Lecture 6,
Morphological image processing I
GW 9
Patrick Karlsson
Suggested problem: 9.7 a
Sketch the result of A first eroded by B4 and then dilated by B2

Morphology—form and structure
mathematical framework used for:
• pre-processing
• noise filtering, shape simplification, ...
• enhancing object structure
• skeletonization, convex hull...
• segmentation
• quantitative description
• area, perimeter, ...

Some set theory
A is a set in \( \mathbb{Z}^2 \):
If \( a = (a_1, a_2) \) is an element in A: \( a \in A \).
If \( a = (a_1, a_2) \) is not an element in A: \( a \notin A \).
empty set: \( \emptyset \)
set specified using \( \{ \} \), e.g., \( C = \{ w | w = -d, \text{ for } d \in D \} \)
every element in A is also in B (subset): \( A \subseteq B \)
union of A and B:
\( C = A \cup B = \{ c | c \in A \text{ or } c \in B \} \)
intersection of A and B:
\( C = A \cap B = \{ c | c \in A \text{ and } c \in B \} \)
disjoint/mutually exclusive: \( A \cap B = \emptyset \)

Logical operations
• pixelwise combination of images (AND, OR, NOT, XOR)

Some more set theory
complement of A: \( A^c = \{ w | w \notin A \} \)
difference of A and B:
\( A - B = \{ w | w \in A, w \notin B \} = A \cap B^c \)
reflection of A:
\( \hat{A} = \{ w | w = -a, \text{ for } a \in A \} \)
translation of A by a point \( z = (z_1, z_2) \):
\( (A)_z = \{ c | c = a + c, \text{ for } a \in A \} \)

structuring element (SE)
• small set to probe the image under study
• for each SE, define origo
  – SE in point \( p \): origo coincides with \( p \)
• shape and size must be adapted to geometric properties for the objects

Patrick Karlsson
basic idea

- in parallel for each pixel in binary image:
  - check if SE is "satisfied"
  - output pixel is set to 0 or 1 depending on used operation

how to describe SE

- many different ways!
- information needed:
  - position of origo for SE
  - positions of elements belonging to SE

- line segment (origo is not in SE)
- line segment (origo is not in SE)
- pair of points (separated by one pixel)

Five binary morphological transforms

- Erosion
- Dilation
- Opening
- Closing
- Hit-or-Miss transform

Erosion (shrinking)

does the structuring element fit the set?

erosion of a set X by structuring element B, $\varepsilon_B(X)$:
all $x$ in X such that $B$ is in X when origin of $B=x$

$$\varepsilon_B(X) = \{ x \mid B_x \subseteq X \}$$

Gonzalez-Woods:

$$X_\Theta B = \{ x \mid (B)_x \subseteq X \}$$

shrink the object

Dilation (growing)

does the structuring element hit the set?

dilation of a set X by structuring element B, $\delta_B(X)$:
all $x$ in X such that the reflection of $B$ hits X when origin of $B=x$

$$\delta_B(X) = \{ x \mid (\hat{B})_x \cap X \neq 0 \}$$

Gonzalez-Woods:

$$X \otimes B = \{ x \mid (\hat{B})_x \cap X \neq 0 \}$$

grow the object
Example: dilation

\[
\text{SE} = \begin{bmatrix}
1 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 1
\end{bmatrix}
\]

Duality

Erosion and dilation are dual with respect to complementation and reflection:

\[
(A \Theta B)^c = A^c \oplus B^c
\]

Useful

- **Erosion**
  - Removal of structures of certain shape and size, given by SE (structure element)
    
    Example: 3x3 SE

- **Dilation**
  - Filling of holes of certain shape and size, given by SE
    
    Example: 3x3 SE

Input:

- Squares of size 1x1, 3x3, 5x5, 7x7, 9x9, and 15x15 pixels

Erosion result:

SE = square of size 13x13

Dilation of erosion result:

SE = square of size 13x13

Figure 9.7
Use dilation to bridge gaps of broken segments.

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "98" as 1998 rather than the year 2008.

combining erosion and dilation

WANTED: remove structures / fill holes without affecting remaining parts

SOLUTION: combine erosion and dilation (using same SE)

- Opening
- Closing

opening

erosion followed by dilation, denoted •

\[ A \circ B = (A \oplus B) \ominus B \]

- eliminates protrusions
- breaks necks
- smooths contour

Example opening

opening: roll ball (=SE) inside object

see B as a "rolling ball"
boundary of \( A \) = points in \( B \) that reaches closest to \( A \) boundary when \( B \) is rolled inside \( A \)

\[ \text{fig 9.8} \]

closing

dilation followed by erosion, denoted •

\[ A \bullet B = (A \oplus B) \ominus B \]

- smooth contour
- fuse narrow breaks and long thin gulfs
- eliminate small holes
- fill gaps in the contour
Example closing (fill in!)

Closing: roll ball(=SE) outside object

boundary of $A \circ B = \text{points in } B \text{ that reaches closest to } A \text{ boundary when } B \text{ is rolled outside } A$

Fill in true border after closing with ball as SE

hit-or-miss transformation (HMT)

hit-or-miss transformation (HMT)

find location of one shape among a set of shapes

\[ A \otimes B = (A \Theta B_1) \cap (A^c \Theta B_2) \]

composite SE: object part ($B_1$) and background part ($B_2$)

does $B$, fits the object while, simultaneously, $B_2$ misses the object, i.e., fits the background?

Example: Hit-or-Miss transform (HMT)

Search for: $B_1$, $B_2$

basic morphological algorithms

use erosion, dilation, opening, closing, hit-or-miss for

- boundary extraction
- region filling
- extraction of connected components (labelling)
- defining the convex hull
**boundary extraction**

\[ \beta(A) = A - (A \Theta B) \]

**region filling**

fill a region \( A \) given its boundary \( \beta(A) \)

\[ x = x_0 \text{ is known and inside } \beta(A) \]

\[ X_k = (X_{k-1} \oplus B) \cap A^c, k = 1, 2, 3, ... \]

continue until \( X_k = X_{k-1} \)

filled region \( A - X_k \)

use to fill holes!

**CONDITIONAL DILATION**

**compare with removing holes using two-pass labeling algorithm**

( segmentation lecture )

**connected component labelling:**
- label the inverse image
- remove connected components touching the image border
- output = holes + original image
  \[ \rightarrow 2 \text{ scans + 1 scan (straightforward...)} \]

**mathematical morphology:**
- iterate: dilation, set intersection
  \[ \text{dependent on size & shape of the hole} \]
  \[ \text{needed: initialization!!} \]

**convex hull**

- convex region \( R \)
  - for any \( x_1, x_2 \in R \), straight line between \( x_1 \) and \( x_2 \) is in \( R \)
- convex hull \( H \) of a region \( R \)
  - smallest convex set containing \( R \)
- convex deficiency \( D = H - R \)

**to compute convex hull**

\[ X_k^i = (X_{k-1} \ominus B^i) \cup A, i = 1, 2, 3, 4, k = 1, 2, 3, ... \]

\[ X_0^i = A \]

converges to \( D^i \) \( (X_k \cap X_{k-1}) \)

\[ \text{CONDICAL DILATION} \]
Alternative approach (fill in!)

• Count the number of neighbors of a pixel, if more than three, mark the pixel! Gives octagon instead of rectangle (by Gunilla Borgefors, CBA)