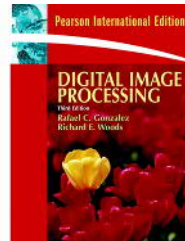
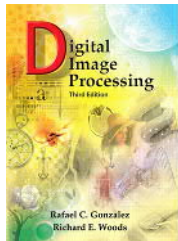


Lecture 2 – Point processes

Wednesday, November 1

Ch. 2.6-2.6.4
3.1-3.3 in
Gonzales & Woods



Damian Matuszewski
damian.matuszewski@it.uu.se
Centre for Image analysis
Uppsala University

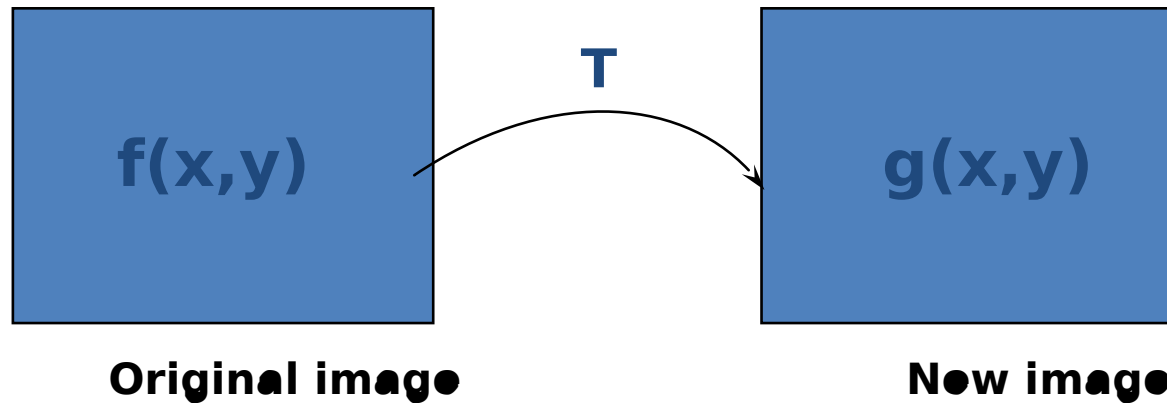
Image enhancement



https://youtu.be/LhF_56SxrGk

- an image processing technique to enhance certain features of the image

Image processing



- We want to create an image which is "better" in some sense.
 - For example
 - Image restoration (reduce noise)
 - Image enhancement (enhance edges, lines etc.)
 - Make the image more suitable for visual interpretation
 - **Image enhancement does NOT increase image information**

Image processing

- can be performed in the:

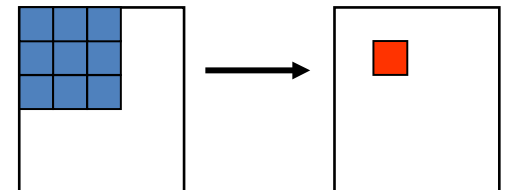
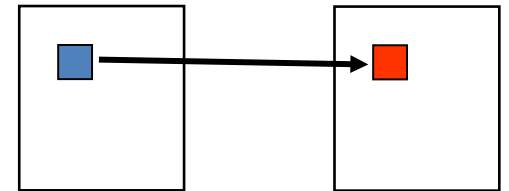
- **Spatial domain**

- Point processes → **Lecture 2**

- Works per pixel

- Spatial filtering → **Lecture 3** (*Filip*)

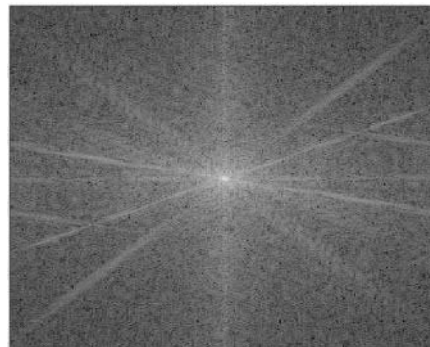
- Works on small neighborhood



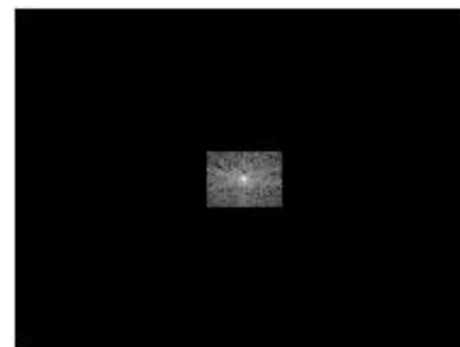
- **Frequency domain** → **Lecture 4** (*Filip*)



Original image
in spatial domain



Original image in
frequency domain



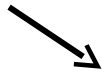
Processed image in
frequency domain



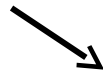
Processed image
in spatial domain

Problem solving using image analysis: fundamental steps

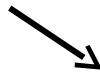
image acquisition



preprocessing,
enhancement



segmentation



feature extraction,
description



classification,
interpretation,
recognition



result



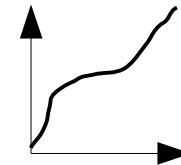
Overview

i. repetition

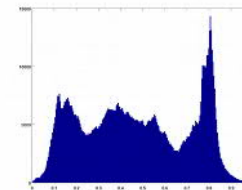
ii. image arithmetics

'+', '-', '*'

iii. intensity transfer functions



iv. histograms and histogram equalization



Last lecture

- Digitization
 - Sampling in space (x,y)
 - Sampling in amplitude (intensity)
- Pixel/Voxel
- How often should you sample in space to see details of a certain size?

M



N

Bit depth

*2 gray levels,
1bit/pixel*



- Number of bits that are used to store the intensity information
- Images are typically of 8- or 16-bit
 - 1bit = $2^1 \rightarrow 2$ steps (0,1)
 - 2 bit = $2^2 \rightarrow 4$ steps
 - 8 bit = $2^8 \rightarrow 256$ steps
 - 16 bit = $2^{16} \rightarrow 65\,536$ steps
- Not continuous!

*64 gray levels,
6bit/pixel*

AUT01



AUT01

*256 gray levels,
8bit/pixel*



AUT01

I. Image arithmetics in the spatial domain

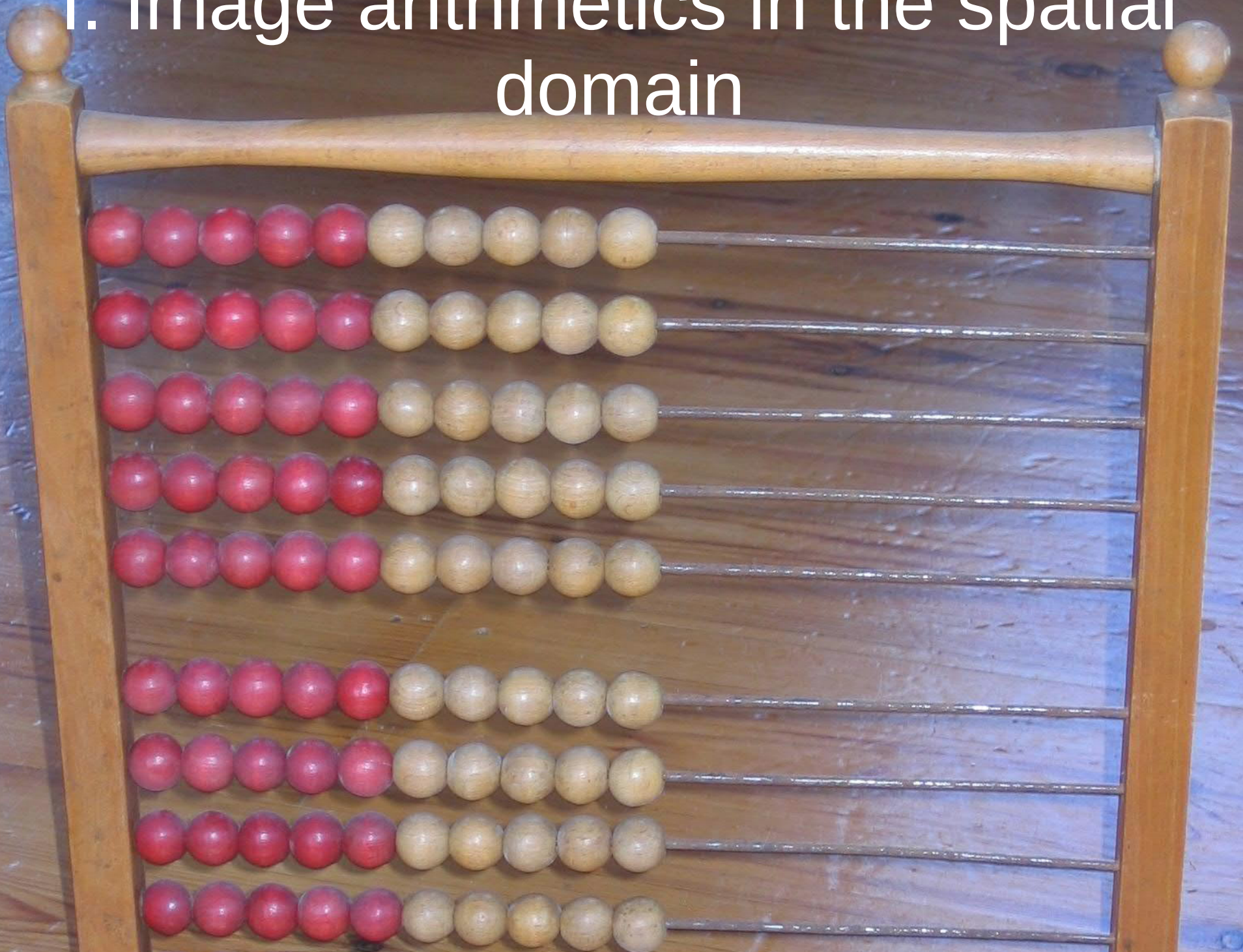


Image arithmetics

- **$A(x,y) = B(x,y) \circ C(x,y)$** for all x,y .
B, C \rightarrow images with the same (spatial) dimensions
 \rightarrow images + constant value
 - can be
 - Standard arithmetic operation: **+**, **-**, *****.
 - Logical operator (binary images): **AND**, **OR**, **XOR**,...
- Any pitfalls? \rightarrow bit depth, negative values and pixel saturation

Arithmetics with binary images

■ min value
□ max value

image1

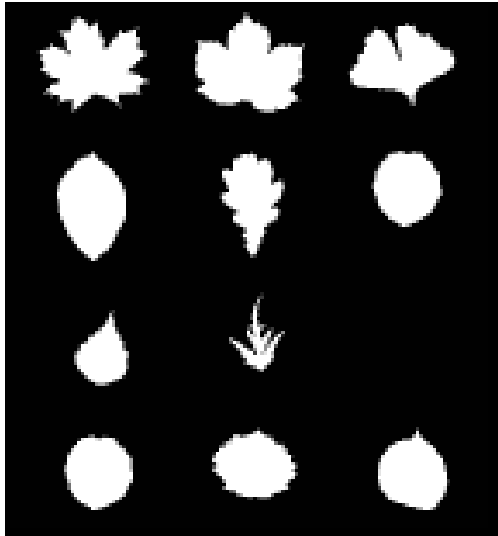


image2

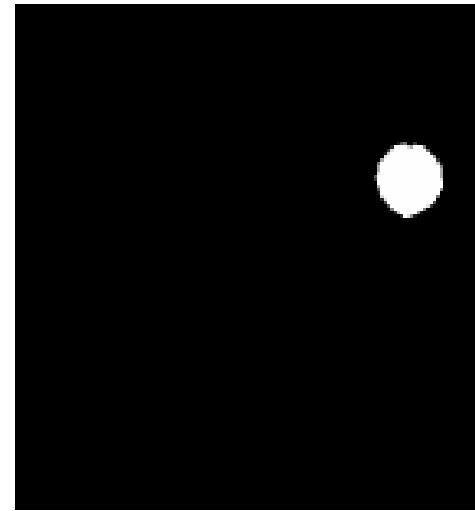
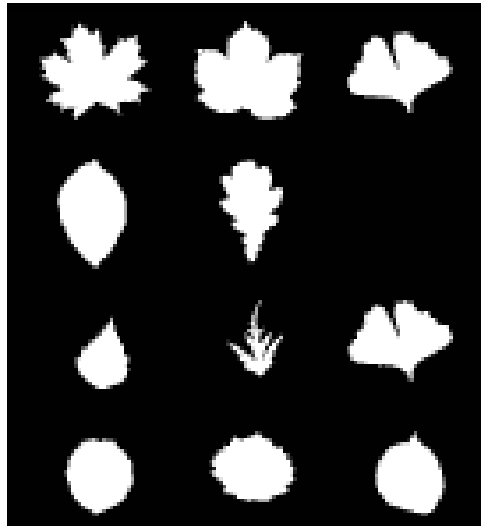


image1-image2

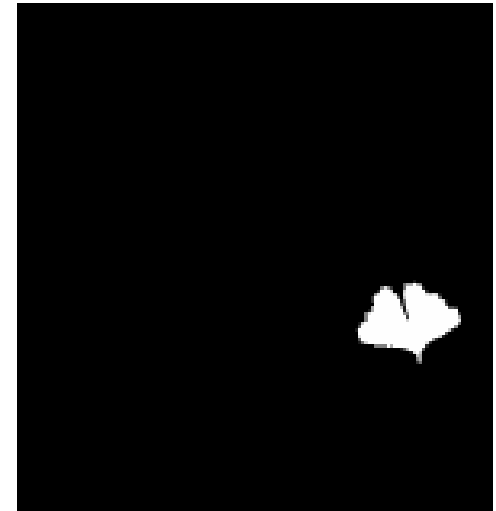
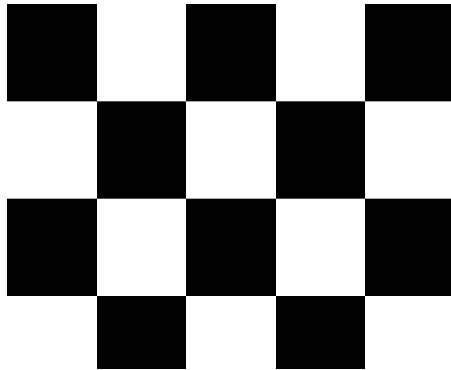


image2-image1

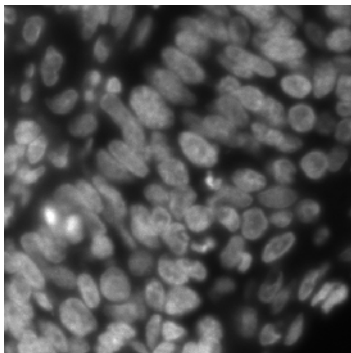
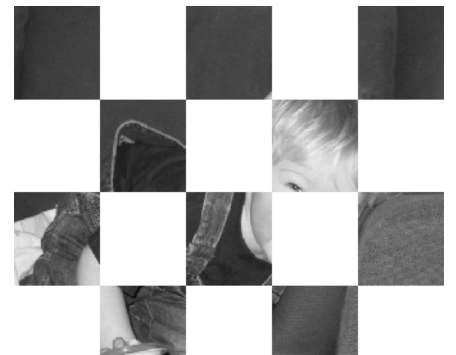
Arithmetics with greyscale images



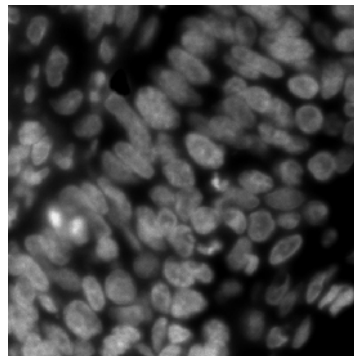
+



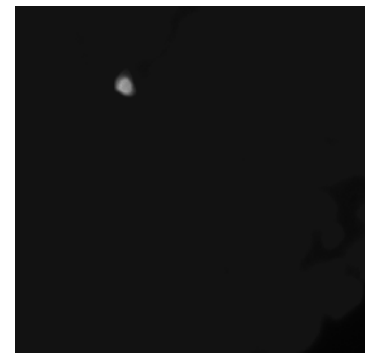
=



-



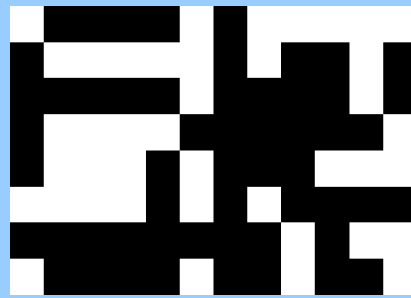
=



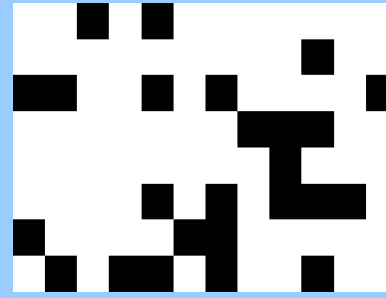
Logical operations on binary images



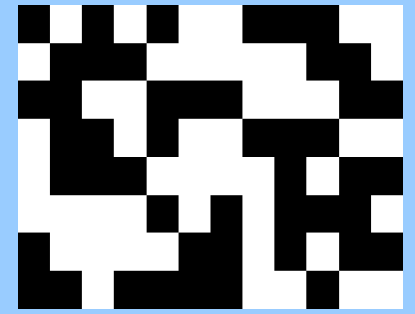
A



B



A OR B



A XOR B

| INPUT | | OUTPUT |
|-------|---|--------|
| A | B | A OR B |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

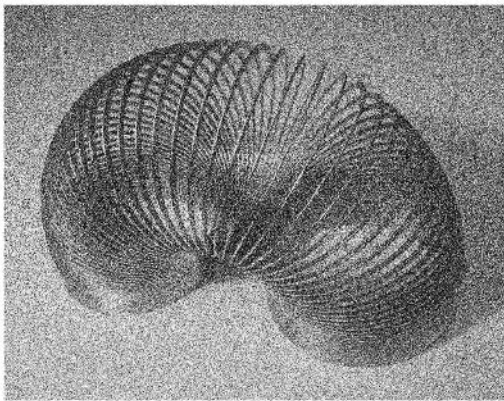
| INPUT | | OUTPUT |
|-------|---|---------|
| A | B | A XOR B |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Applications

