Lecture 7
Object representation and description

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GW 11.1-11.4
Suggested problems:
11.19,11.25
Image analysis fundamental steps

- image acquisition
  - preprocessing, enhancement
    - segmentation
      - Representation, description, feature extraction
        - classification, interpretation, recognition
          - result
Farmed vs wild salmon
Distinguishing between salmon and trout
Identifying melanoma
Analysing drug effects on cell cultures
Commonly after segmentation one needs to **represent** objects in order to **describe** them

- **External (boundary):**
  - Representation: Polygon of the boundary
  - Description: The circumference

- **Internal (regional):**
  - Representation: Pixels inside the object
  - Description: The average color
Representations and descriptors

• The Representation of the Object
  • An encoding of the object
  • Truthful but possibly approximate

• A Descriptor of the Object:
  • Only an aspect of the object
  • Suitable for classification
  • Consider invariance to e.g. noise, translation,
Shape Representation

• Sometimes necessary/desirable to represent an object in a less complicated or more intuitive way
• Simple descriptions like enclosing circle, enclosing rectangle, inscribed circle etc.
• The boundary or boundary segments
• Divide an object into regions or parts
• Represent by ”skeleton”
Scale, rotation and translation

- Often we want descriptors that are invariant of scale, rotation and translation:

- However, not always. In Optical Character Recognition (OCR) rotation and scale is important (e.g. ‘P’ and ‘d’)

Centre for Image Analysis
Swedish University of Agricultural Sciences
Uppsala University
Chain code: a contour based shape descriptors

Chain code – describe the sequence of steps generated when walking around the boundary

Chain code can be defined for 4 and 8 neighbours
Chaincode example

4-connected:
0003030303232
1122232110111

8-connected:
0007776542344542212

[Diagram of chaincode examples]
Chain Coding issues/drawbacks

- Code becomes very long and noise sensitive
  - Use larger grid spacing, smooth/edit the code
- Scale dependent
  - Choose appropriate grid spacing
- Start point determines result
  - Treat code as circular (minimum magnitude integer)
    754310  ->  075431
- Depends on rotation
  - Calculate difference code (counterclockwise)
    075431  ->  767767
Example: editing the chain code

replace 0710 with 0000
Polygonal Approximations

- A digital boundary can be approximated (simplified)
- For closed boundaries:
  - Approximation becomes exact when no. of segments of the polygons is equal to the no. of points in the boundary
- Goal is to capture the essence of the object shape
- Approximation can become a time consuming iterative process
Polygonal Approximations

- **Minimum Perimeter Polygons (MPPs)**
  - Cover the boundary with cells of a chosen size and force a rubber band like structure to fit inside the cells
Polygonal Approximations

- **Merging techniques**
  1. Walk around the boundary and fit a least-square-error line to the points until an error threshold is exceeded
  2. Start a new line, go to 1
  3. When the start point is reached the intersections of adjacent lines are the vertices of the polygon
Polygonal Approximations

- **Splitting techniques**
  1. Start with an initial guess
  2. Calculate the orthogonal distance from lines to all points
  3. If maximum distance > threshold, create new vertex there
  4. Repeat until no points exceed criterion

![Initial guess](image1)

- $e_1, e_2, e_4, e_5 < T$
- $e_3 > T$
Boundary representation: signatures
Signatures

• A 1D representation of a boundary
• Could be implemented in different ways
  • Distance from centre point to border as a function of angle
  • Angle between the tangent in each point and a reference line (histogram of this is called slope density function)
• Independent of translation, but not rotation & scaling.
  -> Select unique starting point (e.g. based on major axis)
  -> Normalize amplitude of signature (divide by variance)
Boundary segments

• When a boundary contains major concavities that carry shape information it can be worthwhile to decompose it into segments
• A good way to achieve this is to calculate the convex Hull of the region enclosed by the boundary = minimal enclosing convex region

