Digital geometry
Representations of objects
Digital Geometry

“The geometry of the computer screen”
The elements are points with integer coordinates.

Which primitives do we use?
Grids (2D)

Square grid

Hexagonal grid
Tessellation

A *tessellation* of the plane is a collection of plane figures that fills the plane with no overlaps and no gaps.

- A tessellation of triangles
- A tessellation of squares
- A tessellation of hexagons
Voronoi diagram
(giving dirichlet tessellation)
of a discrete set of points (generating points):
partition the plane into cells so that each cell contains exactly one generating point and the locus of all points which are nearer to this generating point than to other generating points.
Picture elements (pixels)
Voronoi regions

Square grid

Hexagonal grid
Connectivities (2D)

4-connectedness
objects connected through edge-neighbors

8-connectedness
objects connected through edge- and vertex-neighbors

6-connectedness
for hexagonal grid
Connectivities (3D)

Three different neighborhoods of a voxel

6-connectedness
face neighbors

18-connectedness
face and edge neighbors

26-connectedness
face, edge, and vertex neighbors
Connectivity – object and background

connected object components \((O_i)\)

the object \(O\) is the union of all \(O_i\)

the complement of \(O\) \((O^c)\) consists of

background

connected to border of image / image limits

holes
Connectivity paradox

Euclidean geometry:
A closed (simple) curve divides the plane into two (distinct) connected components.

Digital geometry:

How many background components?!?
Connectivity paradox: connected or intersecting lines?

Solution 1
Use 4-connectedness for background and 8-connectedness for object (or vice versa)
Connectivity paradox: connected or intersecting lines?

Solution 2
Use hexagonal grid
Connectivity paradox: connected or intersecting lines?

Solution 3
Use cellular complexes:

Elements of dimension 0 (point), 1 (line), and 2 (area) are used.
Connectivities

Connectivity paradox also for 3D images

Solution 1:

26-connectedness for object
6-connectedness for background

or

26-connectedness for background
6-connectedness for object
Connectivities

Connectivity paradox also for 3D images

Solution 2:

Cellular complexes
Connectivities

Connectivity paradox also for 3D images

Solution 3:

Use other grids

face-centered cubic grid (FCC)  body-centered cubic grid (BCC)
Digital Geometry

“The geometry of the computer screen”
The elements are points with integer coordinates.

Different from the Euclidean geometry.

Example: What is a straight line?
Euclidean Geometry

What is a straight line?

Intuitively:

• A curve traced by a point traveling in a constant direction
• A curve of zero curvature
• The distance between two points is the length of the straight line segment between the points
Digital Geometry

What is a straight line?

A set of pixels is a (simple) path if it is connected, and all but two of its points (the “endpoints”) have exactly two neighbors in the set, while those two have exactly one.
Digital Geometry

In Euclidean geometry:
“The distance between two points is the length of the straight line segment between the points”

What about digital geometry?

“City-block distance”
Let $pq$ denote the Euclidean straight line segment between $p$ and $q$. $pq$ lies near a digital object $S$ if, for any (real) point $(x,y)$ of $pq$, there exists a grid point $(i,j)$ of $S$ such that
\[
\max (|i-x|,|j-y|) < 1.
\]
Rosenfeld's Chord-property from 1974:
A digital path $S$ is a digital straight line segment if each point on a Euclidean straight line segment between any two points $p, q$ in $S$ is near $S$. 
Digital Straight Lines
Rosenfeld's Chord-property