- **Noise reduction** using image mean or median

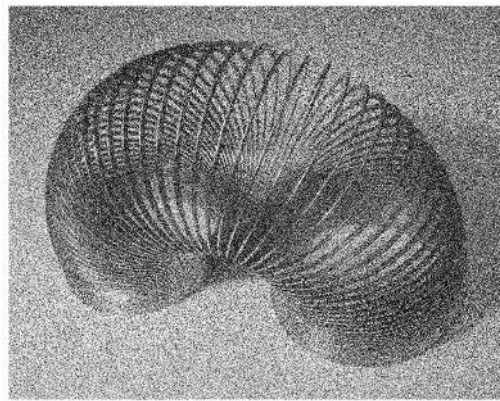
$$I = \frac{1}{n} \sum_{k=1..n} I_k$$

I_1



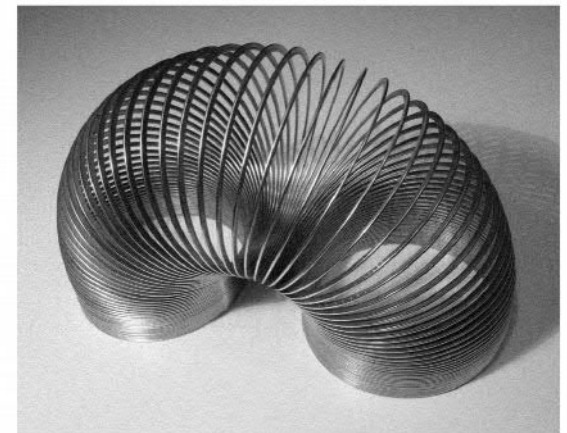
+ ... +

I_n



=

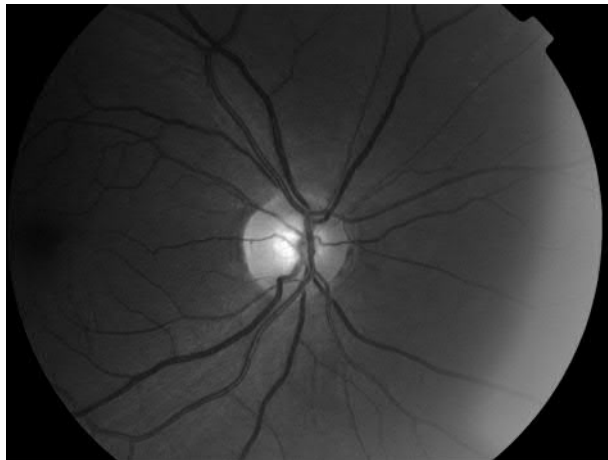
I



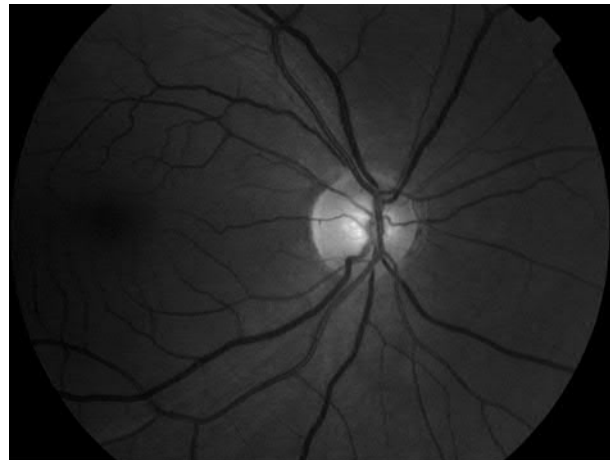
- Useful in astronomy, low light (night) pictures

Applications

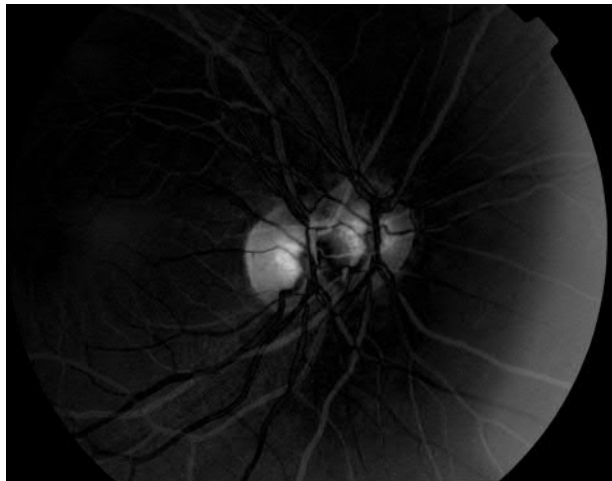
- **Change detection using subtraction**



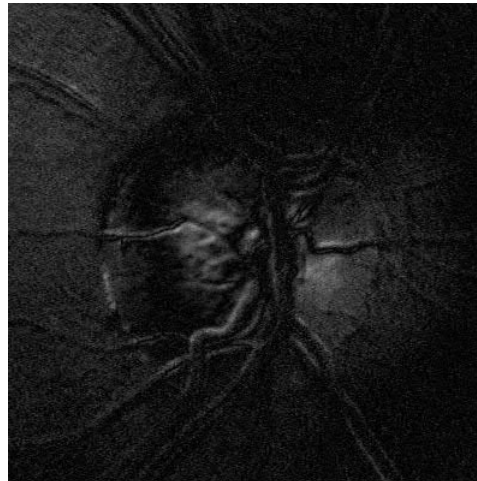
2015



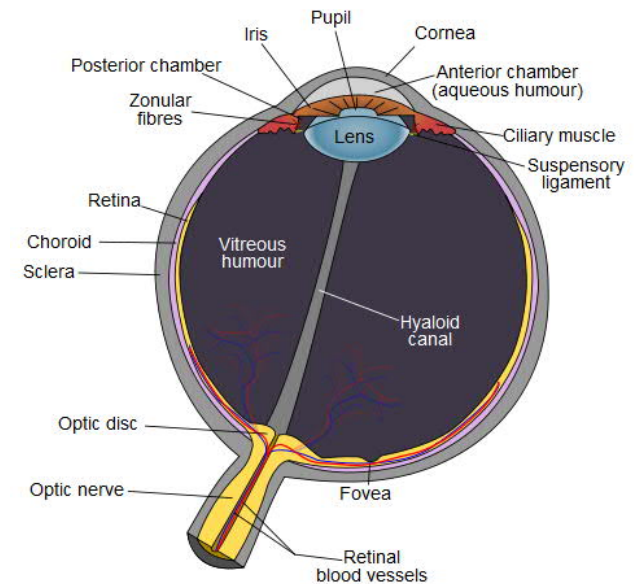
2016



direct difference



difference after registration



Applications

- **Change/motion detection** using subtraction

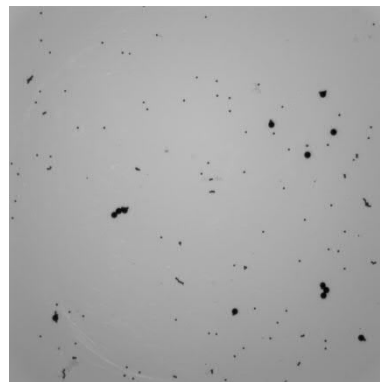
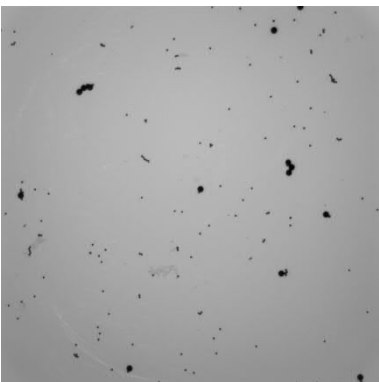
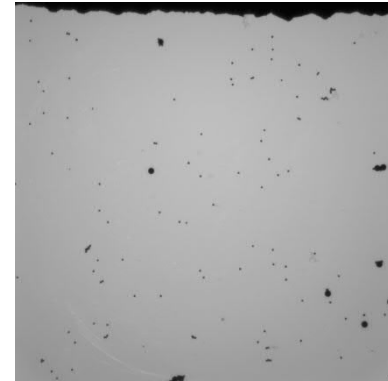
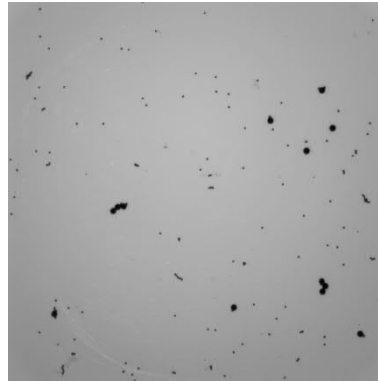
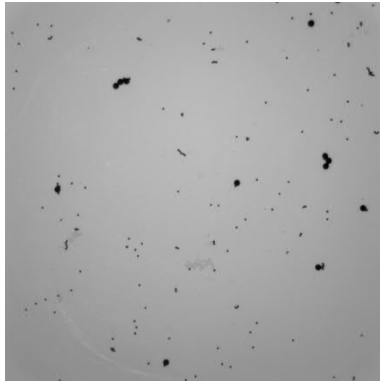
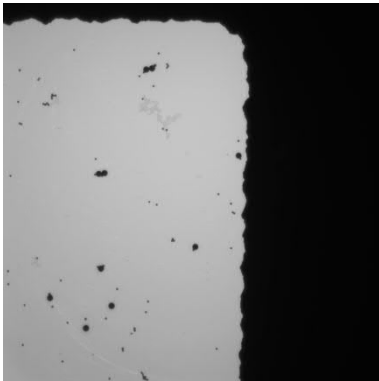


Applications

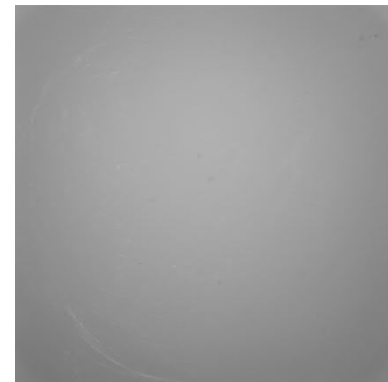
- **Background removal**

image - background image

- Creating a background image

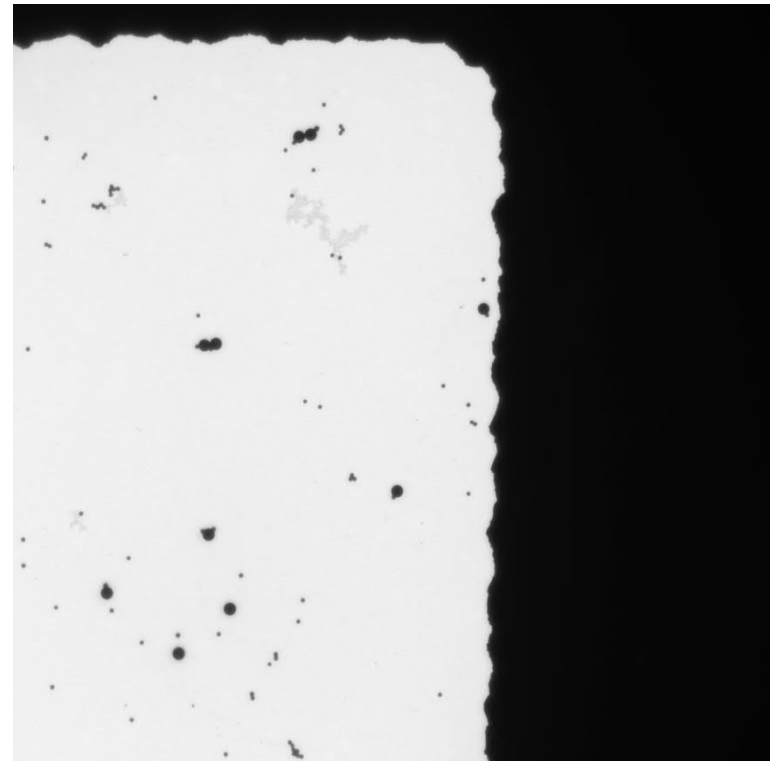
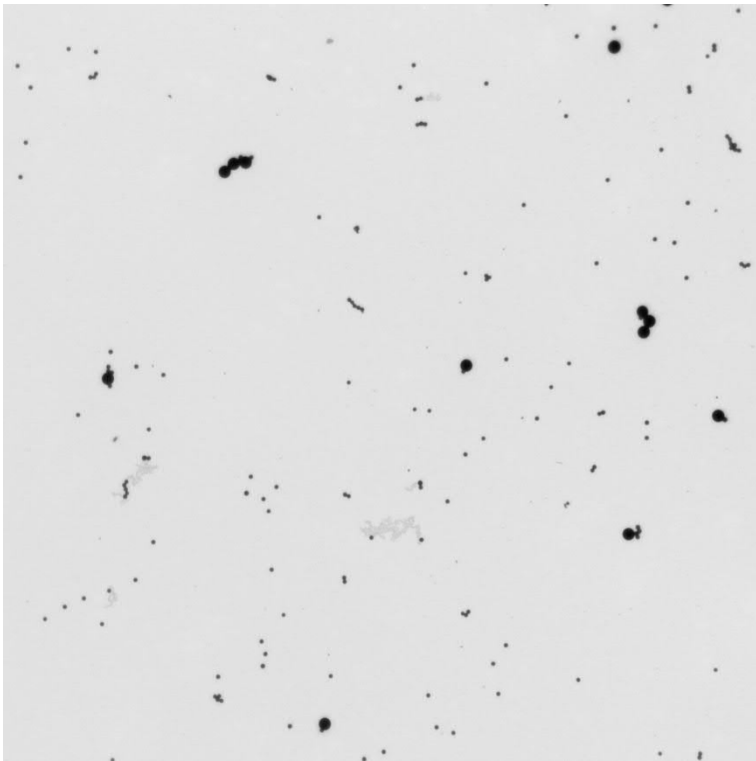


Max or median of
the pixel intensities
at all positions.



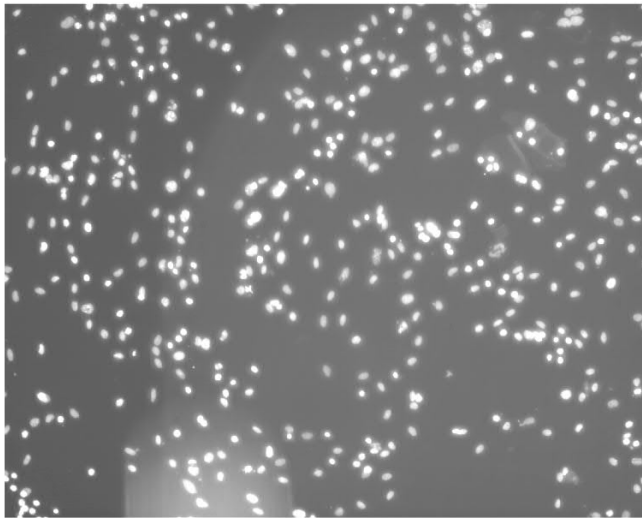
Applications

- **Background removal - result**



Applications

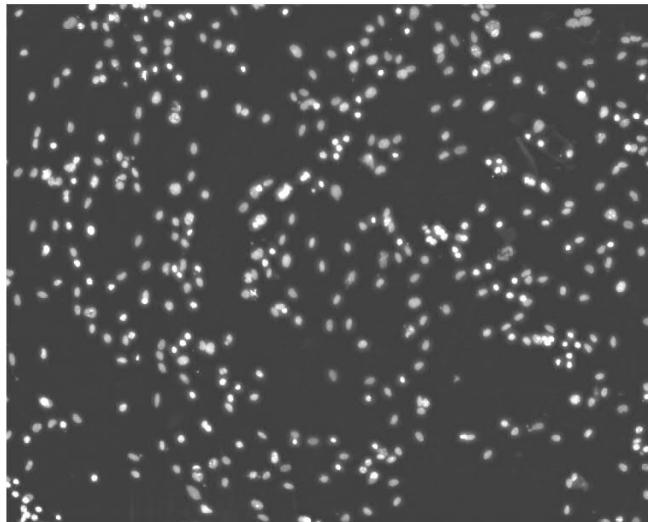
- Subtracting a background image/correcting for uneven illumination



