# Computer Assisted Image Analysis II 

## VT 2015

Exam 2015-03-11

Time: 08:00-13:00
Place: Polacksbacken, Skrivsal
Tools: Pen or pencil and paper, 1 hand-written sheet with notes
Grades: 0-17 fail
18-24 3
25-32 4
33-40 5

OBS: Please use drawings and figures to illustrate your answers when suitable. Answer each question on a different sheet. Make sure that you write your exam code (or name) on each page, and remember that the top left corner will be hidden by the staple. It is OK to use both front and back sides of the paper. You may write your answers in English or Swedish, but not with red ink.

Results Will be forwarded to the student office no later than March 25.

## 1. Mathematical morphology (Ida-Maria)

The image below shows an electron microscopy image (2048 x2048 pixels) of virus vectors. A closeup of part of the image (width=144 pixels) is also shown. When manufacturing virus vectors it is important that they are homogenous.
A virus vector is a modified virus particle used to deliver material (drugs/genetic material) into cells.
a) Describe the tophat algorithm and if/how it would be useful for the image below.
b) Describe how granulometry could be used for the image below to get information about the size distribution.
c) How does preprocessing using the tophat algorithm affect the result of b?

For a) and b) describe the mathematical operations used and describe and motivate what shape and size of SE you would use.


## 2. Computer vision (Anders)

a) Derive the basic projection equation for a pinhole camera, from 3-D to 2-D. You do not need to use projective geometry, but it should include the concept of focal length.
b) A person holds up a paper 0.5 m in front of his/her head. On this paper he/she draws (projects) the contour of an object, which in reality is about 2 m in diameter and positioned 10 meters away from the persons head (along the optical axis). How big will this object appear on the paper? The object is drawn so that if the paper is removed, the contour of the drawn object overlaps with the contour of the real object as seen from the observing persons position.
c) Write down and prove the equations for stereo estimation using disparity. A 1-D example is enough, after all stereo estimation is often done along scan-lines.

## 3. Measurement and Stereology (Cris)

You are to analyse a tumour. You section it and take an image. The tumour is larger than fits in the field of view of your microscope, so the image shows a fraction of the 2D section.
a) You want to obtain a size distribution of the cells in the tumour. Does the section provide enough information?
b) You assume the cells are perfect spheres, and use a statistical method to convert the estimated distribution of diameters of circles in the section to a distribution of diameters of spheres in the volume. Enumerate and discuss three possible sources of bias in this method. Can you overcome these?
c) How would you go about to estimate the distribution of diameters in the section (i.e. you need to measure the diameter of the circular cross-sections of the cells you see in the image). Discuss how you pick which cells to measure, how you measure them, etc.

## 4. Digital geometry (Robin)

a) Give two arguments for using the hexagonal grid instead of the square grid.

Assume that the leftmost pixels of the digital straight lines in the figure below all have coordinate $(x, y)=(0,0)$. The digital straight line a can be defined by all points $(x, y)$ that satisfy the diophantine inequality $0<=(6 / 15) x-y<1$.
b) Diophantine equations can be used to define digital straight lines. Give two other digital straight line definitions.
c) What are the parameters $K$ and $b$ in the Diophantine equation $0<=K x-y+b<1$ for the digital straight line $b$ in the figure?
c) Give a definition of digital straight lines on the hexagonal grid. (Hint: Adjust the definition for digital straight lines on the square grid to the hexagonal grid.)


## 5. Fuzzy sets (Natasa)

a) The fuzzy c-means method uses a fuzzy representation of objects. What is characteristic for such a representation? In what way is it different from a crisp representation?
b) Explain the difference between k-means clustering and fuzzy c-means clustering.
c) Let $X=[0,1] \subset \mathrm{R}, \mu_{A}(x)=x^{2}, \mu_{B}(x)=1-x, A=\left\{\left(x, \mu_{A}(x)\right) \mid x \in X\right\}$ and $B=\left\{\left(x, \mu_{B}(x)\right) \mid x \in X\right\}$. What is $A \cap B$, $A \cup B$ and $A^{c}$ ?

## 6. Segmentation (Cris)

For each of the images below, describe a method that might be able to segment the relevant objects. Include necessary pre-processing. Give your reasoning in detail for choosing each method.

(a) Gel matrix, find each of the bubbles.

(b) Pills, find each of the pills, separate from background.

## 7. Registration (Anders)

a) Name three different functionals that measure pixel-wise image similarity and explain how they perform compared to each other.
b) Demonstrate with a simple example in 1-D (i.e. the images are 1-D functions) how local minima (maxima) arise in registration and give some plausible solutions on how to deal with them.

## 8. Filtering (Cris)

The linear filter with uniform weights computes the mean of the values of the pixels in a neighbourhood. The mean is a weighted $L_{1}$ norm. The dilation computes the maximum of the values of the pixels in a neighbourhood. The maximum is the $L_{\infty}$ norm. Given $N$ values $x_{i}$, the norms are defined by

$$
L_{p}=\left(\sum_{i=1}^{N} x_{i}^{p}\right)^{1 / p}
$$

a) What properties would a filter have that computes the $L_{2}$ norm of the pixel values? (Assume $x_{i} \geq 0$.) Think about what happens in a region with uniform intensities and Gaussian noise, what happens at the border of an object, etc.
Let's move a little closer to $p=\infty$, and pick $p=100$. For this value of $p, L_{p}$ is close to the maximal value of $x_{i}$. The filter will produce a result similar, but not identical, to the dilation.

The rank filter with rank $N-1$ sorts the input values and picks the $N-1$ value, the next to largest one (this is equivalent to the percentile filter with a parameter $100(N-1) / N$ ). This filter also produces a result similar to that of the dilation.
b) How do these two filters differ from the dilation? When would you pick one of these filters over the dilation?
c) How do these two filters differ from each other? What are the benefits and drawbacks of each of them?