-> Smooth prior to Convex hull calculation
-> Calculate Convex Hull on polygon approximation
Convex hull, deficiency and concavity tree

Convex Hull = minimal enclosing convex region

Convex region = all points can be connected through a straight line inside the region

Convex deficiency = Convex hull – object

The number and distribution of convex deficiency regions may also be useful

⇒ Concavity tree, generate convex hulls and deficiencies recursively to create a concavity tree

Figure 6.30 Concavity tree construction: (a) Convex hull and concave residual, (b) concavity tree.
Skelettons

“Curve representation” of the object

Should in general be thin, centered, topologically equivalent to original object and reversible

Can be created by thinning = iteratively removing pixels from the border while keeping the overall shape and topology (see book for detailed description)

or by medial axis transform (MAT) = all inscribed circles touching two or more points at the border at the same time

Skelettons are sensitive to small changes in shape

- >smooth first or ”prune” skeleton afterwards
Skeleton from medial axis
Skeleton example

Largest connected component is chosen as object of interest

Skeleton or medial axis representation used for length measurements
Skeleton example: Neurite outgrowth analysis
After representation, the next step is to **describe** our boundaries and regions so that we later can **classify** them (next lecture).

A description is an aspect of the representation.

What descriptor is useful for classification of:
- adults / children
- pears / bananas / tomatoes
Simple boundary (segment) descriptors

- Length (perimeter)
- Diameter = $\max_{i,j} [D(p_i, p_j)] = \text{major axis}$
- Minor axis (perpendicular to major axis)
- Basic rectangle = major $\times$ minor
- Eccentricity = major / minor
- Curvature = rate of change of slope
Fourier descriptors

- Represent the boundary as a sequence of coordinates
- Treat each coordinate pair as a complex number

\[ s(k) = [x(k), y(k)], k = 0, 1, 2, \ldots, K - 1 \]

\[ s(k) = x(k) + iy(k) \]
Fourier descriptors

• From the DFT of the complex number we get the Fourier descriptors (the complex coefficients, $a(u)$)

$$a(u) = \sum_{k=0}^{K-1} s(k) e^{-j2\pi uk/K}, u = 0, 1, 2, \ldots, K - 1$$

• The IDFT from these coefficients restores $s(k)$

$$s(k) = \frac{1}{K} \sum_{u=0}^{K-1} a(u) e^{j2\pi uk/K}, k = 0, 1, 2, \ldots, K - 1$$

• We can create an approximate reconstruction of $s(k)$ if we use only the first $P$ Fourier coefficients

$$\hat{s}(k) = \frac{1}{P} \sum_{u=0}^{P-1} a(u) e^{j2\pi uk/K}, k = 0, 1, 2, \ldots, K - 1$$
Fourier descriptors

- Boundary reconstruction using 546, 110, 56, 28, 14 and 8 Fourier descriptors out of a possible 1090.
Fourier descriptors

- This boundary consists of 64 point, $P$ is the number of descriptors used in the reconstruction.
Statistical moments

- Useful for describing the shape of boundary segments and signatures
- 2\textsuperscript{nd} moment gives spread around mean (variance)
- 3\textsuperscript{rd} moment gives symmetry around mean (skewness)
Statistical moments

• Can also be applied to amplitude histograms
• Let the amplitude of g be a discrete variable v and create an amplitude histogram where A is the number of amplitude increments
• The \( n^{th} \) statistical moment of \( v \) (about its mean) is calculated as:

\[
\mu_n(v) = \sum_{i=0}^{A-1} (v_i - m)^n p(v_i), \quad m = \sum_{i=0}^{A-1} v_i p(v_i)
\]
Simple Regional Descriptors

- Area = number of pixels in a region
- Compactness (P2A) = \( \text{perimeter}^2 / \text{area} \)
- Circularity ratio = \( 4 \times \pi \times \text{area} / \text{perimeter}^2 \)
- Graylevel measures
  - Mean
  - Median
  - Max
  - Etc.
Examples of P2A vs area
Topological descriptors

