Why Representation and Description

- Image acquisition
- Preprocessing
- Segmentation
- Representation
- Classification

Representation and Description

Choose representation
- Boundary (external characteristics)
- Whole region (internal characteristics)

Describe region based on representation

Today

Representations
- Chain coding
- Polygonal approximations
- Signatures
- Boundary Segments
- Skeletons

Boundary Descriptors
- Simple descriptors
- Shape numbers
- Fourier descriptors
- Statistical moments
Chain Coding

- Walk around boundary and describe directional change in each step by a number.

```
1  3  2  1
0  4  0
3  5  6  7
```

Drawbacks with Chain Coding

- Long code
- Small changes on the boundary makes code change
  - Use larger grid spacing (and edit chain code to reduce noise, e.g. 0710 → 0000)
- Depends on scale
  - Chose appropriate grid spacing
- Depends on start point
  - Treat code as circular (minimum magnitude integer)
- Depends on rotation
  - Create difference code (counterclockwise)

Polygonal Approximations

- **Minimum perimeter polygons** cover boundary by cells of chosen size and force a rubber band to fit inside the cells.

Polygons Approximations

- **Merging techniques**: walk around boundary and fit a least-square-error line to the points passed until an error threshold is exceeded → Start new line → When the startpoint is reached the intersections of adjacent lines are the vertices of the polygon.
- **Splitting techniques**: divide a line segment into two until an criterion is reached. For example start by creating two lines between the two endpoints of the boundary. If the max distance from a boundary point exceeds a specified threshold, the point farthest away is a new vertex.
Signatures

- A 1D representation of the boundary.
- Independent of translation, but not rotation and scaling.
  - Select unique start point
  - Normalise amplitude of signature (divide by variance)

Examples

- Distance from centre point to the border as function of angle.
- Angle between the tangent in each point and a reference line (slope density function).

Boundary Segments

- Useful mainly when boundary contains major concavities that carry shape information.
- Convex hull (CH) and convex deficiency (CD) very useful for decomposing the border into segments. Noise sensitive though!
  - Smooth prior to convex hull calculation.
  - Calculate convex hull on polygon approximation.

Figure: Distance from centre point to the border as function of angle.
**Skeletons**

- "Curve representation" of the object.
- Should in general be thin, centred, topologically equivalent to original object and reversible.
- Can be created by thinning or by medial axis transform (MAT) (keep all centres of maximal discs).

**Definitions:**

If $O$ is the object, $B$ is the background, and $S$ is the skeleton, then

- $S$ is topological equivalent to $O$
- $S$ is centered in $O$
- $S$ is one pixel wide (difficult!)
- $O$ can be reconstructed from $S$

**Algorithm 1** from textbook

```
Require: A = binary object
repeat
  for $x, y \in \text{Contour}(A)$ do
    if $2 \leq N(p_i) \leq 6 \& \& T(p_i) == 1 \& \& p_2 \cdot p_4 \cdot p_6 == 0 \& \& p_4 \cdot p_6 \cdot p_8 == 0$ then
      Mark $p_i$ for deletion
    end if
  end for
  Delete marked points
  for $x, y \in \text{Contour}(A)$ do
    if $2 \leq N(p_i) \leq 6 \& \& T(p_i) == 1 \& \& p_2 \cdot p_4 \cdot p_6 == 0 \& \& p_4 \cdot p_6 \cdot p_8 == 0$ then
      Mark $p_i$ for deletion
    end if
  end for
  Delete marked points
  until No points deleted
```

**Skeletons (Centers of Maximal Discs)**

A disc is made of all pixels that are within a given radius $r$. The skeleton of a binary object is the union of all disc centers needed to reconstruct the object using the corresponding discs.