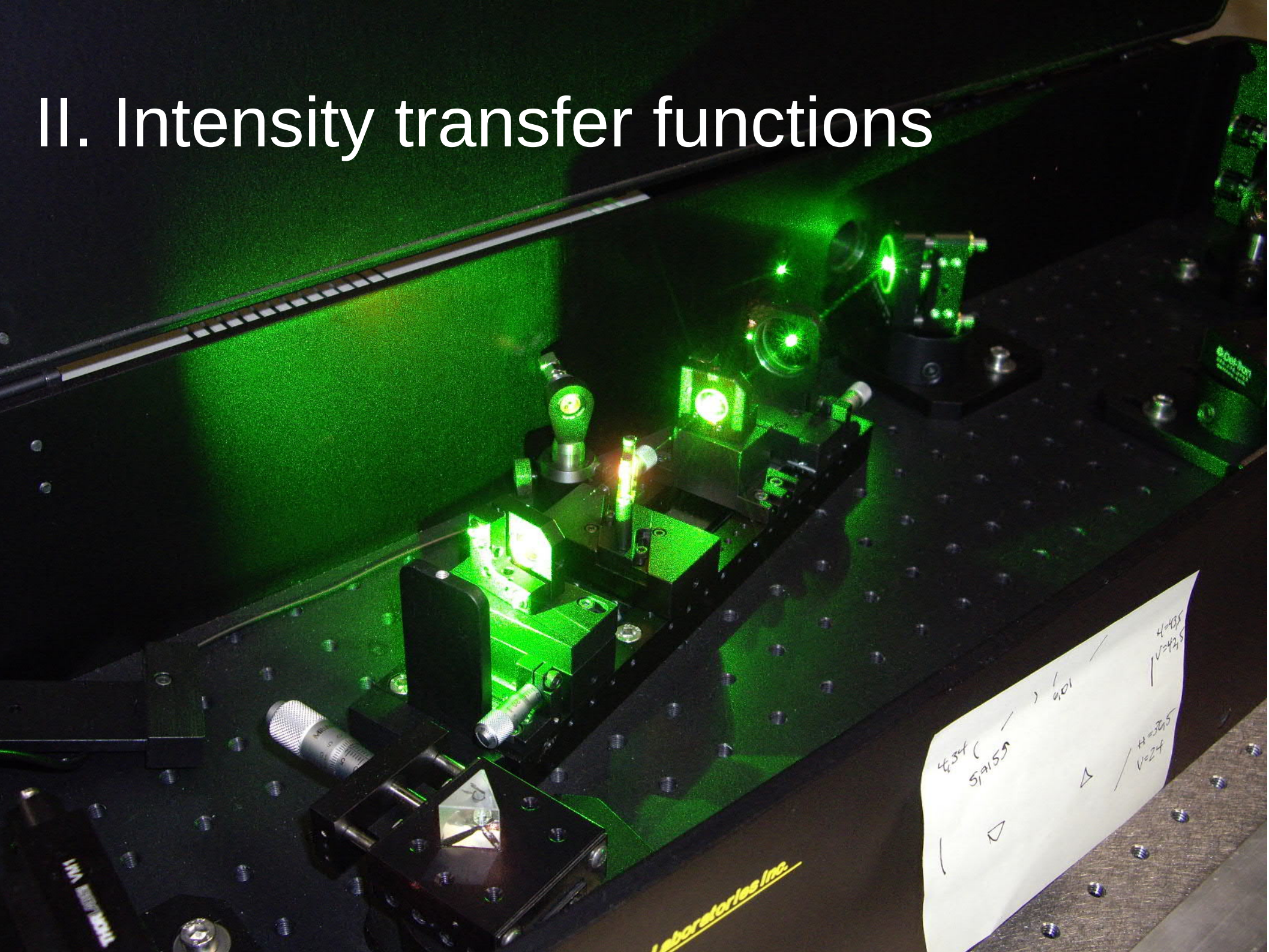
-



=

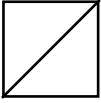


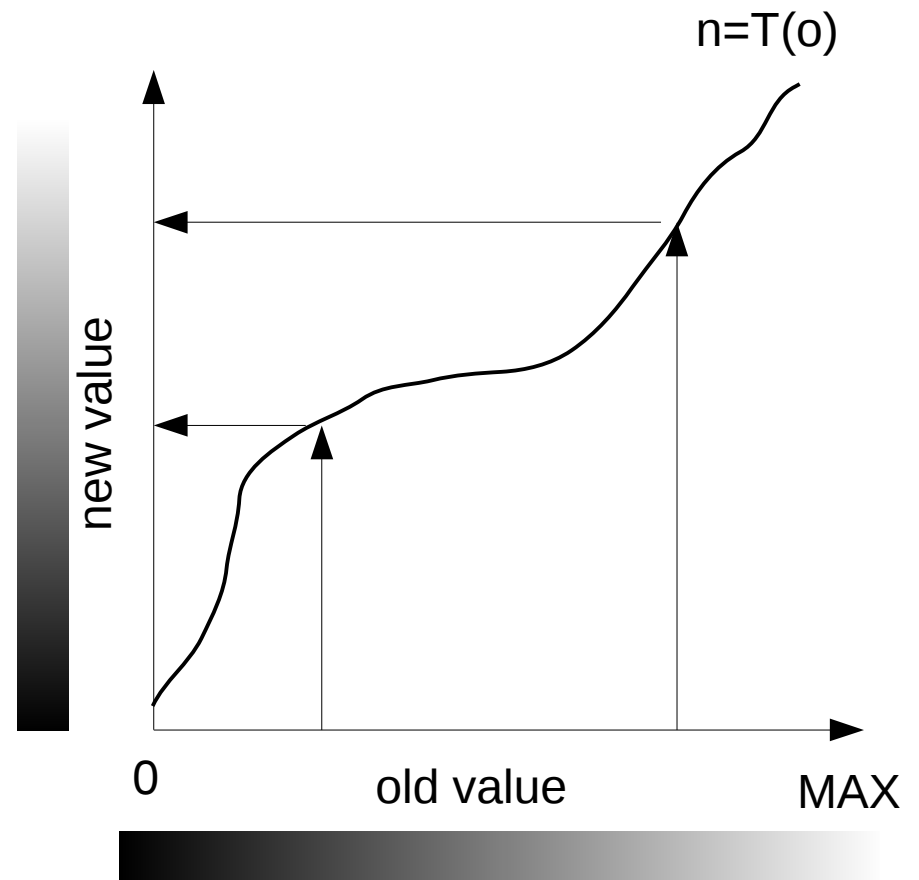
II. Intensity transfer functions



Intensity transfer functions

$$g(x, y) = T f(x, y)$$

- i. **linear** (neutral  ,
negative, contrast,
brightness)
- ii. **smooth** (gamma, log)
- iii. **arbitrarily**

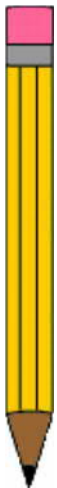


The negative transformation

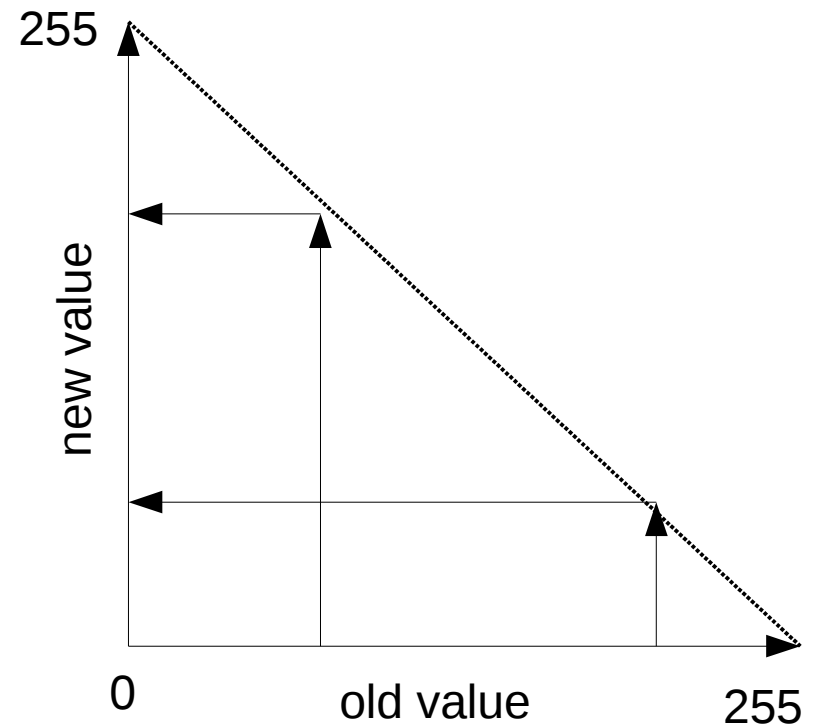
$$g(x, y) = \max - f(x, y)$$

- For eight bit image:

$$g(x, y) = 2^8 - 1 - f(x, y)$$



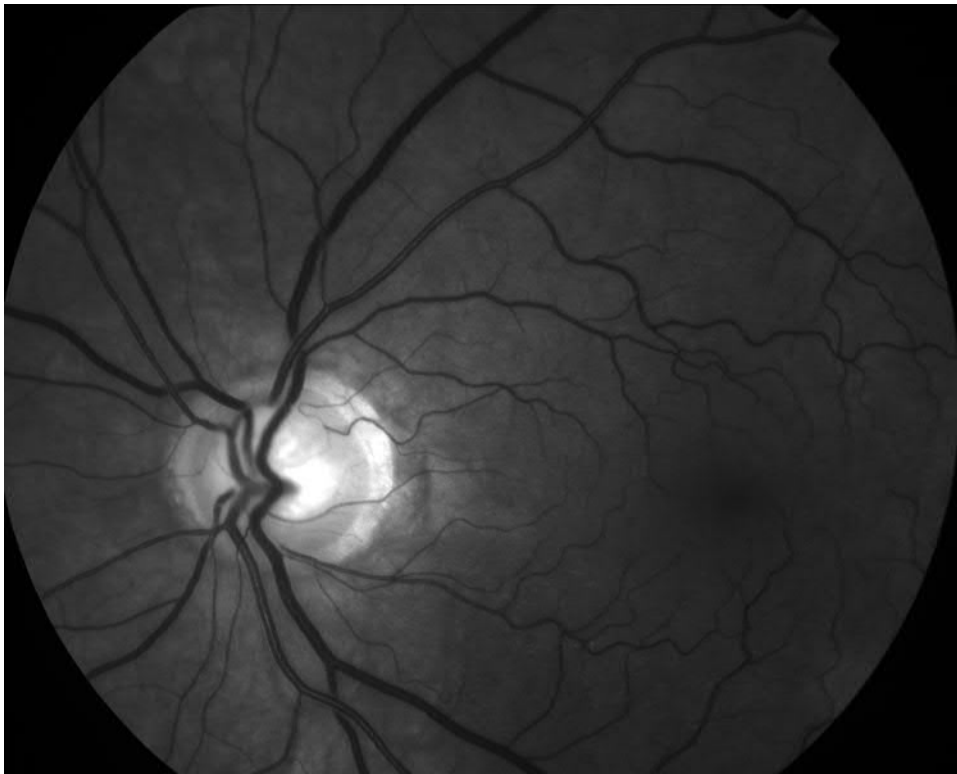
| | | |
|-----|-----|-----|
| 255 | 254 | 253 |
| 125 | 130 | 110 |
| 4 | 3 | 0 |



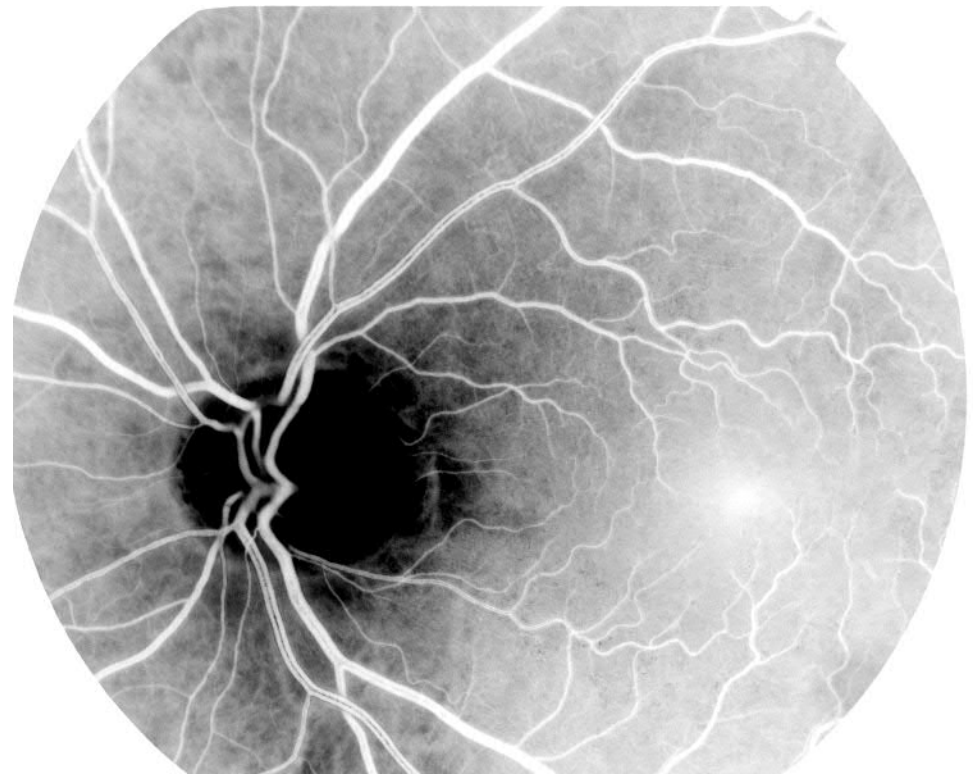
The rules of how to transfer values from the old image to the new one.

The negative transform

- Useful in medical image processing



Original



Negative

The negative transform



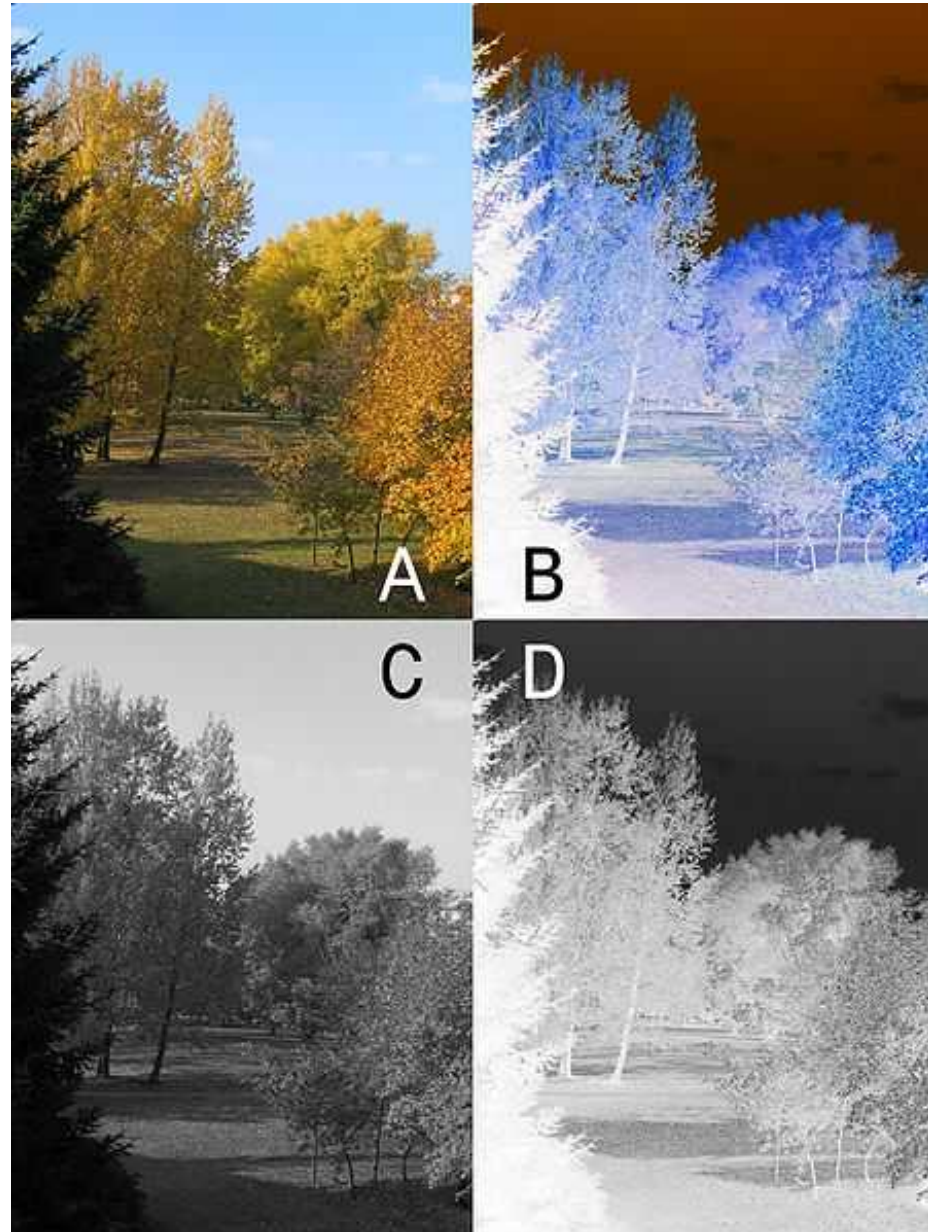
original digital mammogram



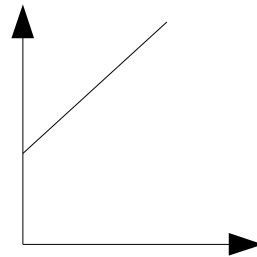
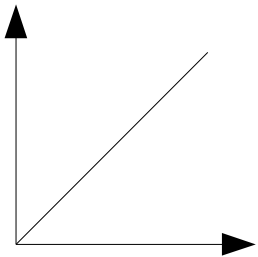
image negative to enhance white or gray details embedded in dark regions

