What does an ideal lowpass filter (ILPF) look like in spatial domain?

In frequency domain, cut-off radius = $D_0$

$$D(u,v) = \text{distance from origin}$$

ILPF $H(u,v) =
\begin{cases} 
1 & \text{if } D(u,v) \leq D_0 \\
0 & \text{if } D(u,v) > D_0
\end{cases}
$$

Find $h(x,y)$
1. $H(u,v)^*(-1)^{u+v}$ (centering)
2. inverse FT
3. multiply real part by $(-1)^{u+v}$

$h(x,y)$ has
1. a central dominant circular component (providing the blurring)
2. Concentric circular components (rings) giving the ringing effects.

Example of $h(x)$
The inverse FT of a disc (ILPF) with radius 5.

Reduce “ringing” with non-ideal filters

Butterworth Lowpass Filter (BLPF)

$$H(u,v) = \frac{1}{1+(D(u,v)/D_0)^{2n}}$$

$n$ is the order of the filter
- A high $n$ will cause “ringing” (approaching ILPF)
- No sharp discontinuity

Gaussian Lowpass filter (GLPF)

$$H(u,v) = e^{-\frac{D^2(u,v)}{2\sigma^2}}$$

$\sigma$ is the $\sigma$, or “spread” of the Gaussian
- The inverse FT of a Gaussian is also a Gaussian, meaning Gaussian smoothing in spatial domain.
- Guarantees no ringing
Same effects for highpass filters

Ideal HPF \( H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases} \)

Butterworth HPF \( H(u,v) = 1/(1+(D_0/D(u,v))^{2n}) \)

Gaussian HPF \( H(u,v) = e^{-D^2(u,v)/2D_0^2} \)

Periodicity and padding

MxN image size
- Periodicity: \( F(u,v) = F(u+M,v) = F(u,v+N) = F(u+M,v+N) \)
- Symmetry: \( |F(u,v)| = |F(-u,-v)| \)

Have to introduce identical periods at convolution to avoid “wrap-around error”: padding

The same is true in FD: convolution in SD is multiplication in FD
The filter and the image must have the same size at multiplication!

The padding is removed after the filtering.
At padding, the image borders (if not black) become sharp edges, leading to “ringing” at the image borders after filtering using ideal filters (remember computer exercise 1).
There is a strong similarity between convolution and correlation.