**Algorithm**

Find the skeleton with Centers of Maximal Discs (CMD)
Completely reversible situation :-)

- Generate DT of object
- Identify CMDs (smallest set of maxima)
- Link CMDs

"Pruning" is to remove small branches (no longer fully reversible.)
### Simple Descriptors

- **Length**
- **Area**
- Diameter = $\max_{ij} [D(p_i, p_j)] = \text{major axis}$
- Basic rectangle = major $\cdot$ minor
- Eccentricity = major / minor
- Curvature, rate of change of slope (unstable)
  - Instead: difference between slopes of adjacent boundary segments as a descriptor of the point of intersection between the two segments → convex (positive difference) or concave (negative difference) point.

### Shape Numbers

- **Shape number**
  - The shape number of first difference of chain code is the integer of smallest magnitude.
  - The order, $n$, of the shape number is the number of digits in the chain code.
  - The order limits the number of possible different shapes.

- $n = 4$
- $n = 6$
- $n = 8$
Shape Numbers

- Difference code is independent of rotation, but depends on orientation of the grid.
- Align chain code grid with basic rectangle.
  1. Choose order \( n \).
  2. Find rectangle of order \( n \), whose eccentricity best matches basic rectangle of object. This rectangle (of order \( n \)) establishes grid size.
  3. Check that the order of the object in the new grid equals \( n \).

\( n = 18 \)

\[ s(k) = [x(k), y(k)], \quad k = 0, 1, 2, \ldots, K - 1 \]

 Fourier Descriptors

- Useful for comparing objects. Very difficult to interpret geometrically. 2D problem reduced to 1D problem.
- Write \( x \) and \( y \) coordinates as two vectors, and use each coordinate pair as a complex number. \( \Rightarrow \) \( x \)-axis treated as real axis and \( y \) as imaginary axis.
- Fourier transforming the new coordinates generates the Fourier descriptors.
- Inverse transforming all these descriptors regenerates the original coordinates. If only some of the descriptors are used in the inverse transform an approximation of the original object is the result.
Fourier Descriptors
- From the DFT of the complex number we get the Fourier descriptors (the complex coefficients, $a(u)$).

$$a(u) = \frac{1}{K} \sum_{k=0}^{K-1} s(k)e^{-j2\pi uk/K}, \quad u = 0, 1, 2, \ldots, K - 1$$

- The inverse Fourier transform of these coefficients restores $s(k)$.

$$s(k) = \sum_{u=0}^{K-1} a(u)e^{2\pi uk/K}, \quad k = 0, 1, 2, \ldots, K - 1$$

- We can create an approximate reconstruction of $s(k)$ using only the first $P$ Fourier coefficients.

$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u)e^{2\pi uk/K}, \quad k = 0, 1, 2, \ldots, K - 1$$

This boundary consists of 64 points, $P$ is the number of descriptors used in the reconstruction of the boundary.

Statistical Moments
- Useful for describing the shape of boundary segments (or other curves).
- For example, the shape of convex deficiencies can be described by moments.

The histogram of the function (segment curve) can also be used for calculating moments.
- 2nd moment gives spread around mean (variance).
- 3rd moment gives symmetry around mean (skewness).
Statistical Moments as Boundary Descriptor

- $v$ - discrete random variable representing discrete amplitude in the range $[0, A - 1]$.
- $p(v_i)$ - estimate of the probability of value $v_i$ occurring.

The $n$th Statistical Moment of $v$ (about its mean)

$$
\mu_n(v) = \sum_{i=0}^{A-1} (v_i - m)^n p(v_i), \quad m = \sum_{i=0}^{A-1} v_i p(v_i)
$$

$\mu_2(v)$ - spread around mean (variance).
$\mu_3(v)$ - symmetry around mean (skewness).

Example

$$
A = 6, \quad v_i = \{0, 1, 2, 3, 4, 5\}
$$

$$
\begin{align*}
\mu_2(v) &= (0 - \frac{5}{2})^2 \cdot \frac{1}{4} + (2 - \frac{5}{2})^2 \cdot \frac{3}{8} + (4 - \frac{5}{2})^2 \cdot \frac{1}{8} + (5 - \frac{5}{2})^2 \cdot \frac{1}{4} = 3.5
\end{align*}
$$