The negative transformation

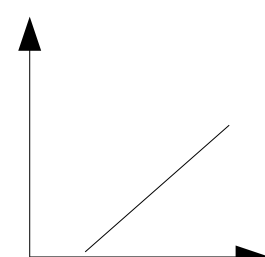
- Careful with color images



Brightness

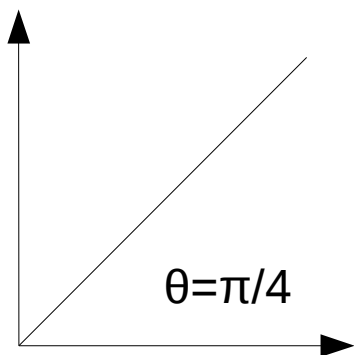


$$g(x, y) = f(x, y) + C$$

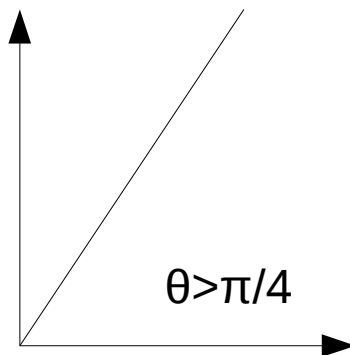


$$g(x, y) = f(x, y) - C$$

Contrast



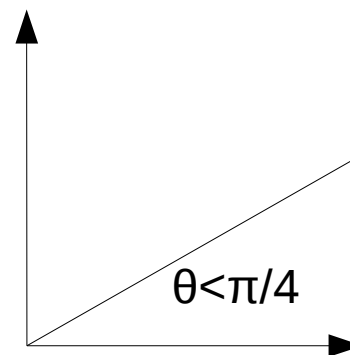
$$\theta = \pi/4$$



$$\theta > \pi/4$$

$$g(x, y) = f(x, y) \times C$$

$$C > 1$$



$$\theta < \pi/4$$

$$g(x, y) = f(x, y) \times C$$

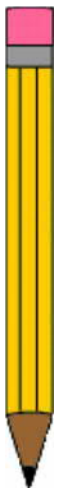
$$C < 1$$

- ♦ Decrease the brightness by 10

$$g(x, y) = f(x, y) - 10$$

- ♦ Decrease the contrast by 2

$$g(x, y) = f(x, y) \times 0.5$$

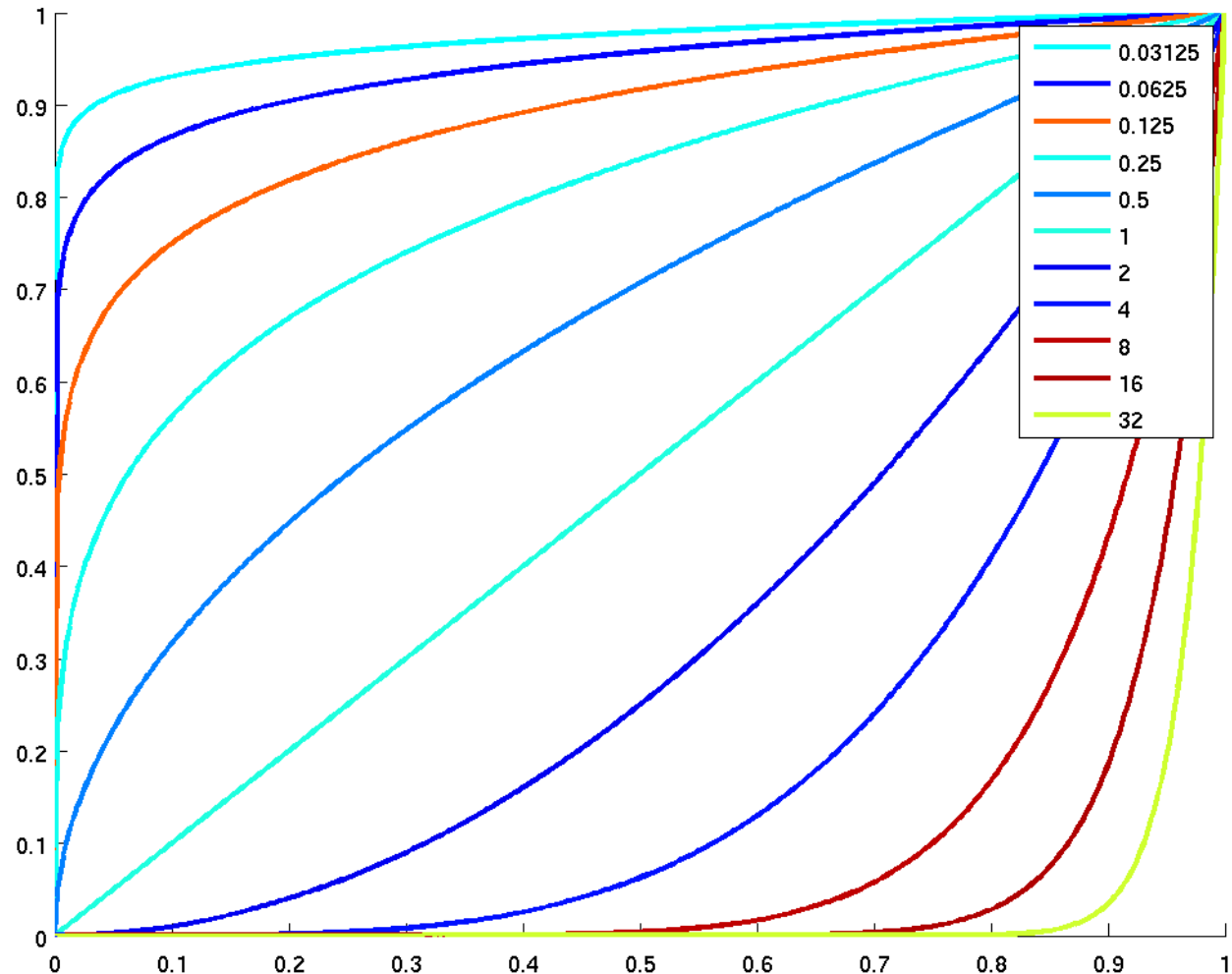


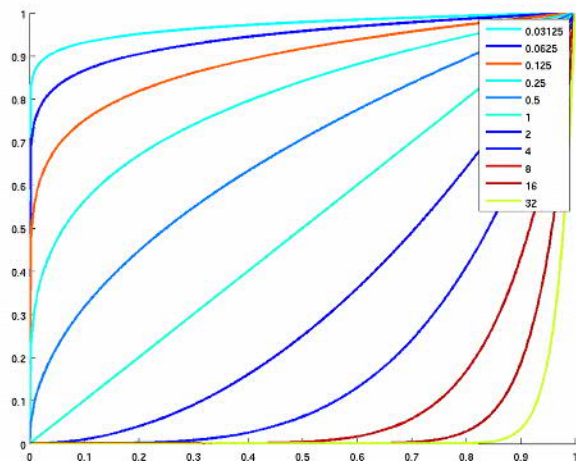
| | | |
|-----|-----|-----|
| 255 | 254 | 253 |
| 125 | 130 | 110 |
| 4 | 3 | 0 |

Gamma transformation

$$g(x, y) = C \times f(x, y)^\gamma$$

- Computer monitors have $\gamma \sim 2.2$
- Eyes have $\gamma \sim 0.45$
- Microscopes should have $\gamma = 1$





$\gamma=0.25$



$\gamma=4$



$\gamma=1$



$\gamma=4$

Log transformations

- Log transformation to visualize patterns in the dark regions of an image

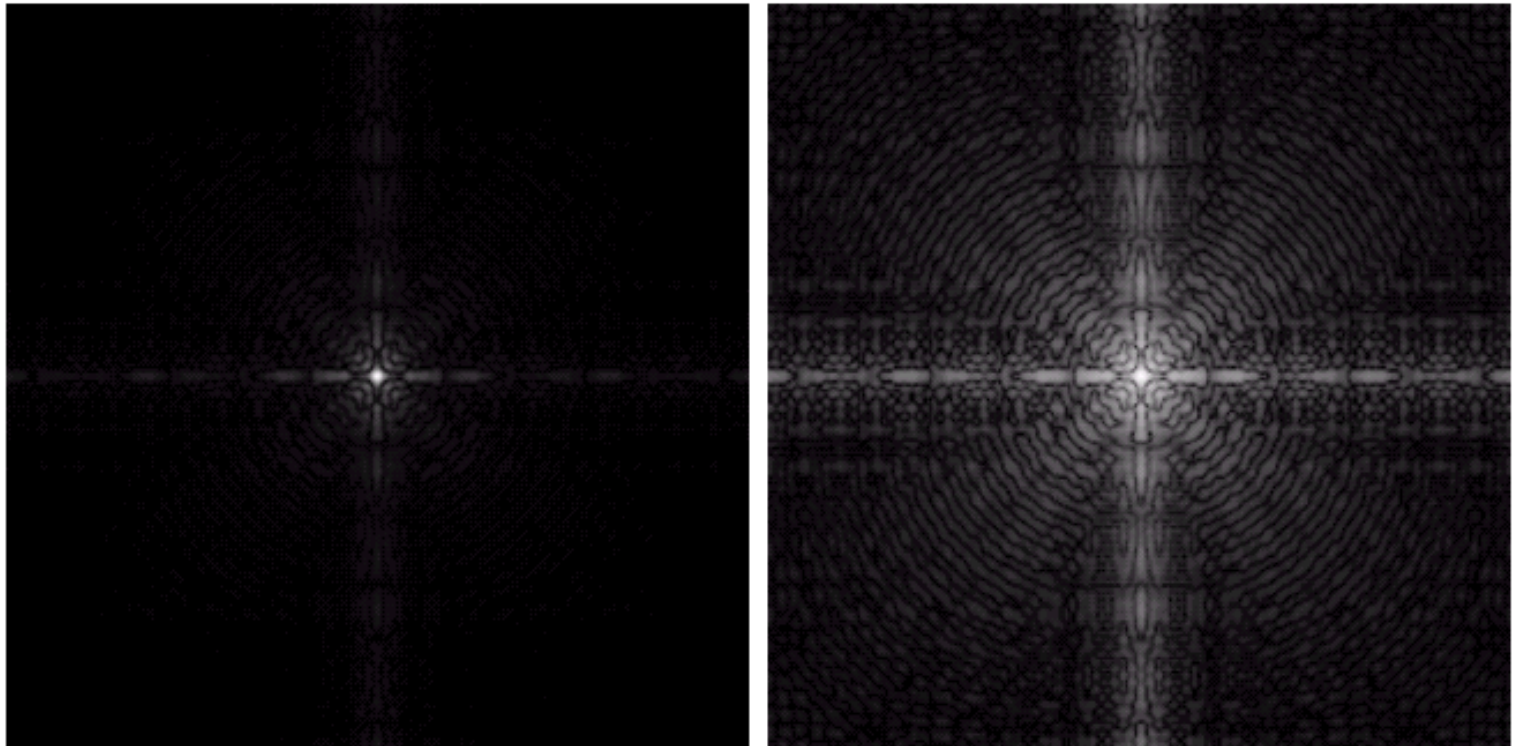
$$g(x, y) = C \log(1 + f(x, y))$$

a b

FIGURE 3.5

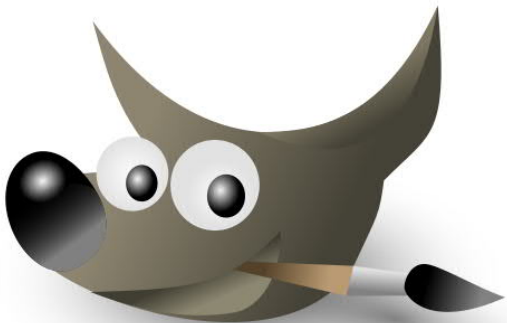
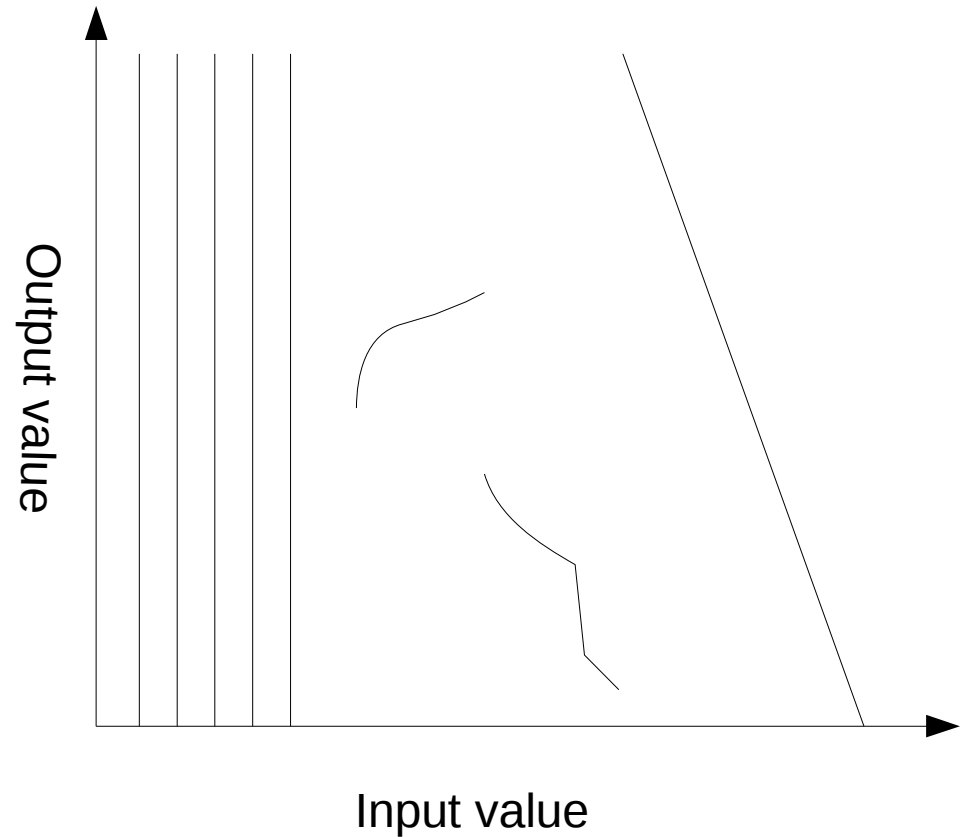
(a) Fourier spectrum.

(b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



Arbitrary transfer functions

- Only one output per input.
- Possibly non-continuous.
- Usually no inverse



III. Histograms and histogram equalization



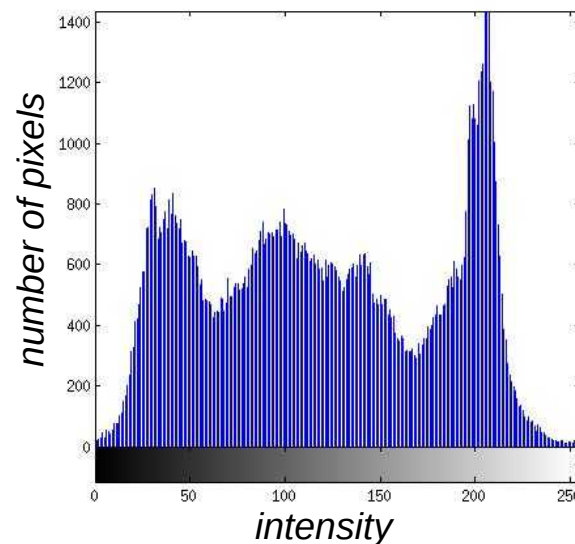
Image histogram

- A gray scale histogram shows how many pixels there are at each intensity level.

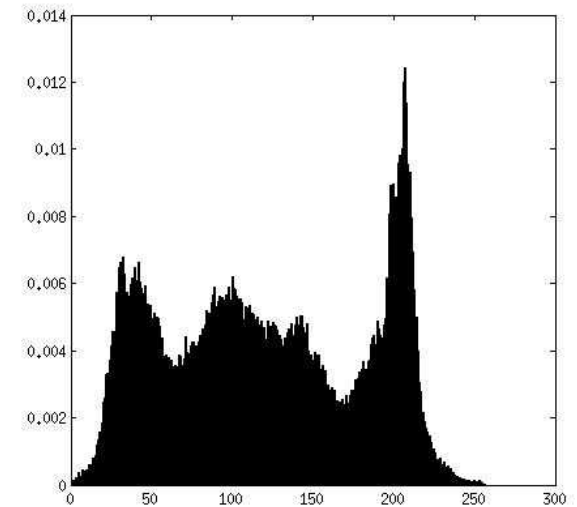


- width = 340 px
- height = 370 px
- bit-depth = 8 bits \rightarrow 0..255

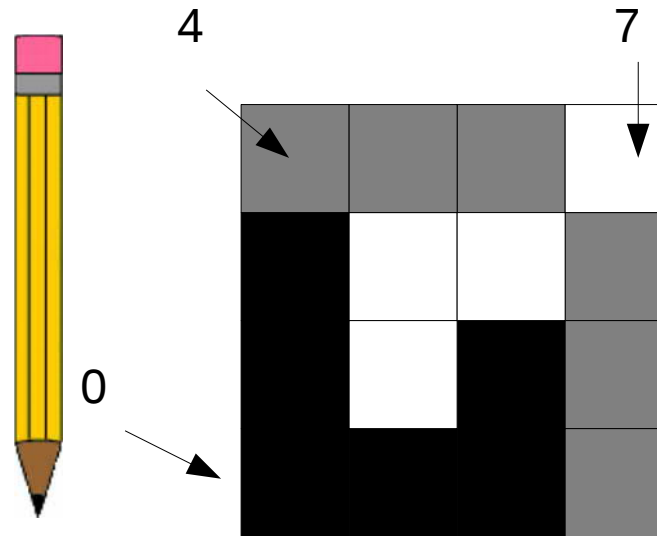
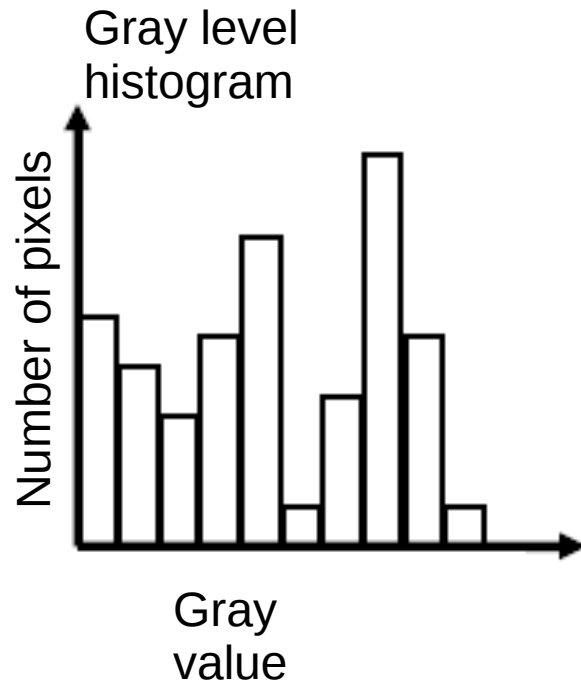
Histogram



