u(2\pi,t), && t \geq 0, \\ u(x,0) &= f(x), && 0 \leq x \leq 2\pi, \end{aligned}$$

in space using fourth order central finite differences. Do not discretise in time.

   (a) Derive and plot the wave speed and the relative error in wave speed as a function of ωh.

   (b) Derive the leading term of the maximum point-wise error at time t given a single-frequency initial condition, i.e. $f = e^{\mathrm{i}\omega x}$.

2. Consider the system of partial differential equations,

$$\begin{aligned} \mathbf{u}_t &= A\mathbf{u}_x + B\mathbf{u}_{xx}, && 0 < x < 2\pi, t > 0, \\ \mathbf{u}(0,t) &= \mathbf{u}(2\pi,t), && t \geq 0, \\ \mathbf{u}(x,0) &= \mathbf{f}(x), && 0 \leq x \leq 2\pi, \end{aligned}$$

where

$$\mathbf{u} = \begin{pmatrix} u \\ v \end{pmatrix}, \; A = \frac{1}{2}\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}.$$

Discretise it in time and space with the Crank-Nicolson method and derive a stability condition for the time step k.