- **Topology** = The study of the properties of a figure that are unaffected by any deformation

- **Topological descriptors**
  - Number of holes in a region, $H$
  - Number of connected components, $C$
  - Euler number, $E = C - H$
Texture

• Textures can be very valuable when describing objects
• Example below: Smooth, coarse and regular textures
Texture

• Statistical texture descriptors:
  • Histogram based
  • Co-occurrence based
    (Statistical moments, Uniformity, entropy, ... )

• Spectral texture descriptor
  • Use fourier transform
Histogram based descriptors

- Properties of the graylevel histogram, of an image or region, used when calculating statistical moments
  - $z$: discrete random variable representing discrete graylevels in the range $[0, L-1]$
  - $P(z_i)$: normalized histogram component, i.e. the probability of finding a gray value corresponding to the $i$:th gray level $z_i$.

$$
\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i), \quad m = \sum_{i=0}^{L-1} z_i p(z_i)
$$

- $2^{nd}$ moment: Variance of $z$ (contrast measure)
- $3^{rd}$ moment: Skewness
- $4^{th}$ moment: Relative flatness
Histogram based descriptors

Two other common histogram based texture measures:

• Uniformity (maximum for image with just one grayvalue):
  \[ U = \sum_{i=0}^{L-1} p^2(z_i) \]

• Average entropy (measure of variability, 0 for constant images)
  \[ e = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i) \]
Intensity histogram says nothing about the spatial distribution of the pixel intensities

Original image 122x122 pixels

Histogram
- Fitted normal distribution
  - mean = 127
  - std = 40
  - skewness = 0.0093
  - kurtosis = 2.5

Arranged as a painting by René Magritte
Co-occurrence matrix

• For an image with N graylevels, and P, a positional operator, generate $A$, a $N \times N$ matrix, where $a_{i,j}$ is the number of times a pixel with graylevel value $z_i$ is in relative position P to graylevel value $z_j$

• Divide all elements in $A$ with the sum of all elements in $A$. This gives a new matrix $C$ where $c_{i,j}$ is the probability that a pair of pixels fulfilling P has graylevel values $z_i$ and $z_j$ which is called the co-occurrence matrix
Building the matrix $A$

P=one pixel to the right

What will the matrix look like for the striped image if P= one pixel down?

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<th>2</th>
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<td>2</td>
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<td>33</td>
</tr>
</tbody>
</table>
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Co-occurrence matrix Descriptors

• Maximum probability (strongest response to $P$)

$$\max_{i,j}(c_{ij})$$

• Uniformity

$$\sum_i \sum_j c_{ij}^2$$

• Entropy (randomness)

$$-\sum_i \sum_j c_{ij} \log_2 c_{ij}$$

How can rotation robust measures be achieved?
Co-occurrence matrix

• Match image with a co-occurrence matrix!

max prob: 0.00006 0.01500 0.0680
Uniformity: 0.00002 0.01230 0.00480
Entropy: 15.75 6.43 13.58
Spectral Analysis

• Peaks in the Fourier spectrum give information about direction and spatial period patterns
• The spectrum can be described using polar coordinates $S(r, \theta)$
• For each angle $\theta$, $S(r, \theta)$ is a 1D function $S_\theta(r)$
• Similarly, for each frequency $r$, $S_r(\theta)$ is a 1D function
• A global description can be obtained by summing $S_\theta(r)$ and $S_r(\theta)$

$$
S'(r) = \sum_{\theta=0}^{\pi} S_\theta(r), \quad S'(\theta) = \sum_{r=1}^{R} S_r(\theta)
$$
Spectral Analysis

\[ S(r) \]

\[ S(\theta) \]
Central Moments

• For a 2D continuous function $f(x,y)$, the moment of order $(p + q)$ is defined as