Normalized histogram



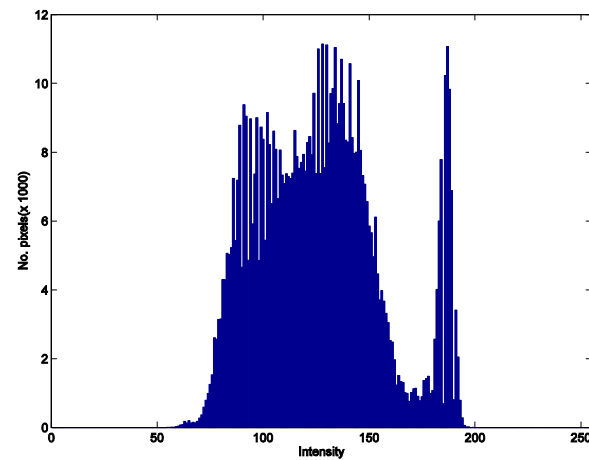
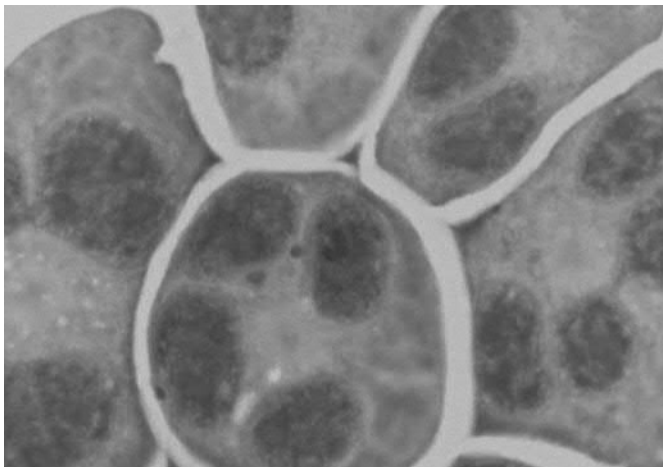
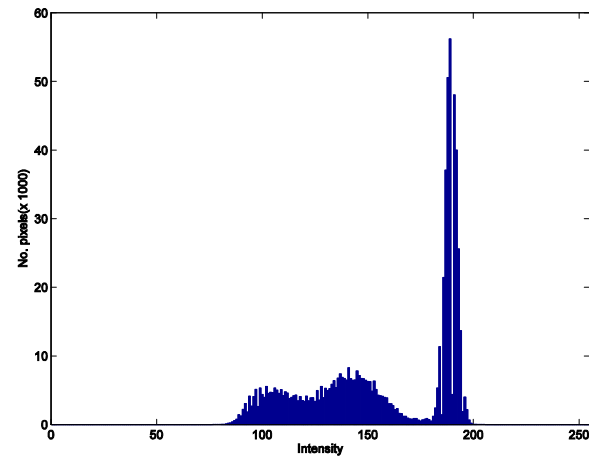
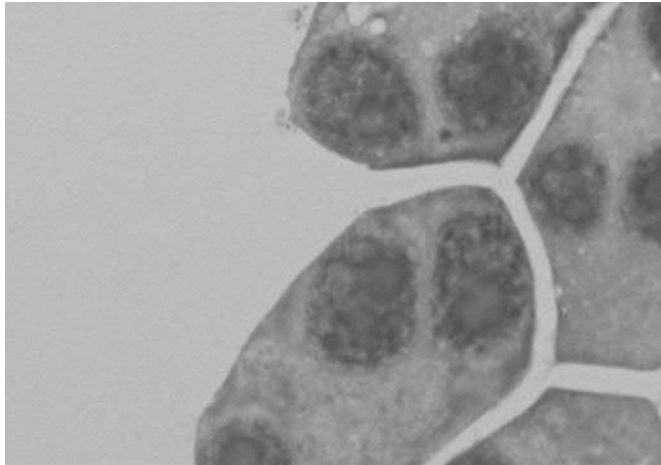
Exercise



- width = 4px
- height = 4px
- bit-depth = 3 bits

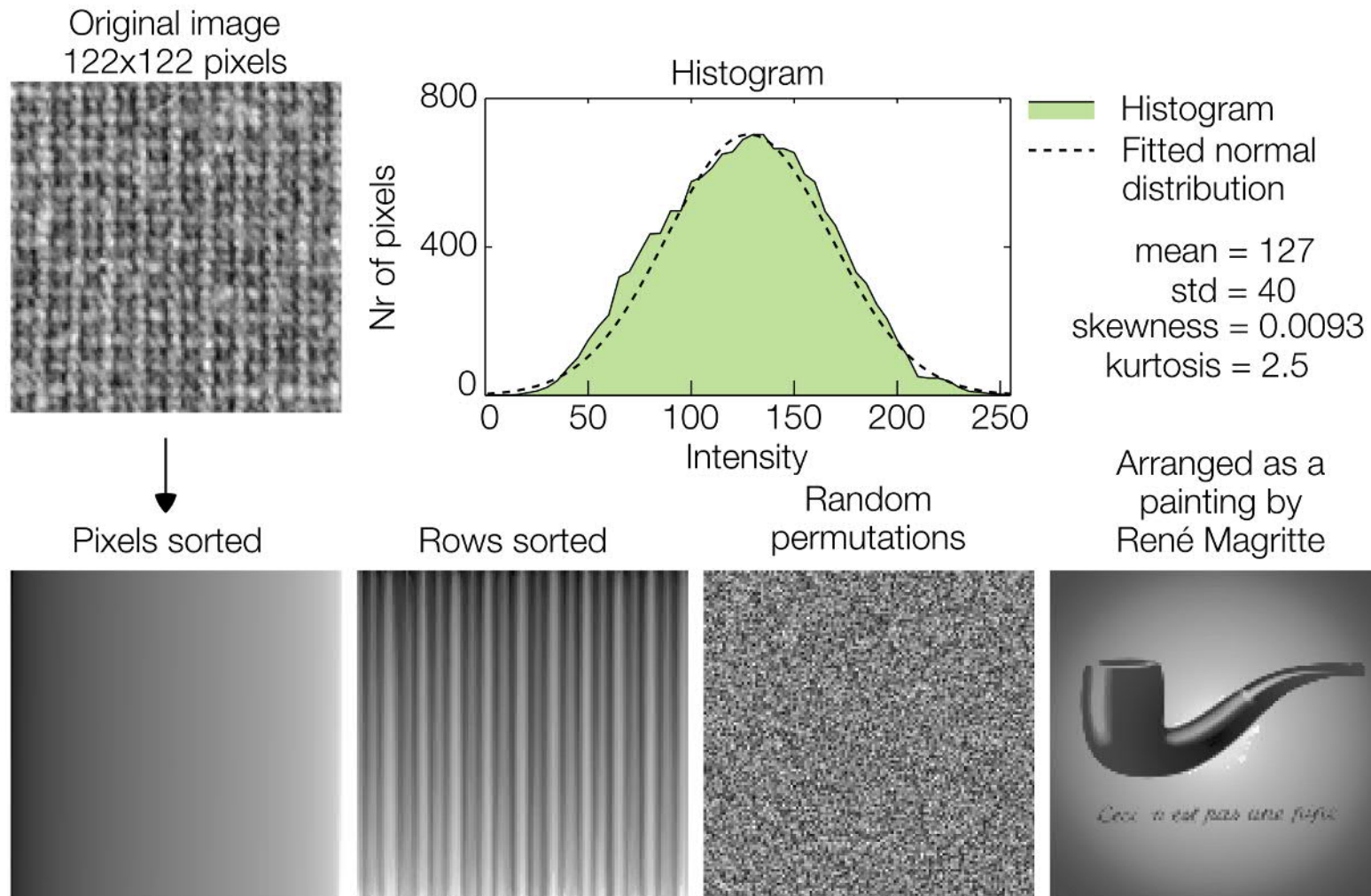
Image histogram

- Gray-level histogram shows intensity distribution



Beware

- Intensity histogram says nothing about the spatial distribution of the pixel intensities





A



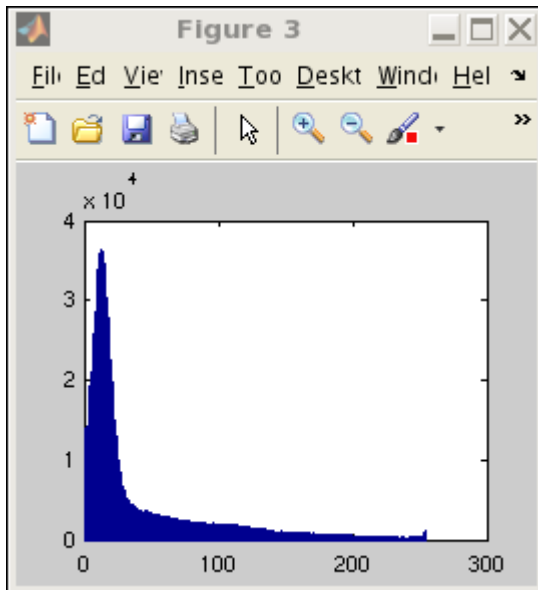
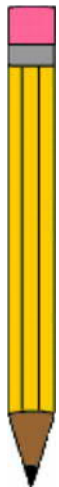
B



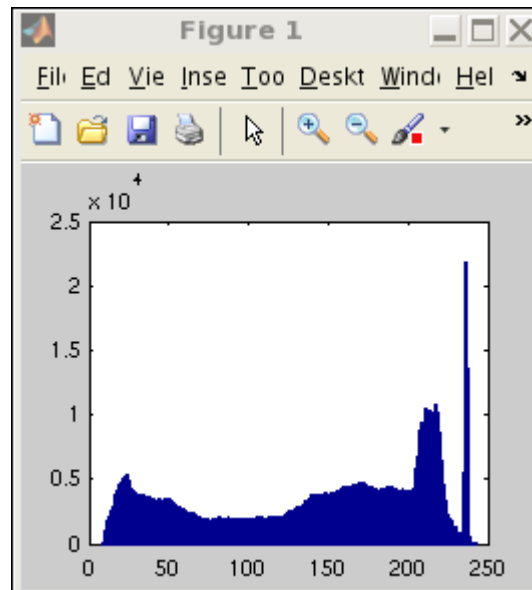
C

Pair images and histograms!

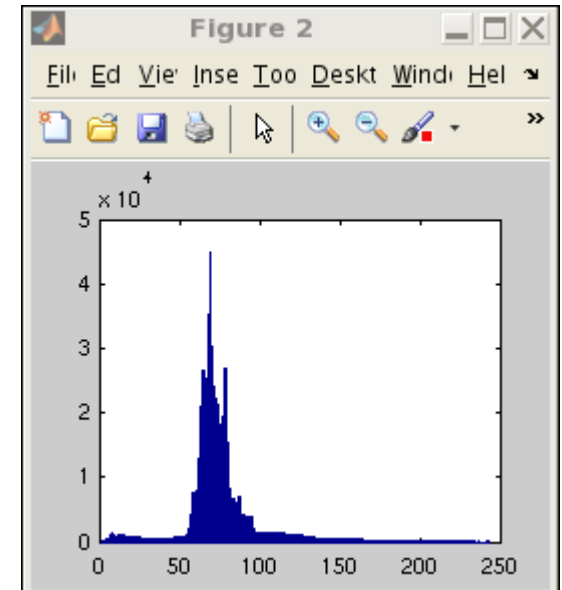
E



F

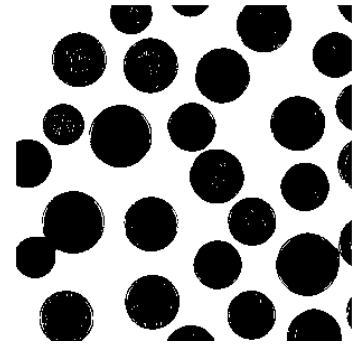
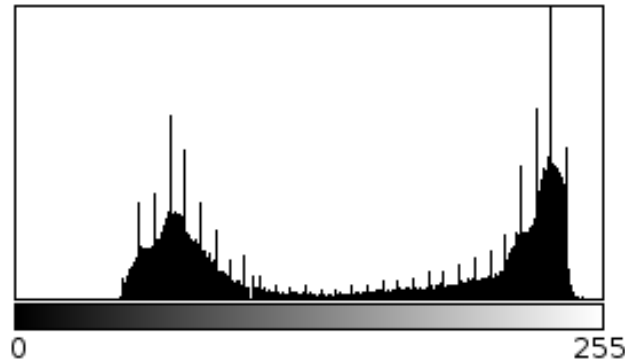
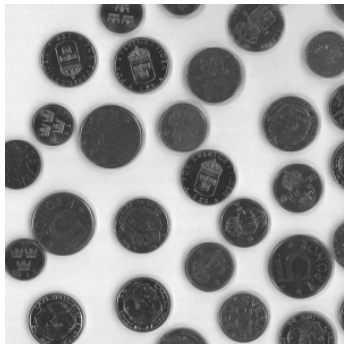


G

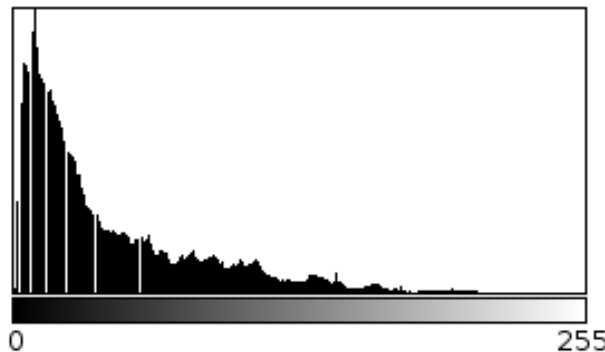
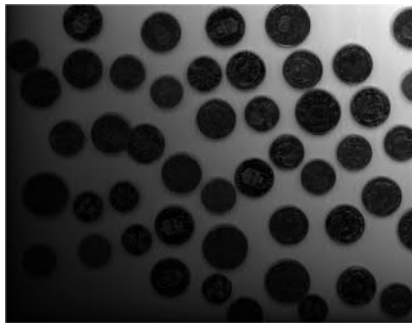


Use of histogram

- Thresholding → decide the best threshold value
- *works well with bi-modal histograms*

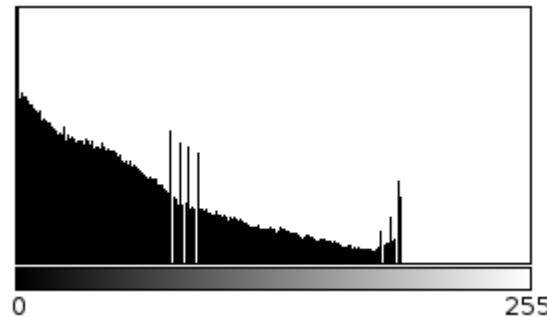
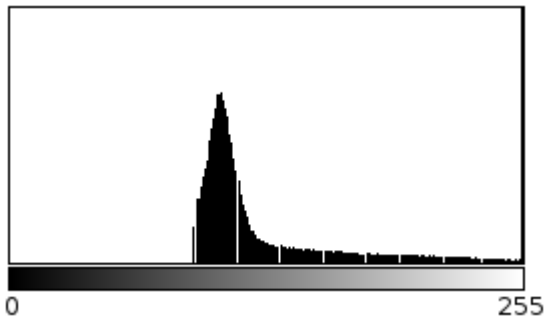


- *does not work with uni-modal histograms*

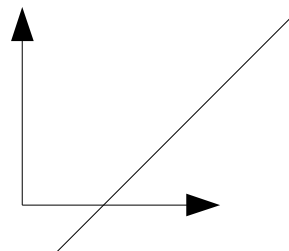
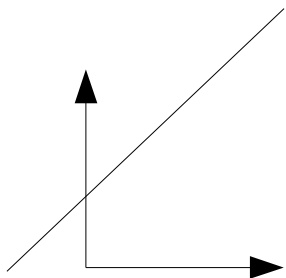


- Analyze the brightness and contrast of an image
- Histogram equalization

Analyze the brightness

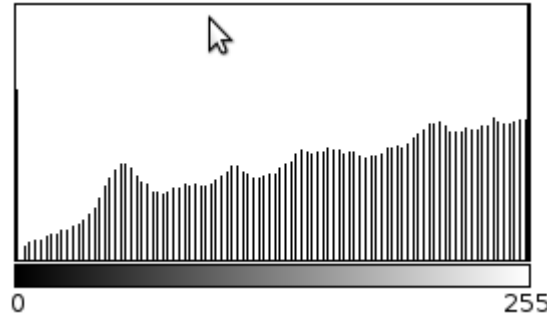
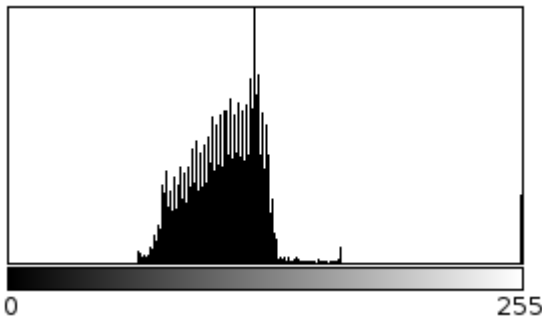


- Increase brightness - shift histogram to the right
- Decrease brightness - shift histogram to the left

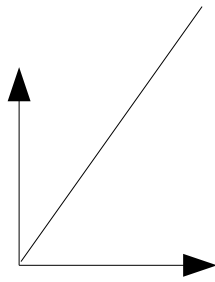
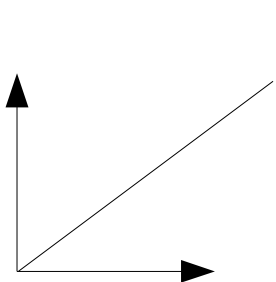


Greylevel transform:
up → increased brightness
down → decreased brightness

Analyze the contrast



- Decreased contrast - compressed histogram.
- When contrast is increased - the histogram is stretched.



