Reading Instructions

Chapters for this lecture

- Chapter 9 in Gonzales-Woods.
Previous Lectures

Image processing
- Point processing (Spatial domain, pixel-wise)
- Local neighborhoods (Spatial domain, filtering)
- Fourier transform (Frequency domain, filtering)
- Morphological image processing

Image analysis
- Segmentation (easier said than done!)

Grayscale morphology
Dilation
Grayscale morphology

Figure 9.27(d) in GW is incorrect

- The result of the dilation of \( f(x) \) with the suggested structuring element \( g(x) \) is shown by the dotted line to the right.
- Compare with the solid line in fig. 9.27(d) in GW.
- Notice the difference at the peak of the dilated function.

Grayscale Dilation

Choose maximum value \( f + b \)

**Effect**

- \( b \) with positive elements \( \rightarrow \) (often) brighter output.
- Dark details are reduced or eliminated.
Grayscale Erosion
Choose minimum value $f - b$

**Effect**
- $b$ with positive elements $\rightarrow$ (often) darker output.
- Bright details are reduced.

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Grayscale morphology
Opening and closing

generical interpretation
1D

opening

closing

fig 9.30
Grayscale morphology
Opening and closing

Effects of opening
- Remove small bright details.
- Leave overall gray levels.
- Leave larger bright features.

Effects of closing
- Remove dark details.
- Leave bright features.

Grayscale morphology
Examples

original
dilation
erosion
closing
opening
Properties

**Binary**
- Duality with respect to complementation and reflection:
  \[(A \bullet B)^C = (A^C \circ \hat{B})\]
- Opening
  - \(A \circ B\) subset/image of \(A\).
  - \(C \subseteq D \rightarrow C \circ B \subseteq D \circ B\)
  - \((A \circ B) \circ B = A \circ B\)
- Closing
  - \(A \circ B\) subset/image of \(A\).
  - \(C \subseteq D \rightarrow C \circ B \subseteq D \circ B\)
  - \((A \circ B) \circ B = A \circ B\)

**Gray level**
- Duality with respect to complementation and reflection:
  \[(f \bullet b)^C = f^C \circ \hat{b}\]
- Opening
  - \(f \circ b\) if
  - if \(f_1 \downarrow f_2\), then \((f_1 \circ b) \downarrow (f_2 \circ b)\)
  - \((f \circ b) \circ b = f \circ b\)
- Closing
  - \(f \downarrow (f \bullet b)\)
  - if \(f_1 \downarrow f_2\), then \((f_1 \bullet b) \downarrow (f_2 \bullet b)\)
  - \((f \bullet b) \bullet b = f \bullet b\)
  - \(e \uparrow r\): domain of \(e\) is subset of domain of \(r\) and \(e(x, y) \leq r(x, y)\) in domain of \(e\).

Application

**Smoothing**
Can be achieved by opening followed by closing \(\rightarrow\) Removal or attenuation of bright and dark artifacts/noise.
Application

Gradient image
Subtract eroded image from the dilated.

\[ g = (f \oplus b) - (f \ominus b) \]

- Similar to boundary extraction for binary case (morphology).
- Compare with edge detectors, ...
- Rather direction dependent.

Distance transforms

Input: Binary image.
Output: In each object (background) pixel, write the distance to the closest background (object) pixel.

Definition

A function \( D \) is a metric (distance measure) for the pixels \( p, q, \) and \( z \) if

- \( a \) \( D(p, q) \geq 0 \) (\( D(p, q) = 0 \) iff \( p = q \))
- \( b \) \( D(p, q) = D(q, P) \)
- \( c \) \( D(p, z) \leq D(p, q) + D(q, z) \)
Different metrics (fill in!)

Euclidean \( D_E(p, q) = \sqrt{x^2 + y^2} \).
City block \( D_4(p, q) = |x| + |y| \).
Chessboard \( D_8(p, q) = \max(|x|, |y|) \).

