Image Restoration

We want to restore an image that has been degraded in some way.
We make a model of the degenerating process and use inverse methods.

In comparison to image processing, which is a subjective way to present the image in a “better” way, image restoration is a more objective method where a priori information of the degradation is used.

Image Restoration

- inverse filtering
- try to model degrading effect
- use Fourier-domain methods and identify which frequencies are related to the degrading effect and which frequencies are related to the original image

Mathematical assumptions (rep)

- Convolution of two continuous functions is defined as
  \[ f(x,y) \ast g(x,y) = \int f(\alpha, \beta)g(x - \alpha, y - \beta)d\alpha d\beta \]
- Convolution by the impulse function is defined as
  \[ f(x) \ast \delta(x - x_0) = f(x_0) \]
  i.e. we get the function value in the point \( x_0 \)

Simplified Model

Assume a model without noise. We can write
\[ g(x,y) = H[f(x,y)] \]
Assume that the operator \( H \) is
- linear:
  \[ H[k_1f(x,y) + k_2f_1(x,y)] = k_1H[f(x,y)] + k_2H[f_1(x,y)] \]
- additive:
  \[ H[f(x,y) + f_1(x,y)] = H[f(x,y)] + H[f_1(x,y)] \]
- homogenous:
  \[ H[kf(x,y)] = kH[f(x,y)] \]
- position invariant:
  \[ H[f(x - a, y - b)] = g(x - a, y - b) \]
then, the following is true
\[ g(x,y) = \int f(\alpha, \beta)H[\delta(x - \alpha, y - \beta)]d\alpha d\beta \]
Impulse Response

The function
\[ h(x, y, \alpha, \beta) = H[\delta(x - \alpha, y - \beta)] \]
is called the impulse response for H. In the imaging system, it is called the point spread function (PSF).

- PSF describes how a point is imaged.

We can write
\[ g(x, y) = \int \int f(\alpha, \beta) h(x, \alpha, y, \beta) d\alpha d\beta \]

This (Fredholm) integral says that if the impulse response is known, the response to any input signal can be calculated.

- If H is position invariant, the integral reduces to
\[ g(x, y) = \int f(\alpha, \beta) h(x, \alpha, y, \beta) d\alpha d\beta \]

In 3D:
\[ g(x, y, z) = \int \int f(\alpha, \beta, \gamma) h(x, \alpha, y, \beta, z, \gamma) d\alpha d\beta d\gamma \]

Inverse filtering

We can thus describe our model as
\[ g(x, y) = f(x, y) \otimes h(x, y) \]
The Fourier transform gives
\[ G(u, v) = F(u, v) \cdot H(u, v) \]
i.e. by modeling the degenerating effect and dividing the FT of the image by the FT of the model, we can get a restored image by taking the inverse transform of the result.

If noise is present, we get
\[ F(u, v) = \frac{G(u, v) - N(u, v)}{H(u, v)} \]

- The inverse transform gives the restored image
- The method is called inverse filtering.

General Fluorescence Microscopy

Example: deconvolution microscopy

A point imaged through a lens system is "smeared" by a PSF

\[ f(x, y) \rightarrow H \rightarrow g(x, y) \]

(profile of g(x, y))

in 3D:

Creating 3D Fluorescence Microscopy Images

By moving the stage of the microscope in z-direction and capturing one image at each step, a stack of images is created. If the steps are small enough, a 3D image can be created from the stack.
Deconvolution Microscopy

\[ \mathbf{b}_j = \mathbf{F} \mathbf{b}_j \]

\( \mathbf{b}_j \) = observed image of plane \( j \)
\( \mathbf{F} \) = true intensity distribution of plane \( j \)
\( \mathbf{b}_j \) = PSF of point spread function of lens system

If the point spread function (PSF) of the lens system is known, the true intensity distribution can be calculated.

Correction for bad focusing

- **Model:**
  \[ G(x,y) = F(x,y) \cdot H(x,y) \]
  \( G(x,y) \) is known (FT of our image)
  \( H(x,y) \) can be found by taking an image of a point (using the same focus setting) and calculating the FT of the image.

- **The restored image is given by**
  \[ f(x,y) = \frac{G(x,y)}{H(x,y)} \]

  \( H \) is the FT of the point spread function (PSF) and we get a sharpening effect. This is called deconvolution (avfältning).

Restoration of image blurred by linear motion (rörelseoskärpa)

Assume that the camera is moving when the image is acquired.

The restored image is given by

\[ f(x,y) = \frac{\frac{F(u,v)}{H(u,v)}}{G(u,v)} \]

where \( H(u,v) \) is the FT of the displacement function.

Problems with inverse filtering

At deconvolution (avfältning), the FT of the image is divided by the FT of the degrading effect.

- **Problem:**
  - Small values of \( H(u,v) \) can cause overflow.
  - If noise is included, it can be dominating.

- **Solutions:**
  - Perform division only in a limited part of the \((u,v)\)-plane.
  - Use weights to limit the effect at division with small numbers.
  - Use Wiener filtering (least mean square filtration).

\[ F(u,v) = H^*(u,v)G(u,v) \]

Comparison of inverse and Wiener filtering at different levels of noise.

Fourier methods

- **Periodic noise in an image (i.e. repeated noise patterns) can cause peaks in the FT of the image.**

- **By setting these peaks to zero and inverse transforming the image, a restored image is achieved.**

(The last problem in computer exercise 1)
The Hotelling transform - principal component analysis (pca)

Example of reduction of periodic noise

(Figure 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). Sinusoidal filters, which suppress noise, 11. (d) Result of filtering. Original image courtesy of Sandia.)

The Hotelling transform - principal component analysis (pca)

(also called the Karhunen-Loeve transform)

Used for:
- multi-spectral images
- to enhance variations and differences
- to maximize the variance
- image compression

The Hotelling transform - principal component analysis (pca)

Example for 2D object:
1. Find the principal directions of variation in the image.
2. Change the coordinate system so that the origin is in the mass center and the axis are in the directions of the eigenvectors.

principal components of a multi-spectral image

principal components

original band 1 band 2 band 3

pc 1 pc 2 pc 3 as RGB image
Geometric correction

Why?
- Correct for distortions due to
  - tilted surface in satellite and aerial images (called rectification).
  - inhomogeneous magnetic field in MR images
  - motion of object between successive images
  - etc

Method:
- original image $f(x,y)$
- transformation $T$
- new image $g(x,y) = Tf(x,y)$

The process can be divided into three steps:
- decide the features to be used for matching (for example known "tiepoints" present in all images).
- calculate the parameters for the transform $T$
- transform the image

Example of method
- Direct method: Mark the corresponding points in the different images, set up an equation system and solve analytically.

\[ \begin{align*}
  x_1 &= c_1 x_1 + c_2 x_2 + c_3 y_1 + c_4 \\
  y_1 &= c_5 x_1 + c_6 x_2 + c_7 y_1 + c_8 \\
\end{align*} \]

8 unknown, 8 equations => the coefficients can be found

Transformation of the image
- original image $f(x,y)$
- transformation $T$
- calculate the new image $g(x,y) = Tf(x,y)$
  This is done by resampling

\[ g(x,y) = f(T(x,y)) \]

Problem: $T(x,y)$ is usually not an integer => interpolation needed

Interpolation
- When images are resampled (geometric transformation, zooming etc.) interpolation is needed to calculate the gray scale values in the new grid.
- Strategies
  - Nearest neighbor interpolation
  - Bi-linear interpolation
  - Cubic spline