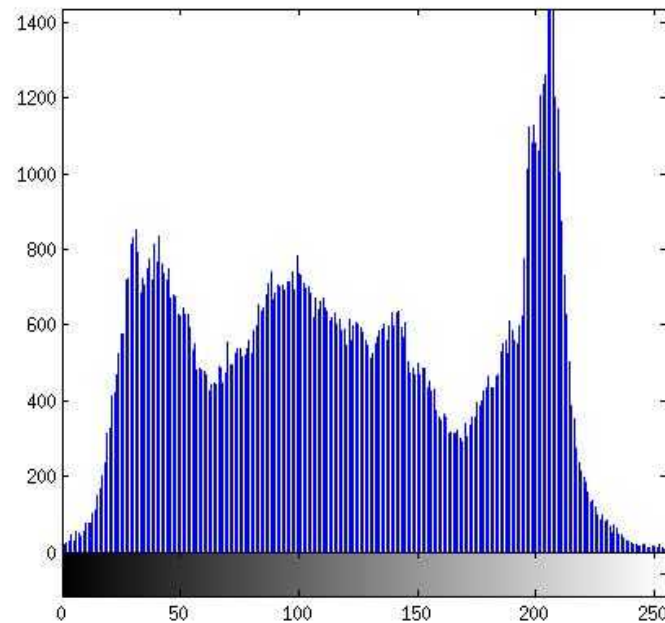
- Greylevel transform:
 - $< 45^\circ \rightarrow$ decreased contrast
 - $> 45^\circ \rightarrow$ increased contrast

Cumulative histogram

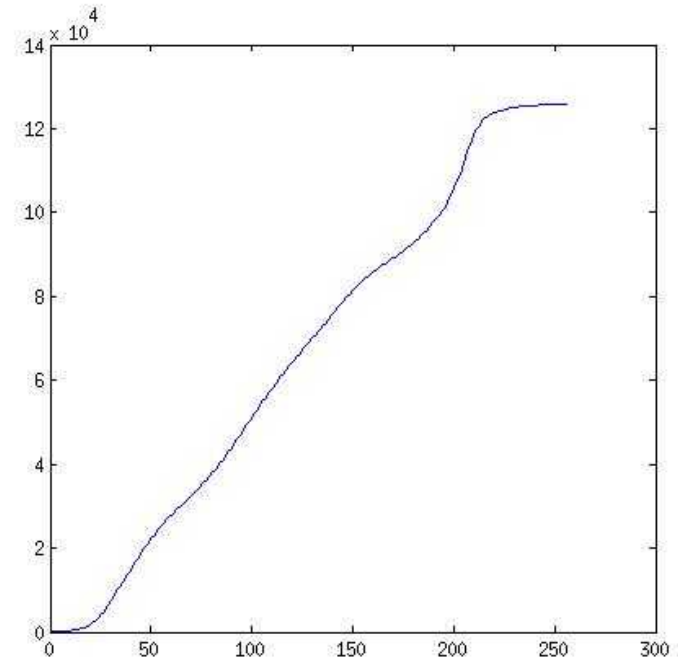
- Easily constructed from the histogram



$$c_j = \sum_{i=0}^j h_i$$



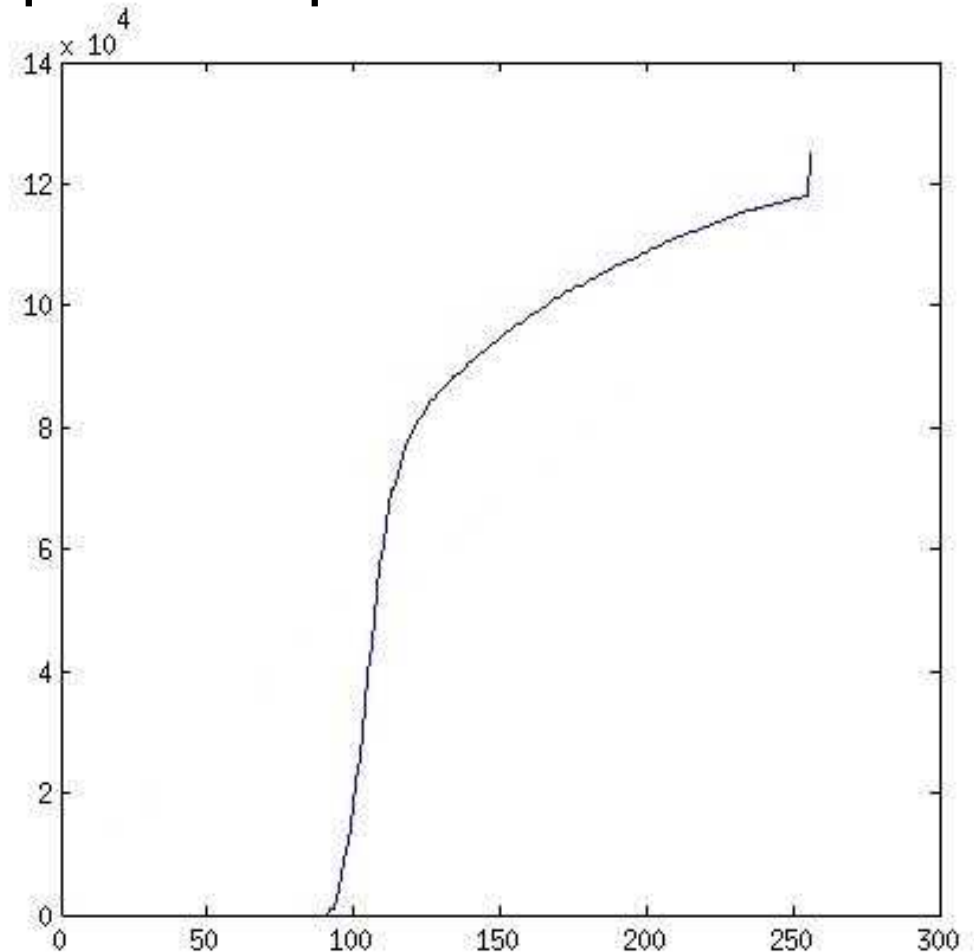
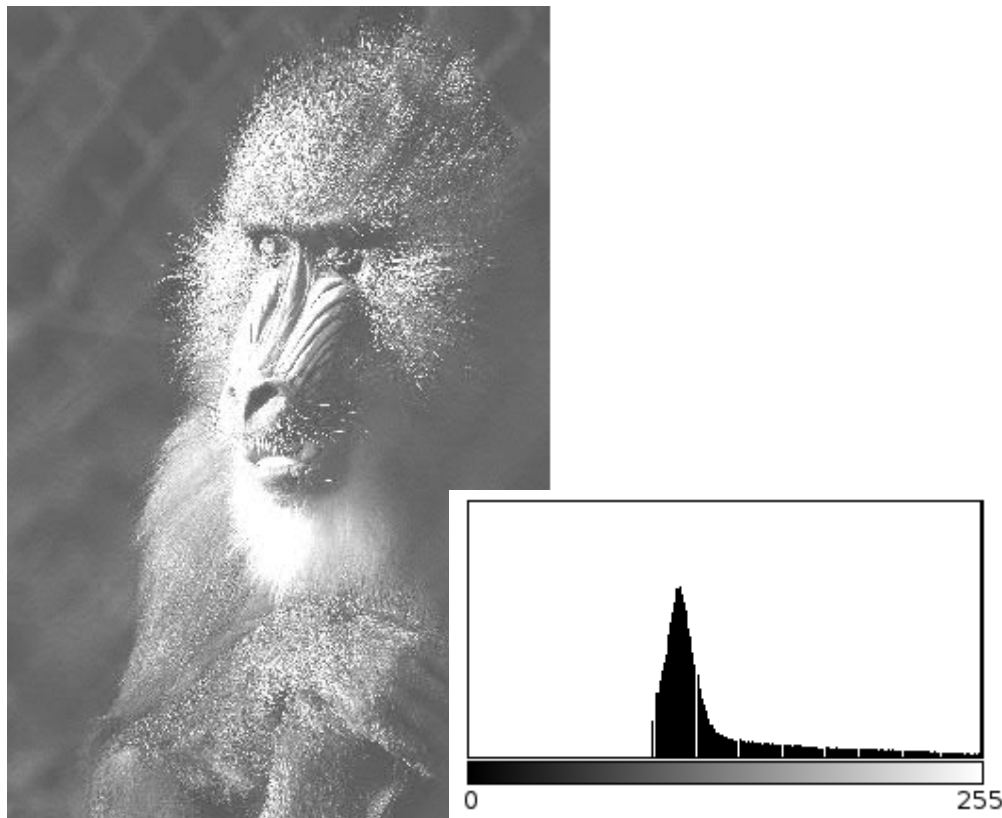
Histogram



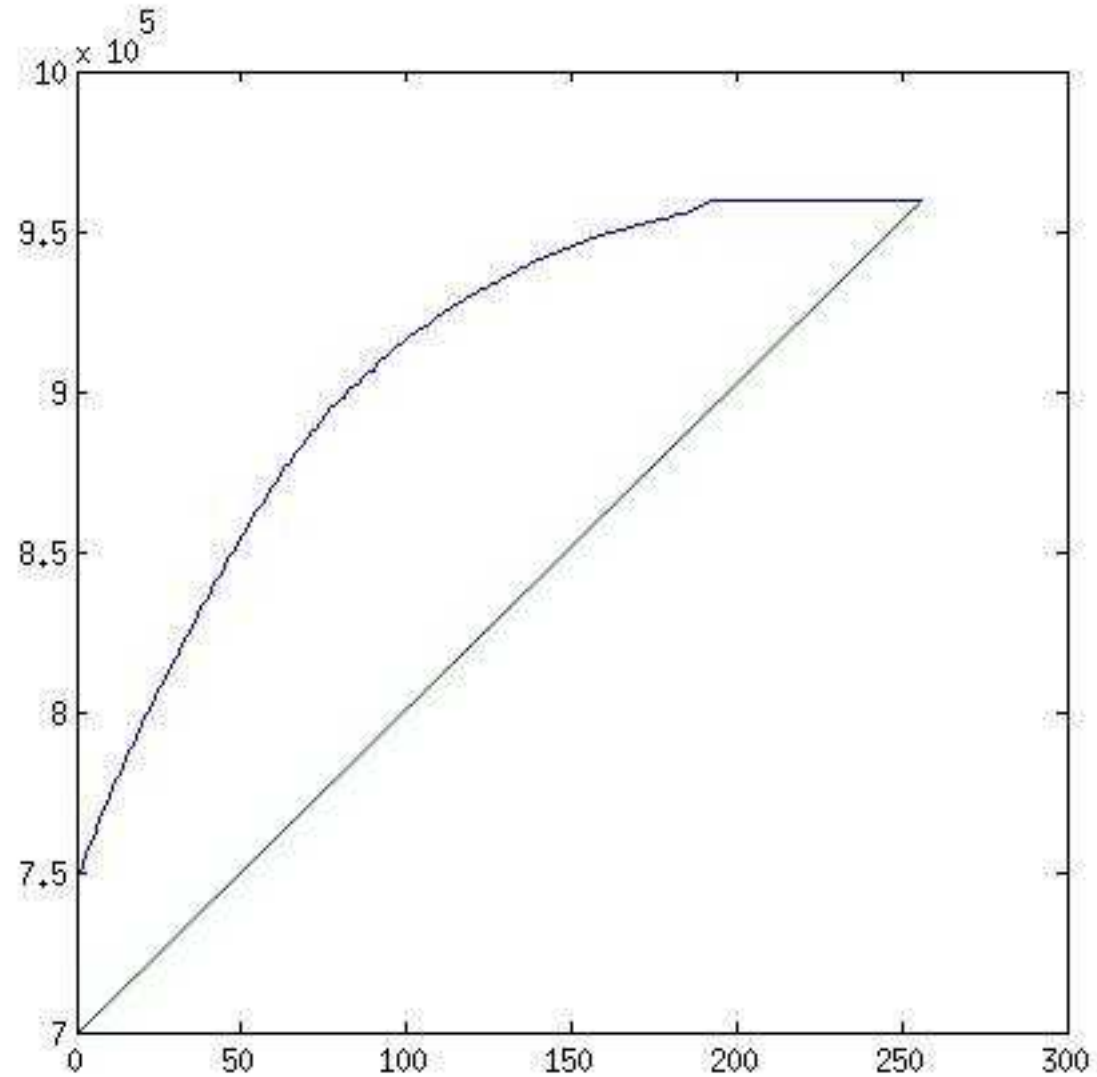
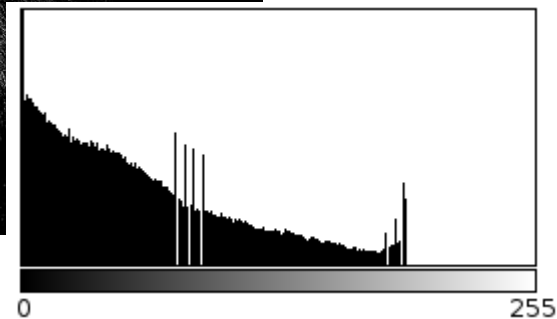
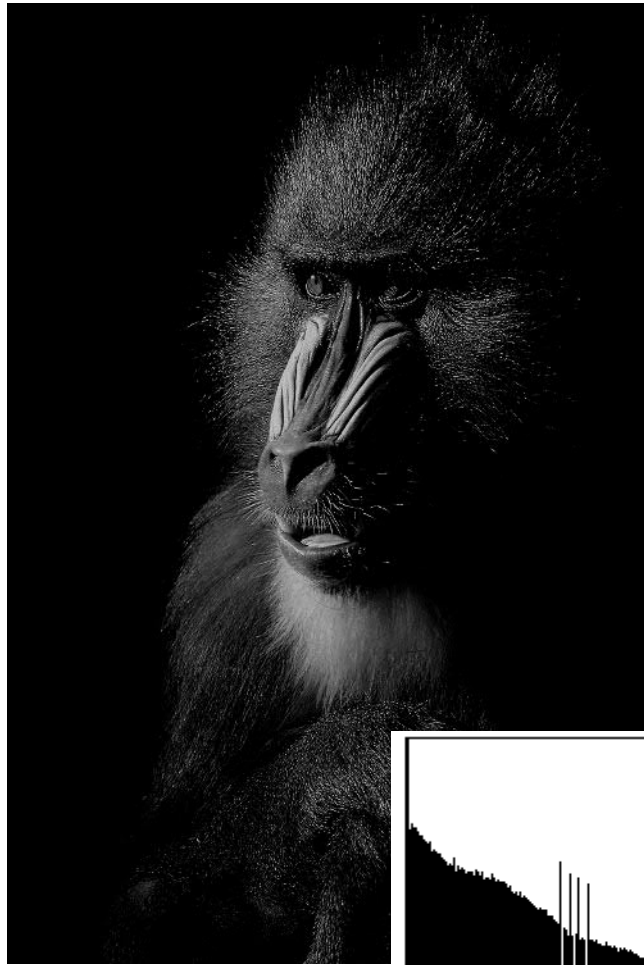
Cumulative histogram

Cumulative histogram

- Slope
 - Steep \rightarrow intensely populated parts of the histogram
 - Gradual \rightarrow in sparsely populated parts of the histogram



Cumulative histogram

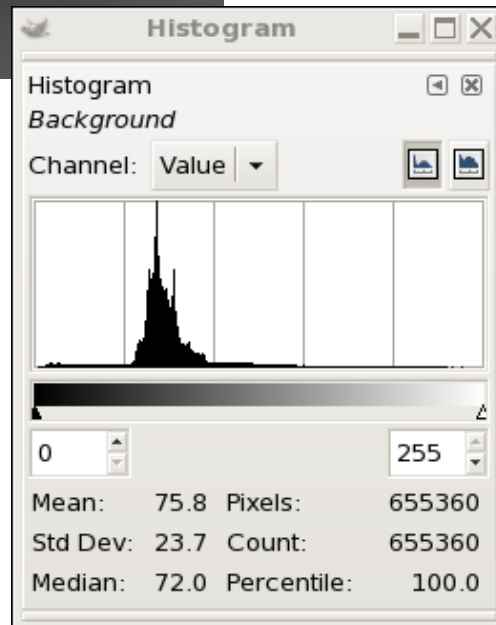
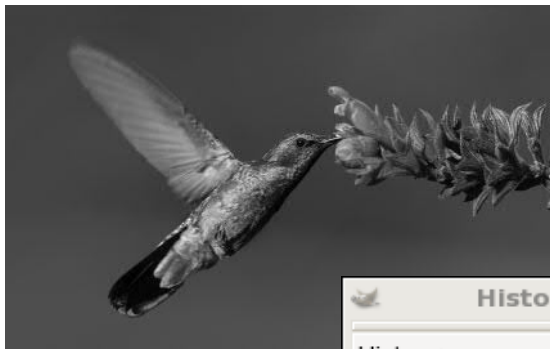


Histogram equalization

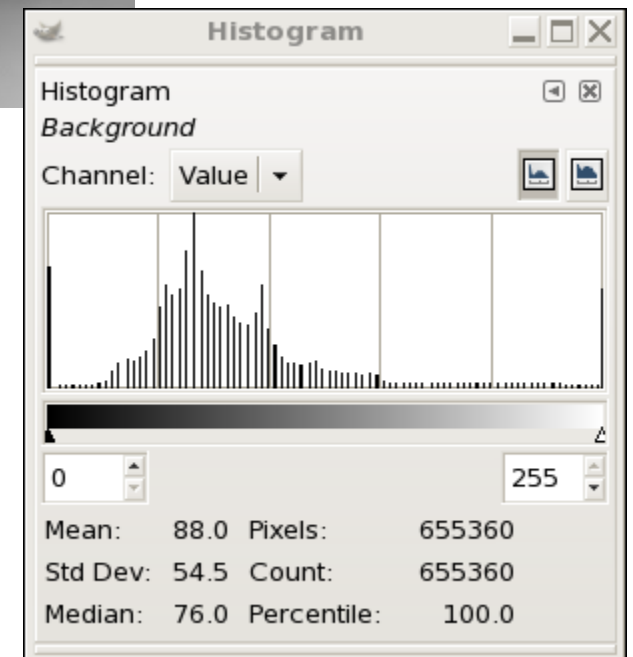
- Idea: Create an image with evenly distributed greylevels, for visual contrast enhancement
- Goal: Find the transformation that produces the most even histogram → cumulative histogram curve
- Equalization flattens the histogram or linearize cumulative histogram
- Automatic contrast enhancement

Histogram equalization

original image



result of histogram equalization

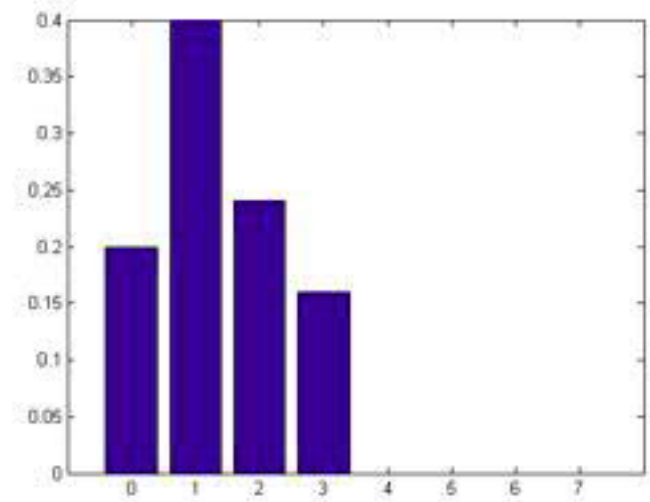


The contrast transform

Hist eq: small example

- Intensity 0 1 2 3 4 5 6 7
- Number of pixels 10 20 12 8 0 0 0 0

- $p(0) = 10/50 = 0.2$, $\text{cdf}(0)=0.2$
- $p(1) = 20/50 = 0.4$, $\text{cdf}(1)=0.6$
- $p(2) = 12/50 = 0.24$, $\text{cdf}(2)=0.84$
- $p(3) = 8/50 = 0.16$, $\text{cdf}(3)=1$
- $p(r) = 0/50 = 0$, $r = 4, 5, 6, 7$ $\text{cdf}(r)=1$