*Weighted measures:*
Chamfer(3-4) since \( 4/3 \approx 1.33 \).

\[
\begin{array}{ccc}
4 & 3 & 4 \\
3 & p & \\
\end{array}
\]

Chamfer(5-7-11) since \( 7/5 = 1.4 \), and \( 11/5 = 2.2 \).

Algorithm for distance transformation

Distance from each object pixel to the closest background pixel

- \( p \) current pixel
- \( g_1 \) – \( g_4 \) neighboring pixels
- \( w_1 \) – \( w_4 \) weights (according to choice of metric)

1. Set background pixels to zero and object pixels to infinity (or maximum intensity, e.g., 255).
2. Forward pass, from \((0, 0)\) to \((\max(x), \max(y))\):
   - if \( p > 0 \), \( p = \min(g_i + w_i) \), \( i = 1, 2, 3, 4 \).

\[
\begin{array}{ccc}
w_1 & w_2 & w_3 \\
w_4 & p & \\
\end{array}
\]

3. Backward pass, from \((\max(x), \max(y))\) to \((0, 0)\):
   - If \( p > 0 \), \( p = \min(p, \min(g_i + w_i)) \), \( i = 1, 2, 3, 4 \).

\[
\begin{array}{ccc}
p & w_2 & w_3 \\
w_1 & w_4 & \\
\end{array}
\]
Chamfer (3 – 4) distance

Binary original image

1. Starting image
Chamfer \((3 - 4)\) distance

2. First pass from top left down to bottom right

\[
\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 3 & 3 & 0 & 3 & 0 & 0 & 0 \\
0 & 3 & 4 & 3 & 4 & 3 & 3 & 0 \\
0 & 3 & 6 & 6 & 7 & 6 & 4 & 0 \\
0 & 3 & 6 & 9 & 10 & 8 & 4 & 0 \\
0 & 0 & 0 & 3 & 6 & 8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Chamfer \((3 - 4)\) distance

3. Second pass from bottom right down to top left

\[
\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 3 & 3 & 0 & 3 & 0 & 0 & 0 \\
0 & 3 & 4 & 3 & 4 & 3 & 3 & 0 \\
0 & 3 & 6 & 6 & 7 & 6 & 3 & 0 \\
0 & 3 & 4 & 6 & 4 & 3 & 0 & 0 \\
0 & 0 & 0 & 3 & 3 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
Applications using the distance transform (DT)

I. Find the shortest path between two points a and b.
   1. Generate the DT with a as the object.
   2. Go from b in the steepest gradient direction.

II. Find the radius of a round object
    1. Generate the DT of the object.
    2. The maximum value equals the radius.

   • Segmentation using Watershed.

Applications using the distance transform (DT)

III. Dirichlet tesselation (Voronoi diagram). Given a number of seeds $p_1, p_2, ..., p_m$, associate with each seed $p_i$ all regions closer to $p_i$ than any other seed.
    1. Generate the DT with the seeds as background.
    2. The connected maxima of the distance is the Voronoi-diagram, and the region associated with a point is the Dirichlet tesselation for the point.
Applications using the distance transform (DT)

IV. Skeletons

Definitions: If $O$ is the object, $B$ is the background, and $S$ is the skeleton, then
- $S$ is topological equivalent to $O$.
- $S$ is centered in $O$.
- $S$ is one pixel wide (difficult!).
- $O$ can be reconstructed from $S$.

Skeletons (Centers of Maximal Discs)

A disc is made up of all pixels that are within a given radius $r$. The skeleton of a binary object is the union of all disc centers needed to reconstruct the object using the corresponding discs.

Algorithm

Find the skeleton, Centers of Maximal Discs (CMD) *reversible.

1. Generate DT of object.
2. Identify CMDs (smallest set of maxima).
3. Link CMDs.

“Pruning” is to remove small branches (no longer fully reversible.)
**Skeleton**

Skeleton using Chamfer(3,4) DT, no pruning (fully reversible)

Skeletonisation based on thinning (not reversible)

Skeleton using Chamfer(3,4) DT, followed by pruning (not fully reversible)

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**Reading Instructions**

Chapters for next lecture

- Chapter 6 in Gonzales-Woods.