## Lecture 2 Pointwise processing

Ch. 2.6-2.6.4
3.1-3.3 in

Gonzales \& Woods


## Filip Malmberg <br> filip.malmberg@it.uu.se

Centre for Image analysis
Uppsala University

## Image processing



- We want to create an image which is "better" in some sense.
- For example
- Image restoration (reduce noise)
- Image enhancement (enhance edges, lines etc.)
- Make the image more suitable for visual interpretation
- Image enhancement does NOT increase image information


## Image processing

- can be performed in the:
- Spatial domain
- Pointwise processing $\rightarrow$ Lectule 2
- Works per pixel

- Spatial filtering $\rightarrow$ Lecture 3
- Works on small neighborhood

- Frequency domain $\rightarrow$ Lecture 4


Original image in spatial domain


Original image in frequency domain


Processed image in frequency domain


Processed image in spatial domain

## Problem solving using image analysis: fundamental steps

image acquisition

segmentation

classification, interpretation, recognition


## Overview

i. repetition
ii.image arithmetics
iii.intensity transfer functions

iv.histograms and histogram equalization


## Last lecture

- Digitization
- Sampling in space (x,y)
- Sampling in amplitude (intensity)
- Pixel/Voxel
- How often should you sample in space to see details of a certain size?



## Bit depth



- Number of bits that are used to store the intensity information
- Images are typically of 8- or 16-bit
- 1 bit $=2^{\wedge} 1 \rightarrow 2$ steps $(0,1)$
- 2 bit $=2^{\wedge} 2 \rightarrow 4$ steps

64 gray levels, 6bit/pixel

- 8 bit $=2^{\wedge} 8 \rightarrow 256$ steps
- 16 bit $=2^{\wedge} 16 \rightarrow 65536$ steps

256 gray levels, 8bit/pixel


## I. Image arithmetics in the spatial domain

## Image arithmetics

- $A(x, y)=B(x, y) \circ C(x, y)$ for all $x, y$.
$B, C \rightarrow$ images with the same (spatial) dimensions $\rightarrow$ images + constant value
- can be
- Standard arithmetic operation: +, -, *, /
- Logical operator (binary images): AND, OR, XOR,...
- Any pitfalls?


## Arithmetics with binary images

image1
image2

image1-image2
image2-image1

## Arithmetics with binary images

image1
image2

image1-image2
image2-image1

## Arithmetics with binary images

image1

image2

image1-image2

image2-image1

## Arithmetics with greyscale images


-


II


## Logical operations on binary images



| INPUT |  |  |
| :---: | :---: | :---: |
| OUTPUT |  |  |
| A | B | A OR B |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

INPUT OUTPUT

## Applications

- Noise reduction using image mean or median

$$
I=\frac{1}{n} \sum_{k=1 . . n} I_{k}
$$

$$
I_{1}
$$



## Applications

- Change detection using subtraction


2015
Has anything changed?


## Applications

- Change detection using subtraction


2015
2016

direct difference

## Applications

- Change detection using subtraction


2015

direct difference


2016

difference after registration

## Applications

- Change/motion detection using subtraction



## Applications

- Background removal
image - background image
- Creating a background image


Max or median of the pixel intensities at all positions.

## Applications

- Background removal - result



## Applications

- Subtracting a background image/correcting for uneven illumination



## II. Intensity transfer functions

## Intensity transfer functions

$$
g(x, y)=T f(x, y)
$$

i. linear (neutral $\square$, negative, contrast, brightness)
ii.smooth (gamma, log)
iii.arbitrarily


## The negative transformation

$$
g(x, y)=\max -f(x, y)
$$

- For eight bit image:

$$
g(x, y)=2^{8}-1-f(x, y)
$$



The rules of how to transfer values from the old image to the new one.

## The negative transformation

$$
g(x, y)=\max -f(x, y)
$$

- For eight bit image:

$$
g(x, y)=2^{8}-1-f(x, y)
$$

| 255 | 254 | 253 |
| :---: | :---: | :---: |
| 125 | 130 | 110 |
| 4 | 3 | 0 |



The rules of how to transfer values from the old image to the new one.