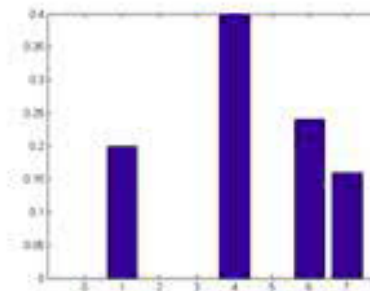
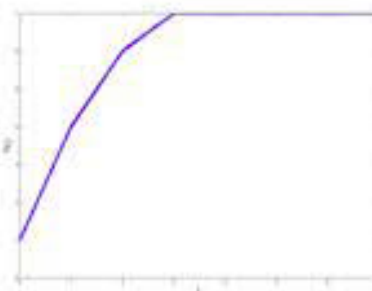
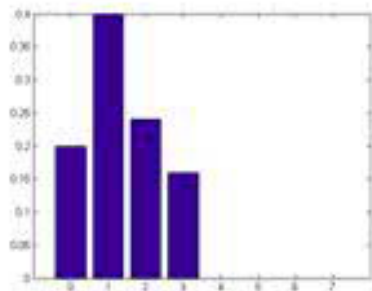


Hist eq: small example

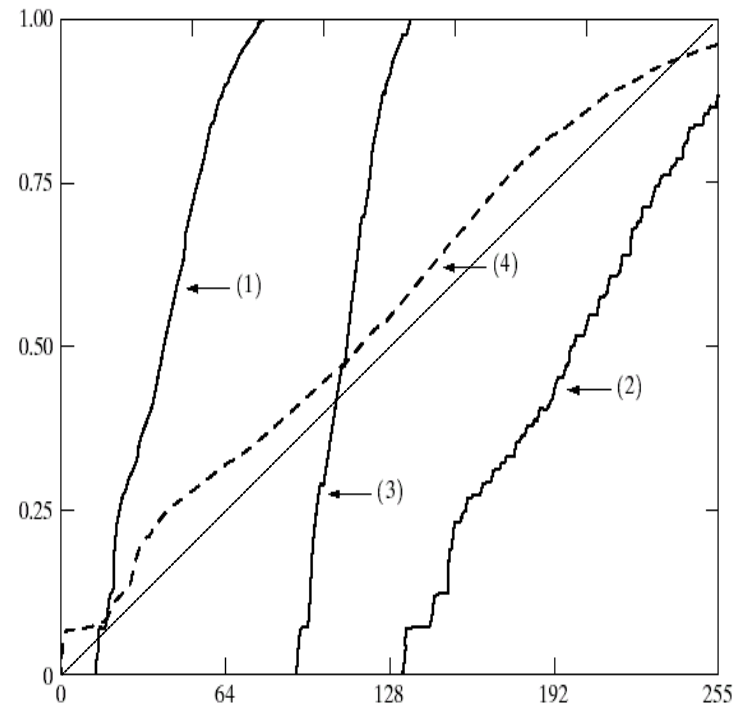
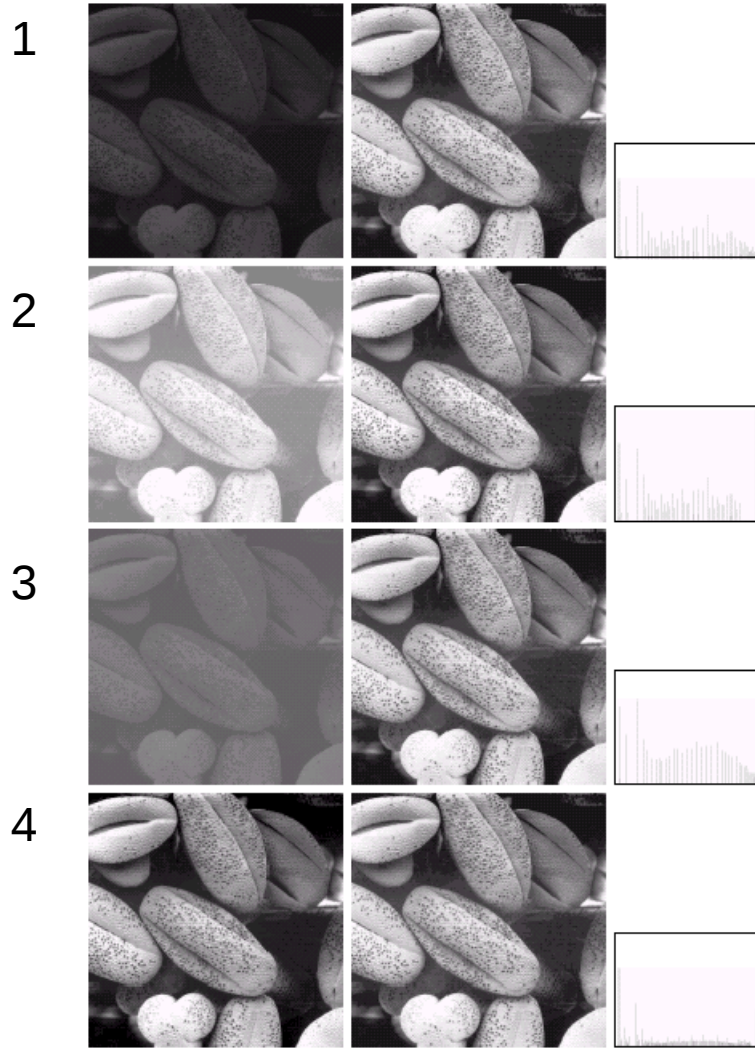
$$T(x) = MAX * cdf(x)$$

- $T(0) = MAX * cdf(0) = 7 * (p(0)) = 7 * 0.2 = 1.4 \approx 1$
- $T(1) = MAX * cdf(1) = 7 * (p(0) + p(1)) = 7 * 0.6 = 3.6 \approx 4$
- $T(2) = MAX * cdf(2) = 7 * (p(0) + p(1) + p(2)) \approx 6$
- $T(3) = MAX * cdf(3) = 7 * (p(0) + p(1) + p(2) + p(3)) \approx 7$
- $T(r) = 7, r = 4, 5, 6, 7$

| | | | | | | | | |
|------------------|----|----|----|---|---|---|---|---|
| Intensity | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Number of pixels | 10 | 20 | 12 | 8 | 0 | 0 | 0 | 0 |

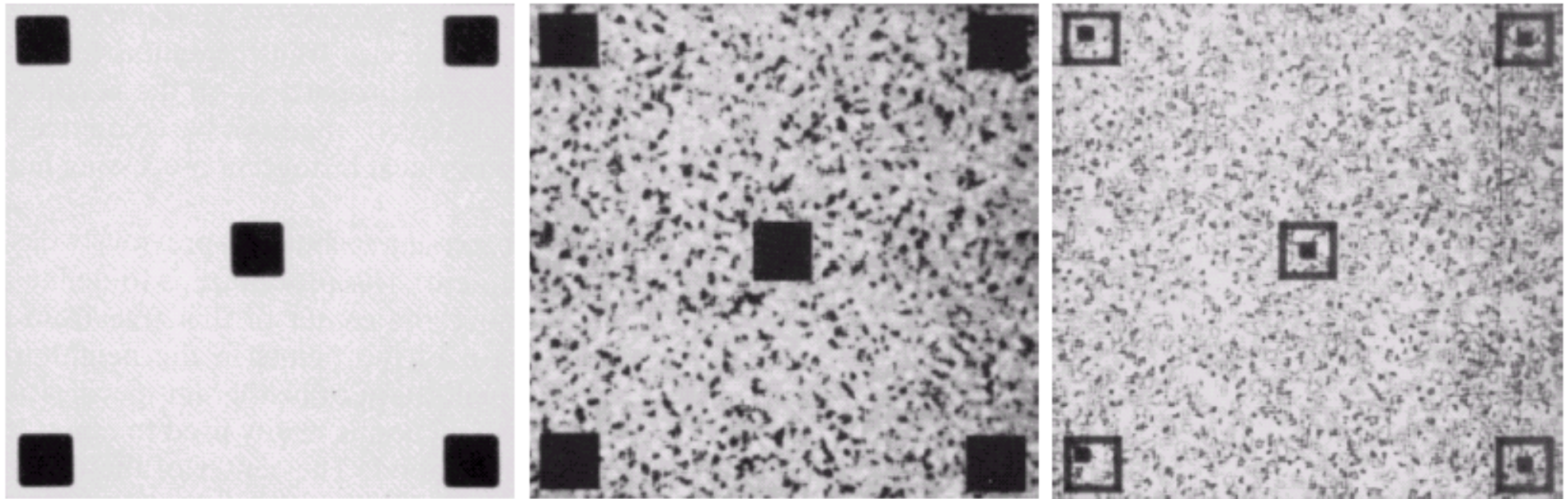


More examples of hist eq



Transformations for image 1-4.
Note that the transform for figure 4
(dashed) is close to the neutral
transform (thin line).

Local histogram equalization

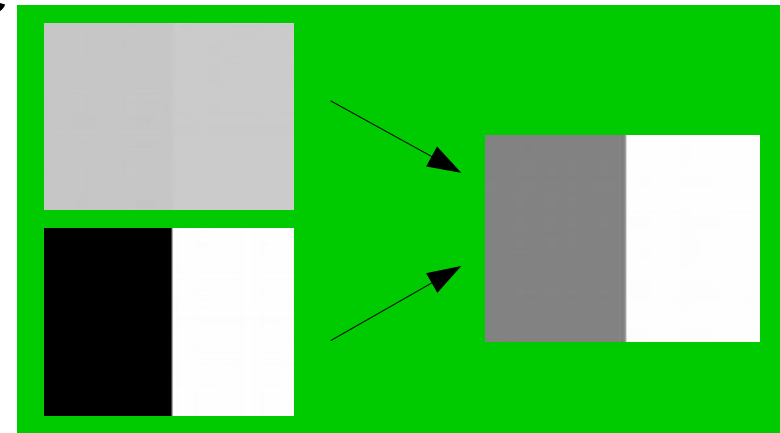


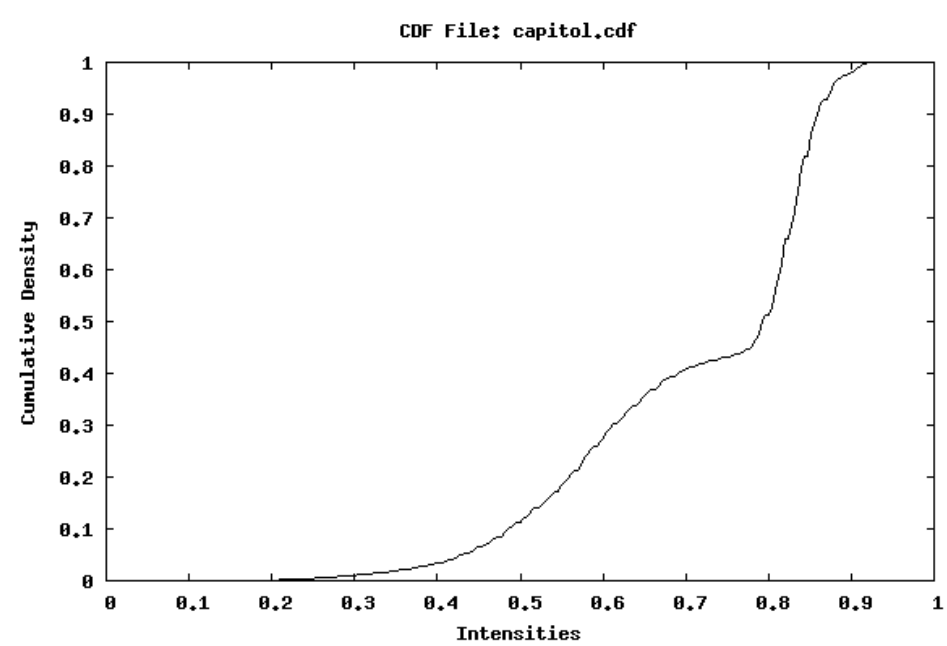
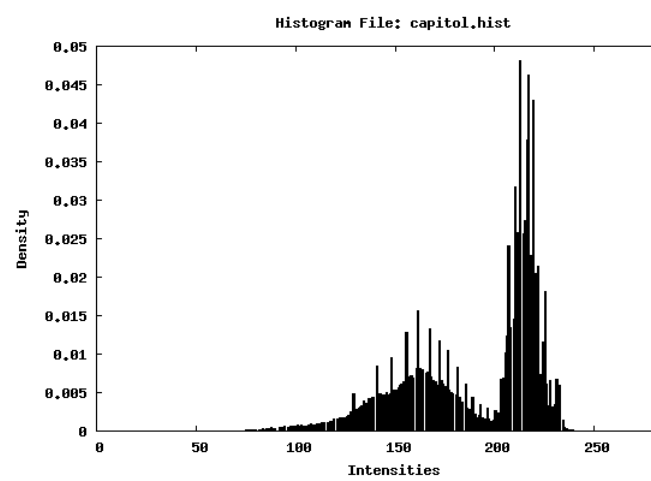
a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

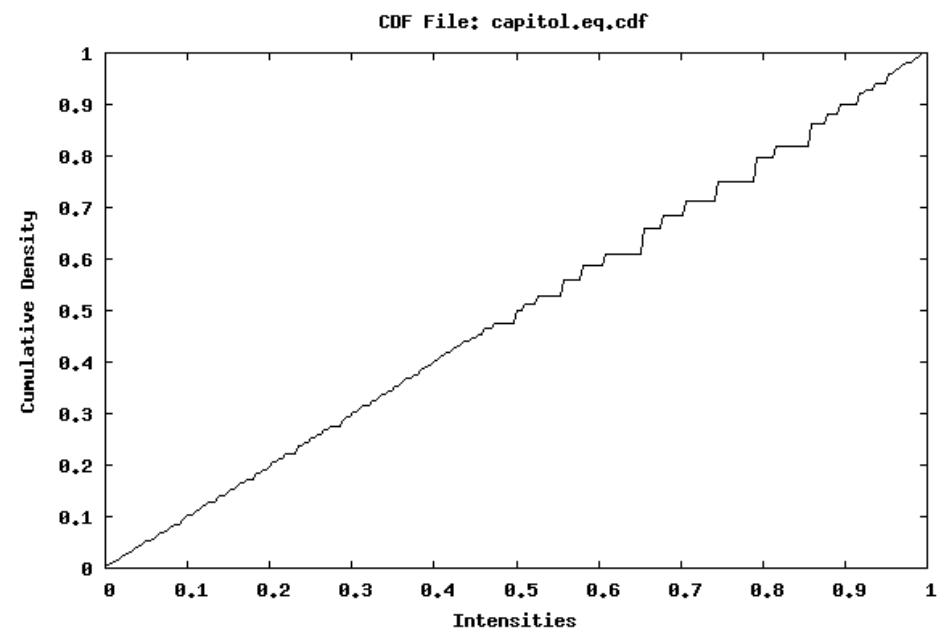
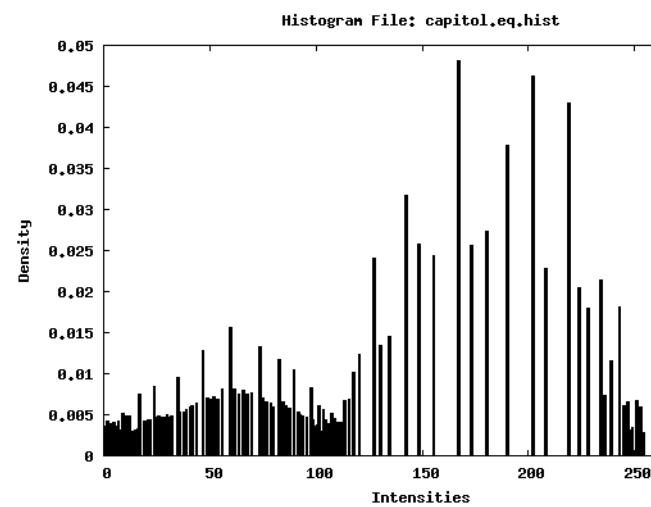
Histogram equalization

- Useful when much information is in a narrow part of the histogram
- Drawbacks:
 - Amplifies noise in large homogenous areas
 - Can produce unrealistic transformations
 - Information might be lost, no new information is gained
 - Not invertible, usually destructive

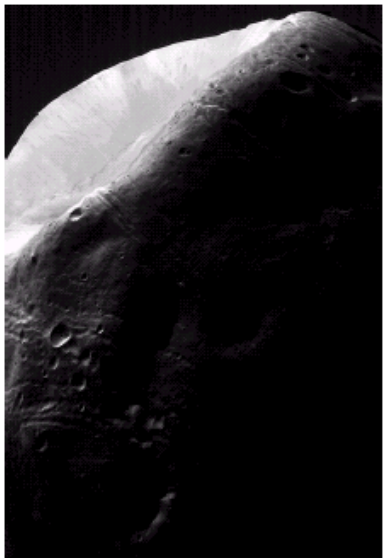




- Does not work well in all cases!



