

Algoritmer och datastrukturer DV3,

Exam June 2007

Rules:

1. You are allowed to bring the textbook: *Cormen T. H., Leiserson C. E., Rivest R. L., Stein C. Introduction to Algorithms, MIT Press*.
2. Write clear answers.
3. Answer at most one question per sheet.
4. Only write on one page per sheet.
5. Think!
6. Good luck!

Name:

Personnr:

Mark which questions you have answered:

Question	Answered?	Grade
1		
2		
3		
4		
5		
6		
7		
8		
TOTAL		

1.

Assuming that you have a subroutine that finds the median of n keys in $O(n)$ time, describe an algorithm that finds the k th key for any k , in $O(n)$ time.

2.

Describe an algorithm that takes as input two sorted arrays, where each array contains n keys in sorted order, and finds the median of the $2n$ keys in $O(\log n)$ time.

Example:

First array: 1 5 16 18 27 45 77 86 99

Second array: 2 3 11 12 13 15 20 23 46

The median in this case is 17 (the average of 16 and 18)

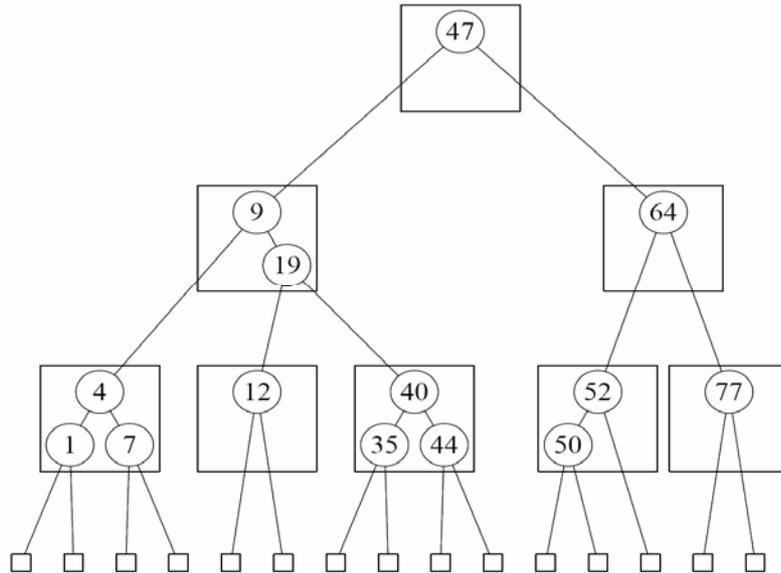
3.

Draw an interval tree representing the following intervals.

(1, 14) (3, 11) (8, 20) (5, 22) (7, 9) (2, 6) (10, 15)

4.

a) Below is a red-black tree (or symmetric binary B-tree). Draw the tree after deletion of element 12.



5.

a)

Describe the following problem as an *integer program* (IP):

We have 4 items (c_1, c_2, c_3, c_4) for sale, and we want to maximize our income, given the following bids:

Bidder A: 100 Euro for c_1 , 100 Euro for c_2 , 100 Euro for c_3 , 100 Euro for c_4

Bidder B: 202 Euro for the combination (c_1, c_2)

Bidder C: 311 Euro for the combination (c_1, c_3, c_4).

Bidder D: 201 Euro for the combination (c_2, c_4).

Bidder E: 120 Euro for c_3 .

We also have the following constraint:

Bidder A can win at most 2 items

b)

Show how to modify the IP if we add the following constraint:

Only one of A and E can win anything.

6

A spanning tree of a graph is an acyclic subset of the edges that connects all vertices. Consider the following problem:

Degree Bounded Spanning Tree Problem: Given a graph $G = (V, E)$ and a positive integer $K \leq |V|$, is there a spanning tree for G in which no vertex has degree larger than K ?

Prove that the Degree Bounded Spanning Tree Problem is NP-complete.

Hint: You may use the fact that the Hamiltonian Path Problem is NP-complete.

Hamiltonian Path Problem: Given a graph $G = (V, E)$, is there a path that visits each vertex exactly once?

7.

Given a computer with word size $w = \theta(\log^2 n)$, we wish to store n keys in a data structure so that we can find a key or its nearest neighbor. Describe briefly which data structure to use, and give the search cost.

8.

a) In the book, there is a description on how to maintain a dynamic table at $O(1)$ amortized cost per insert/delete in such a way that the load factor is bounded below by a constant. Now, assume that we wish to have the load factor close to 1, say that the load factor should be at least $1 - \epsilon$ for some arbitrary small constant ϵ . Describe how this can be done, still at $O(1)$ amortized cost per insert/delete.

b) As the value of ϵ gets smaller and smaller, the constant factor within the $O(1)$ amortized cost per insert/delete gets larger. Describe how. That is, give the amortized cost per insert/delete as a function of ϵ (in big-O notation).

c) We wish to let the load factor depend on n , and grow toward 1 as n grows, by setting it to at least $1 - 1/\log \log n$. What is the amortized cost per insert/delete?