Today

Regional Descriptors
- Simple
- Topological
- Texture
  - Statistical
  - Spectral
- Moments

Other Descriptors
- Principal components
Simple Descriptors

**Simple Descriptors**
- Area - number of pixels in region.
- Perimeter - boundary length.
- Compactness - $P^2/A$ (circularity).
- Graylevel measures: mean, median, max etc.

Topological Descriptors

**Topological Descriptors**
- topology The study of properties of a figure that are unaffected by any deformation.
- Number of holes in region, $H$.
- Number of connected components, $C$.
- Euler number, $E = C - H$. 
Topological Descriptors
Using Connected Components in Segmentation

IR image

Thresholded image

Largest connected component

Skeleton

Texture

Figure: Smooth, coarse and regular textures

Statistical and spectral approaches.
Statistical Texture

Statistical Texture Descriptors

- Histogram based
  - Statistical moments
  - Uniformity
  - Average entropy
- Co-occurrence matrix based
  - Maximum probability
  - etc.

A drawback with using the histogram for texture analysis is that no positional information is used. Only gives information such as smooth or coarse not if “patterned”.

Histogram Based Descriptors

Statistical Moments
Properties of the graylevel histogram, of an image or region, used when calculating statistical moments.

- \( z \) - discrete random variable representing discrete graylevels in the range \([0, L - 1]\).
- \( p(z_i) \) - normalised histogram component corresponding to the \( i \)th value of \( z \).

The \( n \)th Statistical Moment (about its mean)

\[
\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i), \quad m = \sum_{i=0}^{L-1} z_i p(z_i)
\]

- \( \mu_2(z) \) - variance of \( z \) (i.e., \( \sigma^2(z) \)).
- \( \mu_3(z) \) - histogram skewness.
- \( \mu_4(z) \) - relative flatness of histogram.
Histogram Based Descriptors

Uniformity and Average Entropy

Also use the histogram properties \( z \) and \( p(z_i) \).

**Uniformity**

\[
U = \sum_{i=0}^{L-1} p^2(z_i)
\]

\( U \) is maximum for an image in which all graylevels are equal (maximally uniform).

**Average Entropy**

\[
e = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)
\]

Measure of variability. Is 0 for constant images.

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Co-occurrence Matrix

- For an image with \( N \) graylevels, and \( P \), a positional operator, generate \( A \), a \( N \times N \) matrix, where \( a_{i,j} \) is the number of times a pixel with graylevel value \( z_i \) is in relative position \( P \) to graylevel value \( z_j \).

- Divide all elements in \( A \) with the sum of all elements in \( A \) (the number of times \( P \) was satisfied in the image). This gives a new matrix \( C \), where \( c_{i,j} \) is the probability that a pair of pixels fulfilling \( P \) has graylevel values \( z_i \) and \( z_j \), which is called the co-occurrence matrix.
### Co-occurrence Matrix

Examples

\[ P = \text{“one pixel down”} \]

**Ex. 1)**

\[
\begin{array}{cccc}
2 & 0 & 1 & 0 \\
1 & 1 & 2 & 1 \\
2 & 0 & 1 & 2 \\
1 & 1 & 0 & 1 \\
\end{array}
\]

\[
A_1 = \begin{pmatrix}
0 & 2 & 0 \\
3 & 0 & 4 \\
0 & 3 & 0 \\
\end{pmatrix}, \quad C_1 = \begin{pmatrix}
0 & \frac{1}{6} & 0 \\
\frac{1}{4} & 0 & \frac{1}{3} \\
0 & \frac{1}{4} & 0 \\
\end{pmatrix}
\]

**Ex. 2)**

\[
\begin{array}{cccc}
1 & 0 & 1 & 2 \\
1 & 0 & 2 & 1 \\
0 & 1 & 2 & 1 \\
2 & 1 & 0 & 1 \\
\end{array}
\]

1\st row: pixel value 0 in pos P to other values
2\nd row: pixel value 1 in pos P to other values
3\rd row: pixel value 2 in pos P to other values.