- Histogram equalization is not always “optimal” for visual quality



original image



image after histogram
equalization

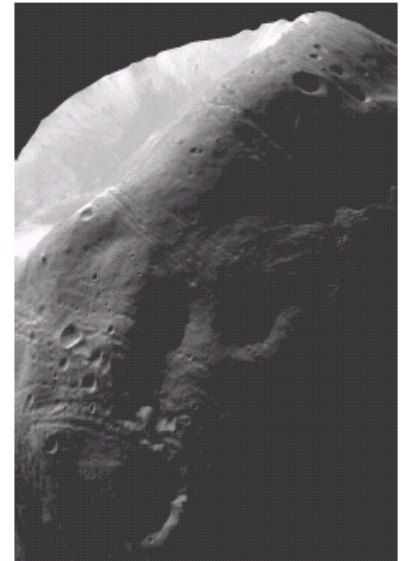
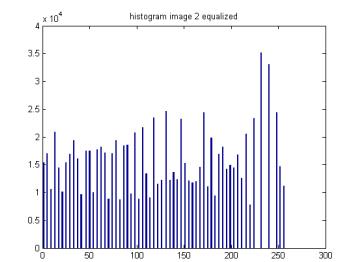
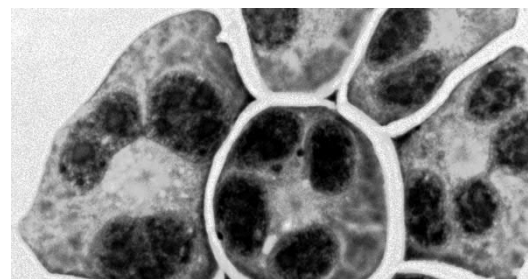
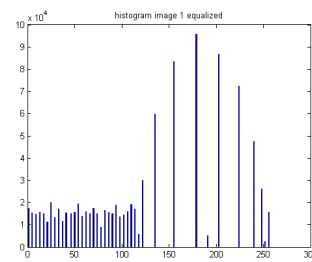
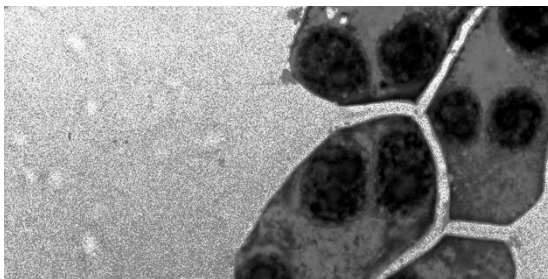
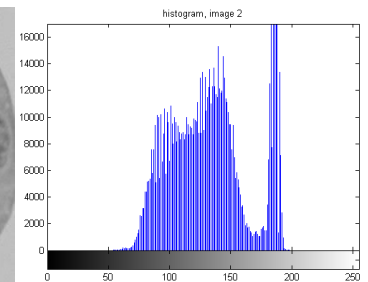
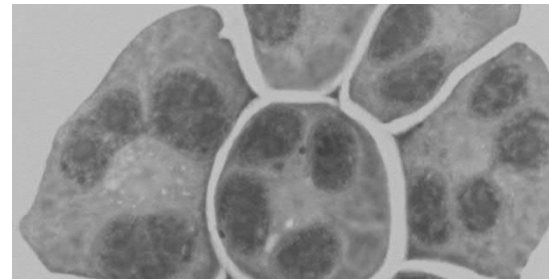
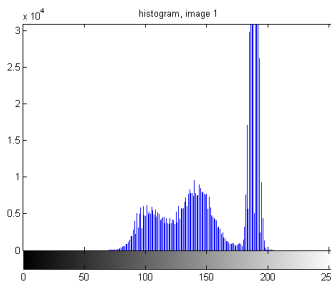
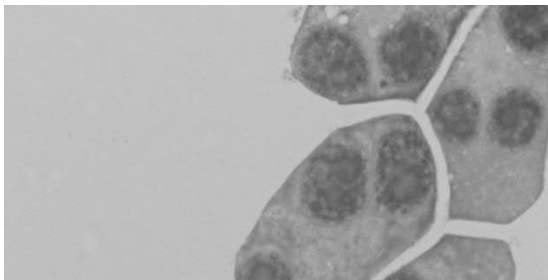


image after
manual choice of
transform

- Histogram eq: the result depends on the amount of different intensities



Histogram matching

- In histogram equalization a **flat distribution** is the goal
- In histogram matching the **distribution of another image** is the goal

For an image, I , find the transformation, T , that gives the histogram some ideal shape, s .

Image 1 histogram matched to image 2

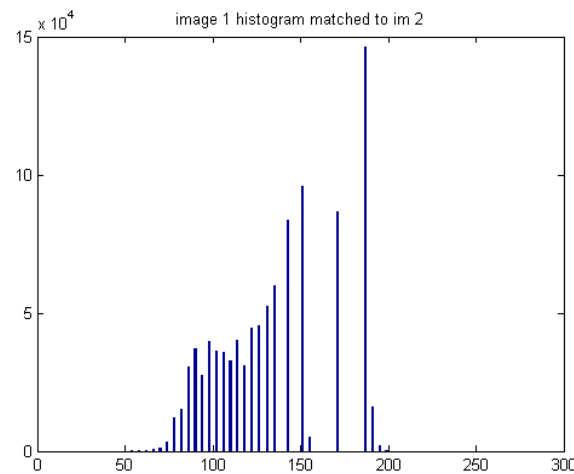
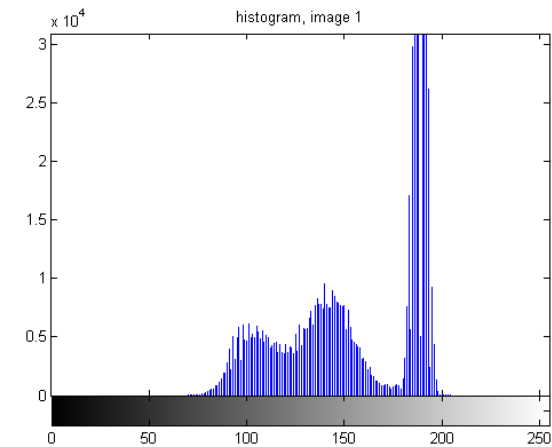
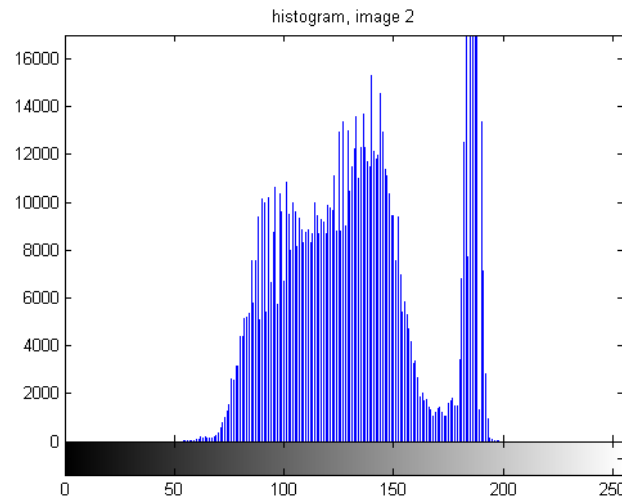
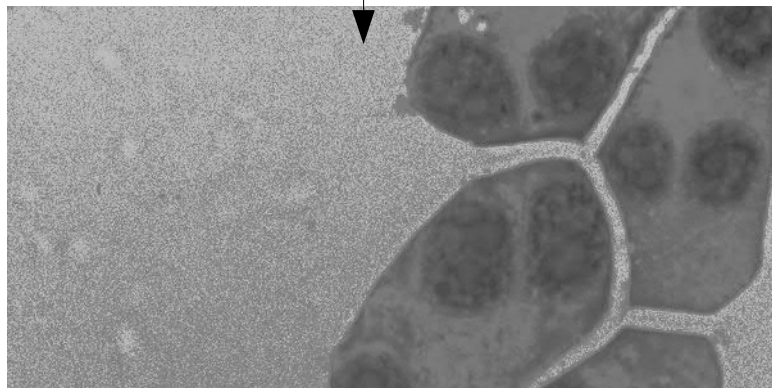
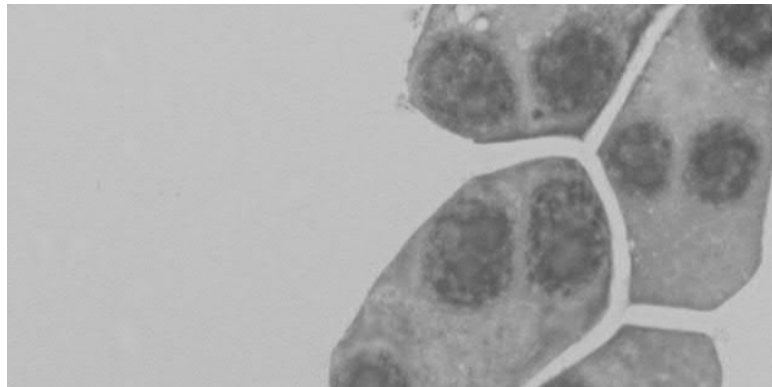
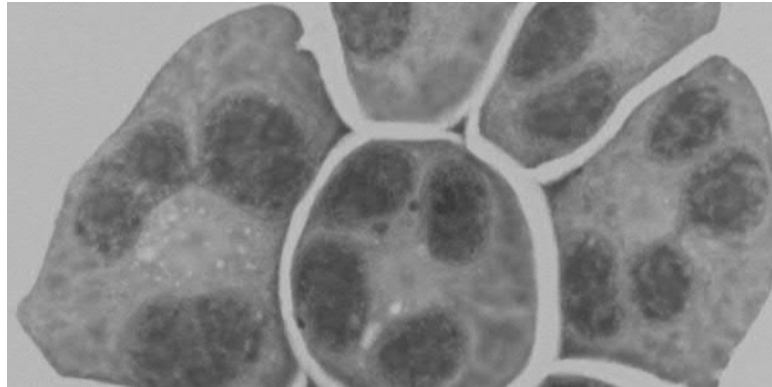
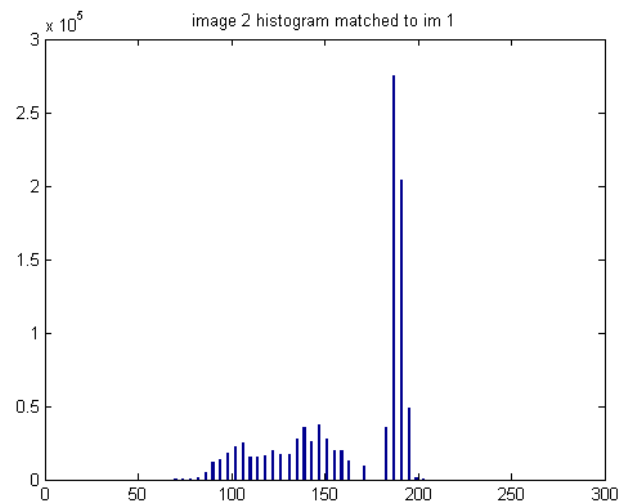
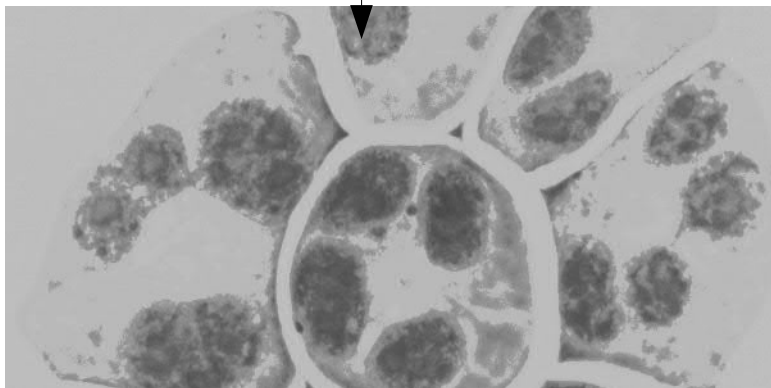
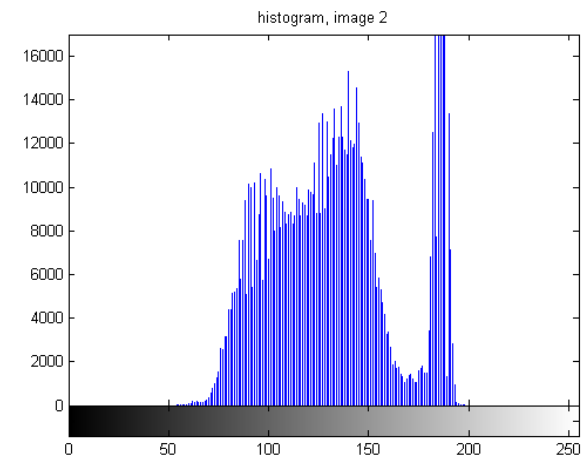
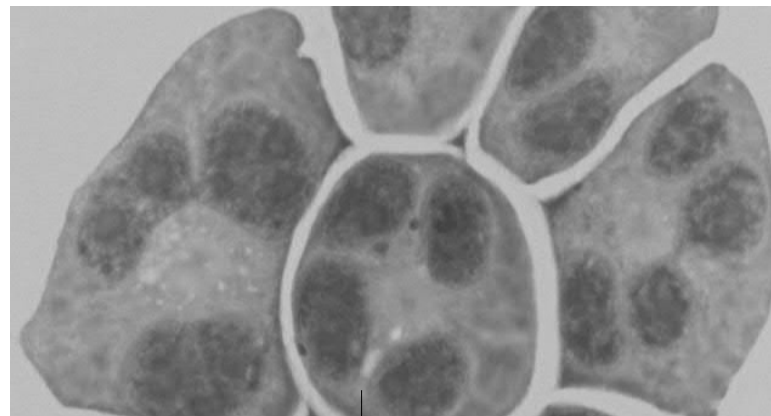
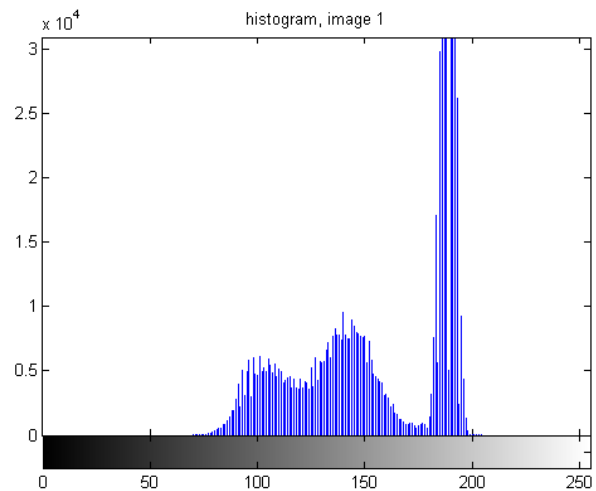
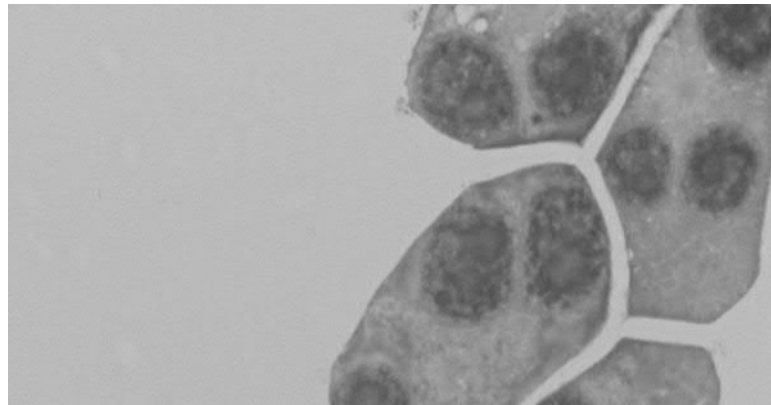
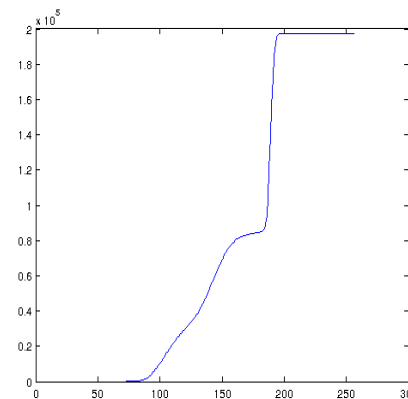
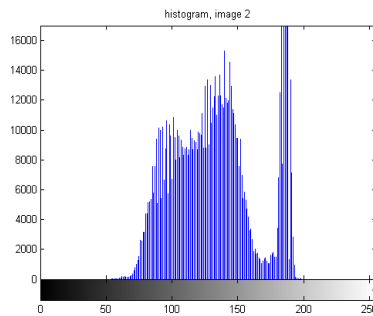
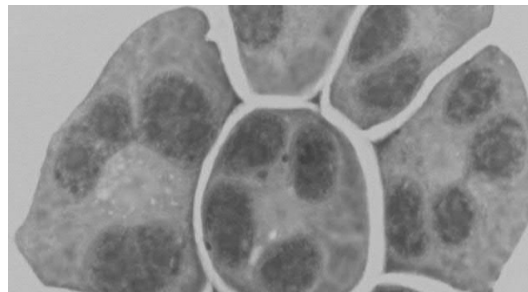
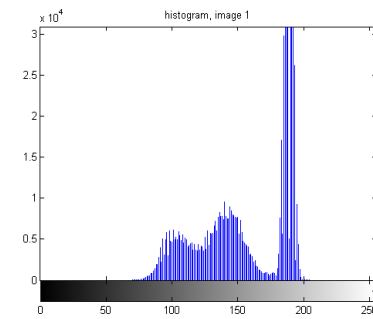
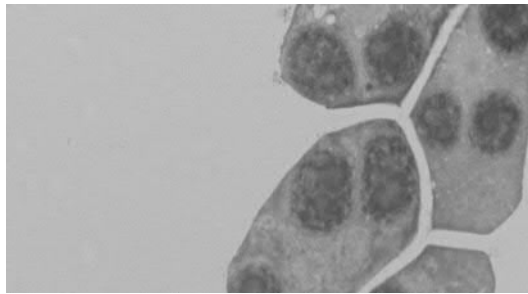


Image 2 histogram matched to image 1

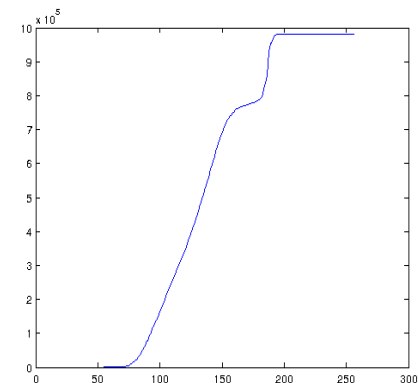


Histogram matching

- Compute the histograms for image I_1 , I_2
- Calculate the cumulative distribution function $F_1()$, $F_2()$
- For each gray level G_1 $[0,255]$ find gray level G_2 for which $F_1(G_1)=F_2(G_2)$
 - Histogram matching function $M(G_1) = G_2$



img1



img2

Try at home!

Summary

- Many common tasks can be described by image arithmetics.
- Histogram equalizations can be useful for visualization.
- Watch out for information leaks!

A few things to think about....

- What is the relation between image arithmetics and linear transfer functions?
- What can you tell about an image from its histogram?
- If you have an 8-bit image, A; how will the 8-bit image $B=255*(A+1)$ look like (exactly!)?
- What conclusions can you draw from the histogram if the first/last column is really high?
- Can you get better resolution by combining multiple images of the same sample?

Suggested problems:

2.22, 2.18, 2.9, 3.1, 3.6

Next lecture:

Spatial filtering (Ch. 3.4-3.8)

