Advanced Algorithmics,

Exam June 2011

<u>Rules</u>:
1. You are allowed to bring the textbook: ***Cormen T. H., Leiserson C. E., Rivest R. L., Stein C. Introduction to Algorithms, MIT Press***. 2. Write clear answers. 3. Answer at most one question per sheet. 4. Only write on one page per sheet. 5. Think! 6. Good luck!

NOTE: There will also be an oral exam. Make sure to book it.

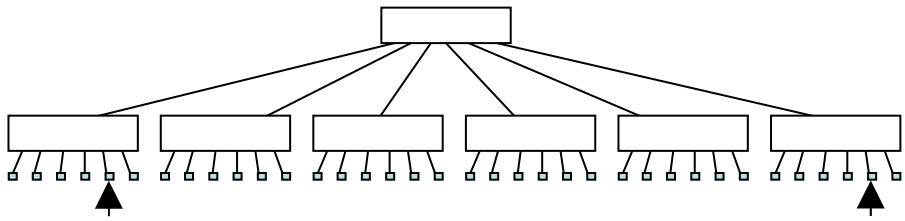
Name: Personnr:

Mark which questions you have answered:

Question	Answered?	Grade
1		
2		
3		
4		
5		
6		
7		
8		
TOTAL		

1.
Describe an algorithm that sorts n integer in O(n) time on a machine with wordlength W = 8 log n.

2.
Below you see a B-tree which is a 3-6 tree. The actual elements are shown as small bullets at the bottom, the other nodes are used for searching. Draw the tree as it will look after deleting the element at the left arrow, and inserting a new element at the right arrow.
One drawing with the final result is enough.



3.
a) Specify how to randomly select a hash function h(x) such that the following is true:
Given two different keys x_1 and x_2, if the range of h(x) is 0..m-1, the probability of x_1 and x_2 colliding is at most 1/m.

b) Describe a data structure that has the following properties:
- The expected cost for Insert and Delete is O(1)
- The worst-case cost for Find is O(1),
- The worst-case space required for storing n elements is O(n).

4.
Given

- a set S of small keys that are packed together in one machine word, the keys in S are sorted
- a key x that is stored in multiple copies in another machine word.

Describe how the position of x among the keys in S can be computed in O(1) time.

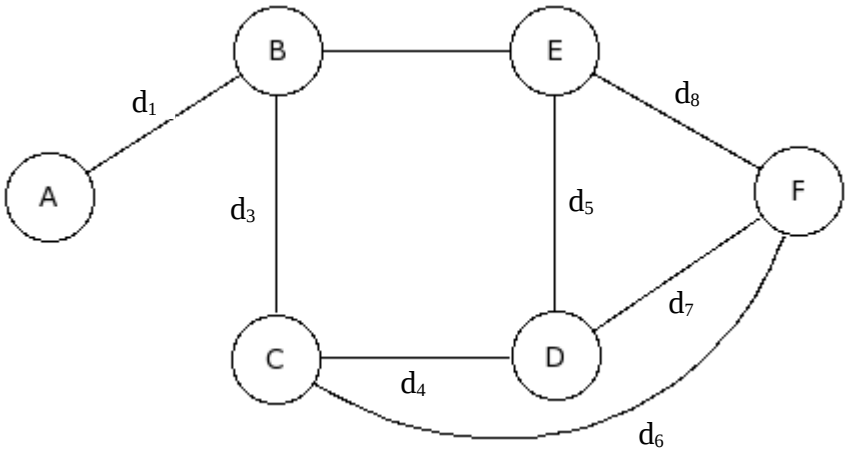
5.
We would like to solve the following problem with linear/integer programming.

On next page is a simple (transportation) network. We want to transport goods from A to F. We have the following facts.

- On each edge, there is a distance d_1, ..., d_8, as marked in the picture below.
- Each vertex in the network shall be visited exactly once.

We wish to minimize the total distance between A and F, fulfilling the constraints above.

a) Describe the problem as an objective function with linear constraints
b) Assume that, for some reason, the distance between B and E has to be at least 20. Modify the equations such that that is modeled properly. All distances are the same.



6.
Describe a decision version of Integer Programming, and prove that this version of integer Programming is NP-complete.

7.
In Assignment 2, you wrote executive summaries of three papers (Simulated Annealing, Tabu search, Genetic Algorithms). Pick one of these and write a short summary.

8.
Describe a data structure that performs the following operations on a set S of numbers:

- **Insert (x, S):** insert x into S
- **DeleteLargestHalf (S):** Delete the largest half of the elements in S.

The data structure should perform in O(1) amortized cost per operation. Prove the amortized complexity.