Chapter 7 (only briefly) in Gonzales-Woods.
There is no assignment for this lecture.
Constant Spectra

- Stationary signal

\[ x(t) = \cos(5t \cdot 2\pi) + \cos(10t \cdot 2\pi) + \cos(20t \cdot 2\pi) + \cos(50t \cdot 2\pi) \]

- Signal in spatial domain

- Fourier transform of signal

(Images taken from http://users.rowan.edu/~polikar/WAVELETS/WTtutorial.html)
Time Varying Spectra

- Non-stationary signal
- Signal in spatial domain
- Fourier transform of signal
Frequency Components

- Stationary signal
- Non-stationary signal

Both signals constitute of the same frequency components.
The signals are undistinguishable in the Fourier domain.
Fourier transform is not suitable for analysis of non-stationary signals.
Spectral and Temporal Information
Short-Time Fourier Transform

- Assume some portion of non-stationary signal as stationary.
- Divide signal into small segments.
- Choose windowing function \( w \) of width equal to signal segment.
- Position window at \( t = 0 \), multiply signal with window, then Fourier transform as with any other signal (\( t \) - new information).
- Position window at \( t = \ldots \) a.s.o. \( \Rightarrow \) one Fourier transform for each time step.
Short-Time Fourier Transform

Short-Time Fourier Transform (STFT)

\[ STFT_x^{(\omega)} (t', f) = \int_{t} [x(t) \omega^* (t - t')] e^{-j2\pi ft} dt \]

- STFT of signal \( \Rightarrow \) FT of signal multiplied by window function.

- Non-stationary signal

- STFT spectra
Drawback with Short-Time Fourier Transform

- Window of finite length, covers only portion of signal.
  - Narrow window $\Rightarrow$ Good time res, bad freq res.
  - Wide window $\Rightarrow$ Good freq res, bad time res.
  - Heisenberg's uncertainty principle.

Example of using a wide window.
Wavelet Transform

Continuous Wavelet Transform

\[ CWT_X^{\psi} (\tau, s) = \psi_X^{\psi} (\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi^* \left( \frac{t - \tau}{s} \right) dt \]

- Function of two variables:
  - \( \tau \) - translation.
  - \( s \) - scale (frequency\(^{-1}\)).
- \( \psi(t) \) is the transform function, and is called the **mother wavelet**.
Wavelet Transform

- Non-stationary input signal.
- Wavelet transform.
Varied Scale Shifting

- High frequency (small scale).
- Low frequency (large scale).
Wavelet Pyramid and System Block Diagram
Wavelet Pyramids and Statistics

- Gaussian (approximation) pyramid.
- Laplacian (prediction residual) pyramid.
Filter Bank and Spectrum Splitting

- Two-band filter bank for one-dimensional subband coding and decoding.
- Its spectrum splitting properties.
Wavelet Transform using Haar Basis Functions

- Wavelet transform using Haar basis functions.
- Approximations \((64 \times 64, 128 \times 128 \text{ and } 256 \times 256)\) that can be obtained from wavelet transform.
- Analysis filter bank.
- Resulting decomposition.
- Synthesis filter bank.
Wavelets in MATLAB

1. Start MATLAB.
2. Run `wavemenu` (part of wavelet toolbox).
3. Select Wavelet-2D.
4. Load an image from 
   
   /it/sw/matlab/current/... 
   .../toolbox/wavelet/wavedemo/*.*.mat.
5. Experiment with different families, types, levels and results of different compression levels.
6. Look at the families of wavelets by going to Display → Wavelet Display.