Segmentation

Appendix
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The Hough transform for finding straight lines uses the formula for the straight line \( y = ax + b \), but expressed for the \( a \) & \( b \) parameter space,

\[ b = -ax + y \]

In the image to the right there are six non-zero pixels, such that:

- \((2,0)\) give line \( b = -2a \)
- \((1,1)\) give line \( b = -a + 1 \)
- \((0,2)\) give line \( b = 2 \)
- \((2,2)\) give line \( b = -2a + 2 \)
- \((3,3)\) give line \( b = -3a + 3 \)
- \((4,4)\) give line \( b = -4a + 4 \)
Parameter space

The six straight lines in the parameter space will cross each other IF they are connected in a line. Discrete artifacts may create spurious responses. This time we select the two strongest responses in the a,b parameter space at positions

(a,b) = (-1,2)
(a,b) = (1,0)

Which corresponds to two lines in the x,y image space,

y = -x + 2
y = x
Lines in image space

Input image

Output image
Hough transform for circles, with radius 1

The equation for a circle in the x,y space is given by

\[(y-a)^2 + (x-b)^2 = r^2\]

where \((a,b)\) is the center of the circle.

The non-zero pixels give rise to the following equations,

- \((0,2)\) give the circle \((0-a)^2 + (2-b)^2 = 1\)
- \((1,1)\) give the circle \((1-a)^2 + (1-b)^2 = 1\)
- \((1,3)\) give the circle \((1-a)^2 + (3-b)^2 = 1\)
- \((2,2)\) give the circle \((2-a)^2 + (2-b)^2 = 1\)
- \((3,5)\) give the circle \((3-a)^2 + (5-b)^2 = 1\)
- \((4,4)\) give the circle \((4-a)^2 + (4-b)^2 = 1\)
- \((4,6)\) give the circle \((4-a)^2 + (6-b)^2 = 1\)
- \((5,5)\) give the circle \((5-a)^2 + (5-b)^2 = 1\)
In the parameter space the eight non-zero pixels represent the center of eight circles with radius 1. Drawing the circles on top of each other will yield high responses where the circles intersect. In this case we pick the two strongest signals, 

\[(a,b) = (1,2)\]
\[(a,b) = (4,5)\]

which correspond to two center points for circles with the radius \(i\) in the \(x,y\) space.
Circles in image space

Original image

Output image