## The negative transformation

$$
g(x, y)=\max -f(x, y)
$$

- For eight bit image:

$$
g(x, y)=2^{8}-1-f(x, y)
$$

| 255 | 254 | 253 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 125 | 130 | 110 |  |  |  |
| 4 | 3 | 0 | 0 | 1 | 2 |
| 130 | 125 | 145 |  |  |  |
| 251 | 252 | 255 |  |  |  |
| Negative image |  |  |  |  |  |



## The negative transform

- Example


Original


Negative

## The negative transform


original digital mammogram
image negative to (visually) enhance white or gray details embedded in dark regions

## The negative transformation

- Careful with color images



## Brightness




## Brightness





$$
g(x, y)=f(x, y)+C \quad g(x, y)=f(x, y)-C
$$

## Brightness


$\Delta$



$$
g(x, y)=f(x, y)+C \quad g(x, y)=f(x, y)-C
$$

## Contrast


$\Delta$


## Contrast




## Contrast



## Examples

- Decrease the brightness by 10
- Decrease the contrast by 2

$$
g(x, y)=f(x, y) \times 0.5
$$

| 255 | 254 | 253 |
| :--- | :---: | :---: | :---: |
| 125 | 130 | 110 |
| 4 | 3 | 0 |

## Examples

- Decrease the brightness by 10

$$
g(x, y)=f(x, y)-10
$$

- Decrease the contrast by 2

| 255 | 254 | 253 |
| :---: | :---: | :---: |
| 125 | 130 | 110 |
| 4 | 3 | 0 |


| 245 | 244 | 243 |
| :---: | :---: | :---: |
| 115 | 120 | 100 |
| 0 | 0 | 0 |
| Decreased <br> brightness |  |  |


| 128 | 127 | 127 |  |
| :---: | :---: | :---: | :---: |
| 63 | 65 | 55 |  |
| 2 | 2 | 0 |  |
| Decreased <br> contrast |  |  |  |
|  |  |  |  |

## Gamma transformation

$g(x, y)=C \times f(x, y)^{\gamma}$

- Computer monitors have $\gamma \sim 2.2$
- Eyes have $\gamma \sim 0.45$
- Microscopes should have $\gamma=1$


$\gamma=0.25$

$\gamma=1$


$\gamma=0.25$

$\gamma=4$


$\gamma=1$

$\gamma=4$



## Log transformations

- Log transformation to visualize patterns in the dark regions of an image

$$
g(x, y)=C \log (1+f(x, y))
$$

a b
FIGURE 3.5
(a) Fourier
spectrum.
(b) Result of
applying the log transformation
given in
Eq. (3.2-2) with
$c=1$.


## Arbitrary transfer functions

- Only one output per input.
- Possibly noncontinuous.
- Usually no inverse


Input value

## III. Histograms and histogram equalization

## Image histogram

- A gray scale histogram shows how many pixels there are at each intensity level.

- width $=340 \mathrm{px}$
- height $=370 \mathrm{px}$
- bit-depth $=8$ bits $\rightarrow 0 . .255$

Histogram


Normalized histogram


## Exercise



- width $=4 \mathrm{px}$
- height $=4 \mathrm{px}$
- bit-depth $=3$ bits


## Exercise



## Image histogram

- Gray-level histogram shows intensity distribution





## Beware

## - Intensity histogram says nothing about the spatial distribution of the pixel intensities

Original image
$122 \times 122$ pixels



- Histogram
----. Fitted normal
distribution

$$
\begin{aligned}
\text { mean } & =127 \\
s t d & =40
\end{aligned}
$$

skewness $=0.0093$
kurtosis $=2.5$

Arranged as a painting by René Magritte




## Use of histogram

- Thresholding $\rightarrow$ decide the best threshold value
- works well with bi-modal histograms

- does not work with uni-modal histograms

- Analyze the brightness and contrast of an image
- Histogram equalization


## Analyze the brightness




- Increase brightness shift histogram to the right
- Decrease brightness shift histogram to the left


Greylevel transform: up $\rightarrow$ increased brightness down $\rightarrow$ decreased brightness

## Analyze the contrast



- Decreased contrast compressed histogram.
- When contrast is increased the histogram is stretches.