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) \, dx \, dy$$

for $p, q = 0, 1, 2, \ldots$

• The central moments are defined as

$$\mu_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \bar{x})^p (y - \bar{y})^q f(x, y) \, dx \, dy$$

where

$$\bar{x} = \frac{m_{10}}{m_{00}} \quad \text{and} \quad \bar{y} = \frac{m_{01}}{m_{00}}$$
Central Moments

- If $f(x,y)$ is a digital image, the central moments become

$$
\mu_{pq} = \sum_{x} \sum_{y} (x - \bar{x})^p (y - \bar{y})^q f(x, y)
$$

- The normalized central moments, denoted $\eta_{pq}$, are defined as

$$
\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}}
$$

where $\gamma = \frac{p+q}{2} + 1$ for $p+q = 2, 3, \ldots$
• A set of seven invariant moments can be derived from the 2\textsuperscript{nd} and 3\textsuperscript{rd} moments
• These moments are invariant to changes in translation, rotation and scale

\[
\phi_1 = \eta_{20} + \eta_{02} \\
\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \\
\phi_3 = (\eta_{30} - \eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\
\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\
\ldots \text{(see textbook)}
\]
<table>
<thead>
<tr>
<th>Invariant (Log)</th>
<th>Original</th>
<th>Half Size</th>
<th>Mirrored</th>
<th>Rotated 2°</th>
<th>Rotated 45°</th>
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</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>6.249</td>
<td>6.226</td>
<td>6.919</td>
<td>6.253</td>
<td>6.318</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>17.180</td>
<td>16.954</td>
<td>19.955</td>
<td>17.270</td>
<td>16.803</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>22.655</td>
<td>23.531</td>
<td>26.689</td>
<td>22.836</td>
<td>19.724</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>22.919</td>
<td>24.236</td>
<td>26.901</td>
<td>23.130</td>
<td>20.437</td>
</tr>
<tr>
<td>$\phi_5$</td>
<td>45.749</td>
<td>48.349</td>
<td>53.724</td>
<td>46.136</td>
<td>40.525</td>
</tr>
<tr>
<td>$\phi_6$</td>
<td>31.830</td>
<td>32.916</td>
<td>37.134</td>
<td>32.068</td>
<td>29.315</td>
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<td>$\phi_7$</td>
<td>45.589</td>
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</tbody>
</table>
Principal component analysis (PCA)

- Calculate $C_x$, covariance matrix of data $X$
- Find eigenvectors and corresponding eigenvalues of covariance matrix ($C_x e_i = \lambda_i e_i$)
- Find $A$ which is a matrix with the eigenvectors as rows, ordered corresponding to decreasing eigenvalue
- Use $A$ to transform $x$ to $y$: $y = A(x - mx)$.
- Any vector $x$ can be recovered from $y$ by: $x = ATy + mx$ and approximated by only using some (say $k$) of the eigenvalues and an $Ak$ matrix constructed from the $k$ eigenvectors.
Principal component analysis (PCA)

• The first principal component has the largest possible variance (that is, accounts for as much of the variability in the data as possible), and each succeeding component in turn has the highest variance possible under the constraint that it is orthogonal to the preceding components.
Principal components of a multispectral image
Face recognition using PCA

We need a training data set: bunch of sample images of people we want to recognize.

Using PCA analysis we find eigenfaces (eigenvectors).

Every new image that we have, we project the image on eigenvectors and based on the weights we obtained we can classify it.

\[ \text{Image} = w_1 \text{Face}_1 + w_2 \text{Face}_2 + w_3 \text{Face}_3 \]
How to choose or design representations and descriptors:

- Find/create representations/descriptors that are invariant to things that are unimportant for your task:
  - I.e. Noise, scale, blur, …
- Find/create representations and descriptors that are relevant for your question
  - height, to classify adults / children
  - color and shape to separate bananas, pears and tomatoes
- Be creative
- Stay as simple as possible