### Co-occurrence Matrix Descriptors

**Maximum Probability (strongest response to P)**

\[ \max_{i,j} (c_{ij}) \]

**Uniformity**

\[ \sum_i \sum_j c_{ij}^2 \]

**Entropy (randomness)**

\[ - \sum_i \sum_j c_{ij} \log_2 c_{ij} \]

Characterize “content” of C using these descriptors.
### Example 1

$$\mathbf{C}_1 = \begin{pmatrix} 0 & 1 & 6 & 0 \\ 1/4 & 0 & 1/3 \\ 0 & 1/4 & 0 \end{pmatrix}$$

**Maximum probability:**  $$\max_{i,j} (c_{ij}) = \frac{1}{3}$$

**Uniformity:**  $$\sum_i \sum_j c_{ij}^2 \approx 0.264$$

**Entropy:**  $$- \sum_i \sum_j c_{ij} \log_2 c_{ij} = \text{undef.}$$

### Example 2

$$\mathbf{C}_2 = \begin{pmatrix} 1/12 & 1/12 & 1/12 \\ 1/12 & 1/3 & 1/12 \\ 1/12 & 1/12 & 1/12 \end{pmatrix}$$

**Maximum probability:**  $$\max_{i,j} (c_{ij}) = \frac{1}{3}$$

**Uniformity:**  $$\sum_i \sum_j c_{ij}^2 \approx 0.167$$

**Entropy:**  $$- \sum_i \sum_j c_{ij} \log_2 c_{ij} \approx -2.918$$
Spectral Texture

Peaks in the Fourier spectrum give information about direction and spatial period of pattern.

Express spectrum in polar coordinates $S(r, \theta)$. For each direction $\theta$, $S(r, \theta)$ is a 1D function $S_\theta(r)$. Similarly, for each frequency $r$, $S_r(\theta)$ is a 1D function.

A global description can be obtained by summing $S_\theta(r)$ and $S_r(\theta)$:

$$S(r) = \sum_{\theta=0}^{\pi} S_\theta(r), \quad S(\theta) = \sum_{r=1}^{R_0} S_r(\theta)$$

Spectral-energy description (entire region).

Spectral Texture

Example

Image

Spectrum

$S(r)$

$S(\theta)$
Moments

For a 2D continuous function $f(x, y)$, the moment of order $(p + q)$ is defined as

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) \, dx \, dy$$

for $p, q = 0, 1, 2, \ldots$. The moment sequence $(m_{pq})$ is uniquely determined by $f(x, y)$, and conversely, $(m_{pq})$ uniquely determines $f(x, y)$.

The central moments are defined as

$$\mu_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \bar{x})^p (y - \bar{y})^q f(x, y) \, dx \, dy$$

where

$$\bar{x} = \frac{m_{10}}{m_{00}} \quad \text{and} \quad \bar{y} = \frac{m_{01}}{m_{00}}.$$
A set of seven invariant moments can be derived from the second and third moments.

\[
\begin{align*}
\phi_1 &= \eta_{20} + \eta_{02} \\
\phi_2 &= (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \\
\phi_3 &= (\eta_{30} - \eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\
\phi_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\
&\ldots \text{(see textbook)}
\end{align*}
\]

The set of moments is invariant to translation, rotation, and scale change.
Principal Components

Image, region or boundary.

Each pixel represented as a 3D vector. Pixel vector and mean vector:

\[ x_{i,j} = \begin{pmatrix} R(i,j) \\ G(i,j) \\ B(i,j) \end{pmatrix}, \quad m_x = \mathbb{E}\{x\} = \begin{pmatrix} \text{mean}(R) \\ \text{mean}(G) \\ \text{mean}(B) \end{pmatrix} = \frac{1}{K} \sum_{k=1}^{K} x_k \]

Covariance matrix

\[ C_x = \mathbb{E}\{(x - m_x)(x - m_x)^T\} = \frac{1}{K} \sum_{k=1}^{K} x_k x_k^T - m_x m_x^T \]

- \( C_x \) is real and symmetric.
- \( e_i \) and \( \lambda_i \) are the eigenvectors and corresponding eigenvalues. \( (C_x e_i = \lambda_i e_i) \)
- \( A \) is a matrix with the eigenvectors as rows, ordered corresponding to decreasing eigenvalue.
- Use \( A \) to transform \( x \) to \( y \): \( y = A(x - m_x) \). This is called the Hotelling transform or principal component transform.
- Covariance matrix \( C_y = A C_x A^T = \) diagonal matrix with eigenvalues on the diagonal.
- Any vector \( x \) can be recovered from \( y \) by: \( x = A^T y + m_x \) and approximated by only using some (say \( k \)) of the eigenvalues and an \( A_k \) matrix constructed from the \( k \) eigenvectors.
Principal Components

This gives a possibility to store the data more efficiently. For a 2D image the Hotelling or principal component transform is used to align an object or boundary according to its eigenaxes. This removes rotational effects, and the eigenvalues can be used for size normalisation. Translational effects are also removed since the object is centered around its mean.