-Greylevel transform:
$<45^{\circ} \rightarrow$ decreased contrast
$>45^{\circ} \rightarrow$ increased contrast


## Cumulative histogram

- Easily constructed from the histogram


$$
c_{j}=\sum_{i=0}^{j} h_{i}
$$



Histogram


Cumulative histogram

## Cumulative histogram

- Slope
- Steep $\rightarrow$ intensely populated parts of the histogram
- Gradual $\rightarrow$ in sparsely populated parts of the histogram




## Cumulative histogram




## Histogram equalization

- Idea: Create an image with evenly distributed greylevels, for visual contrast enhancement
- Goal: Find the transformation that produces the most even histogram $\rightarrow$ cumulative histogram curve
- Equalization flattens the histogram, linearizes cumulative histogram
- Automatic contrast enhancement


## Histogram equalization

## original image



## result of histogram equalization



## Hist eq: small example

- Intensity $\begin{array}{llllllll}0 & 1 & 2 & 3 & 4 & 5 & 6 & 7\end{array}$
- Number of pixels 10201280000
- $p(0)=10 / 50=0.2, \operatorname{cdf}(0)=0.2$
- $p(1)=20 / 50=0.4, \operatorname{cdf}(1)=0.6$
- $p(2)=12 / 50=0.24, \operatorname{cdf}(2)=0.84$
- $p(3)=8 / 50=0.16, \operatorname{cdf}(3)=1$
- $p(r)=0 / 50=0, r=4,5,6,7 \operatorname{cdf}(r)=1$



## Hist eq: small example

- $T(0)=7$ * $(p(0))=7$ * $0.2=1.4 \approx 1$
- $T(1)=7$ * $(p(0)+p(1))=7$ * $0.6=3.6 \approx 4$
- $T(2)=7$ * $(p(0)+p(1)+p(2)) \approx 6$
- $T(3)=7$ * $(p(0)+p(1)+p(2)+p(3)) \approx 7$
- $T(r)=7, r=4,5,6,7$

Intensity
01234567
Number of pixels 01000200128




## More examples of hist eq




Transformations for image 1-4. Note that the transform for figure 4 (dashed) is close to the neutral transform (thin line).

## Local histogram equalization


a b c
FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a $7 \times 7$ neighborhood about each pixel.

## Histogram equalization

- Useful when much information is in a narrow part of the histogram
- Drawbacks:
- Amplifies noise in large homogenous areas
- Can produce unrealistic transformations
- Information might be lost, no new information is gained
- Not invertible, usually destructive



CDF File: capitol.edf

## - Histogram equalization is not always "optimal" for visual quality


original image

image after histogram equalization

image after manual choice of transform

- Histogram eq: the result depends on the amount of different intensities



## Histogram matching

- In histogram equalization a flat distribution is the goal
- In histogram matching the distribution of another image is the goal

For an image, I, find the transformation, $T$, that gives the histogram some ideal shape, s.

## Image 1 histogram matched to image 2





## Image 2 histogram matched to image 1





## Histogram matching

- Compute the histograms for image $I_{1}, I_{2}$
- Calculate the cumulative distribution function $F_{1}(), F_{2}()$
- For each gray level $G_{1}[0,255]$ find gray level $G_{2}$ for which $\mathrm{F}_{1}\left(\mathrm{G}_{1}\right)=\mathrm{F}_{2}\left(\mathrm{G}_{2}\right)$
- Histogram matching function $M\left(G_{1}\right)=G_{2}$


img1



## Summary

- Many common tasks can be described by image arithmetics.
- Histogram equalizations can be useful for visualization.
- Watch out for information leaks!

A few things to think about....

- What is the relation between image arithmetics and linear transfer functions?
- What can you know about an image from the histogram?
- If you have an 8 -bit image, $A$; how will the 8 -bit image $B=255 *(A+1)$ look like (exactly!)?
- What conclusions can you draw from the histogram if the first/last column is really high?
- Can you get better resolution by combining multiple images of the same sample?

Next lecture:
Spatial filtering (Ch. 3.4-3.8)



## Bit plane slicing

- Pixels $\rightarrow$ digital numbers composed of bits
- Computer $\rightarrow$ Binary number system
- Basic unit, bit


## $194=11000010$

One 8-bit byte



- 8 - and 16 - bits are common for file formats. But how many bits are necessary?


## Next slides:

- The eight bit planes for an image.
- The same image using 7,..,0 bit planes.














$[$

$$
1 \mathrm{c}
$$








