Suggested problem: 11.16
Obtain the grey-level co-occurrence matrix of a 5x5 image composed of a checkerboard of alternating 1's and 0's if the position operator $P$ is defined as
a) “one pixel to the right”
b) “two pixels to the right”
Regional Descriptors

• Simple descriptors:
  – area
  – perimeter
  – compactness \((P^2A, \text{circularity})\)
  – grey-level measures: mean, median, max etc.
• Topological descriptors
  – number of holes in region
  – number of connected components
  – Euler number, $E = C - H$
• Texture

Statistical, structural and spectral approaches
• Statistical texture on grey-level histogram of image or region

\[ z: \text{random variable denoting gray-levels} \]
\[ p(z_i) :(\text{normalized})\text{gray-level histogram, } i=0,1,...,L-1 \]

\[ L: \text{number of gray-levels} \]

- Statistical moments
- (Uniformity, entropy measures)

A drawback with using the histogram for texture analysis is that no positional information is used. Only gives information such as smooth or coarse not if "patterned".
• Statistical texture based on information about relative position

Co-occurrence matrix:
For an image with N gray-levels, and P, a positional operator, generate A, a NxN matrix, where \( a_{ij} \) is the number of times a pixel with gray-value \( z_i \) is in relative position P to gray-value \( z_j \).
Divide all elements in A with the sum of all elements in A (the number of times P was satisfied in the image). This gives a new matrix C where \( c_{ij} \) is the probability that a pair of pixels fulfilling P has gray-values \( z_i \) and \( z_j \) is called the co-occurrence matrix.

Descriptors derived from C can for example be: maximum value, uniformity, entropy (randomness)
P = one pixel down

1st row: pixel value 0 in pos P to other values
2nd row: pixel value 1 in pos P to other values
3rd row: pixel value 2 in pos P to other values.

A =

\[
\begin{bmatrix}
0 & 2 & 0 \\
3 & 0 & 4 \\
0 & 3 & 0 \\
\end{bmatrix}
\]

A =

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 4 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

C =

\[
\begin{bmatrix}
0 & 1/6 & 0 \\
1/4 & 0 & 1/3 \\
0 & 1/4 & 0 \\
\end{bmatrix}
\]

C =

\[
\begin{bmatrix}
1/12 & 1/12 & 1/12 \\
1/12 & 1/3 & 1/12 \\
1/12 & 1/12 & 1/12 \\
\end{bmatrix}
\]
• **Structural texture**

Describe pattern of texels with set of rules combined to a string

Rule $S \rightarrow aS$

a means circle to the right,
b means circle down,

A string $aabbaabbaa$ would give the pattern
Spectral texture

Peaks in Fourier spectrum gives information about direction and spatial period of pattern.
Principal components for description (image, region or boundary)

The mean vector is \( \mathbf{m_x} = \mathbb{E}\{\mathbf{x}\} = \begin{bmatrix} \text{Mean R} \\ \text{Mean G} \\ \text{Mean B} \end{bmatrix} = \frac{1}{K} \sum_{k=1}^{K} (\mathbf{x}_k - \mathbf{m_x} \mathbf{m}_x^T) \) K: no. pixel vect.

First pixel can be written as \( \mathbf{x}_{0,0} = \begin{bmatrix} R_{0,0} \\ G_{0,0} \\ B_{0,0} \end{bmatrix} \)

Covariance matrix \( \mathbf{C_x} = \mathbb{E}\{ (\mathbf{x} - \mathbf{m_x})(\mathbf{x} - \mathbf{m_x})^T \} = \frac{1}{K} \sum_{k=1}^{K} (\mathbf{x}_k \mathbf{x}_k^T) - \mathbf{m_x m}_x^T \)
\( \mathbf{C}_x \) is real and symmetric

\( \mathbf{e}_i \) and \( \lambda_i \) are the eigenvectors and corresponding eigenvalues. \((\mathbf{C}\mathbf{e}_i = \lambda_i \mathbf{e}_i)\)

\( \mathbf{A} \) is a matrix with the eigenvectors as rows, ordered corresponding to decreasing eigenvalue.

Use \( \mathbf{A} \) to transform \( \mathbf{x} \) to \( \mathbf{y}: \mathbf{y} = \mathbf{A}(\mathbf{x} - \mathbf{m}_x) \). This is called the hotelling transform or Principal component transform.

Covariance matrix \( \mathbf{C}_y = \mathbf{A}\mathbf{C}_x\mathbf{A}^T \) = diagonal matrix with eigenvalues on the diagonal.

Any vector \( \mathbf{x} \) can be recovered from \( \mathbf{y} \) by: \( \mathbf{x} = \mathbf{A}^T\mathbf{y} + \mathbf{m}_x \) and approximated by only using some (say \( k \)) of the eigenvalues and an \( \mathbf{A}_k \) matrix constructed from the \( k \) eigenvectors.

This gives a possibility to store the data more efficiently. For a 2-D image the hotelling or principal component transform is used to align an object or boundary according to its eigenaxes. This removes rotational effects, and the eigenvalues can be used for size normalization. Translational effects are also removed since the object is centered around its mean.