Mathematical Morphology

Sonka 13.1-13.6 (13.5.1-13.5.6)+
(13.7 watershed segmentation)

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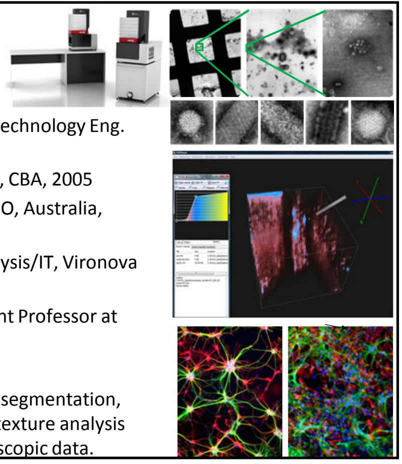
Who am I?



- MSc Molecular Biotechnology Eng. 2000
- PhD Image Analysis, CBA, 2005
- Image Analyst, CSIRO, Australia, 2005-2007
- Head of Image Analysis/IT, Vironova AB since 2007
- Researcher/Assistant Professor at CBA since 2008
- Research interests: segmentation, shape description, texture analysis in 2D and 3D microscopic data.

Today's lecture

- SE, morphological transformations
- Binary MM
- Gray-level MM
- Granulometry
- Geodesic transformations
- (Adaptive SEs)

applications

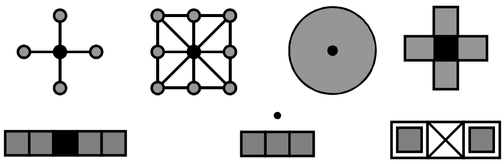
Morphology-form and structure

mathematical framework used for:

- pre-processing
  - noise filtering, shape simplification, ...
- enhancing object structure, describing shape
  - skeletonization, convex hull...
- segmentation

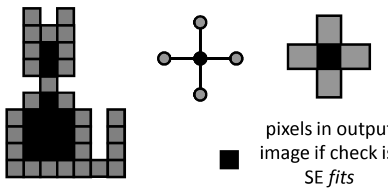
structuring element (SE)

- small set, B, to probe the image under study
- for each SE, define origo & pixels in SE
- shape and size must be suited for the geometric properties for the objects



Morphological Transformation

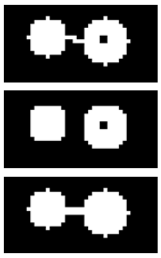
- Ψ is given by the relation of the image (point set X) and the SE (point set B).
- in parallel for each pixel (pixel under SE origo) in binary image:
  - check if SE is "satisfied"
  - output pixel is set to 0 or 1 depending on used operation



pixels in output image if check is: SE *fits*

Five binary morphological transforms

- ⊖ ε Erosion, shrinking
- ⊕ δ dilation, growing
- ○ γ opening, erosion + dilation
- ● φ closing, dilation + erosion
- ⊗ Hit-or-Miss transform

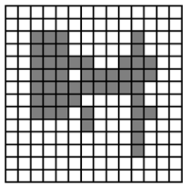


⊖ Erosion (shrinking)

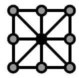
For which points does the structuring element **fit** the set?

erosion of a set X by structuring element B, ε_B(X):
all x in X such that B is in X when origin of B=x

$$X \ominus B = \varepsilon_B(X) = \{x \mid B_x \subseteq X\}$$



SE=B=

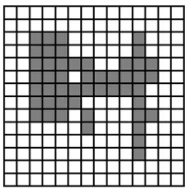


⊕ Dilation (growing)

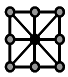
The points the SE **hits** when its origo is in the set?

dilation of a set X by structuring element B, δ_B(X):
all x such that the reflection of B hits X when origin of B=x

$$X \oplus B = \delta_B(X) = \{x \mid (\hat{B})_x \cap X \neq 0\}$$



SE= B=



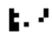
duality

erosion and dilation are dual with respect to complementation and reflection

$$(A \Theta B)^C = A^C \oplus \hat{B}$$

A

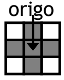
A⊖B



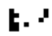
$(A⊖B)^C$

$B = \hat{B}$

origo



A^C



$A^C⊕B$

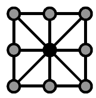
combining erosion and dilation

WANTED:
remove structures / fill holes without affecting remaining parts

SOLUTION:
combine erosion and dilation (using same SE)

- ○ Opening
- ● Closing

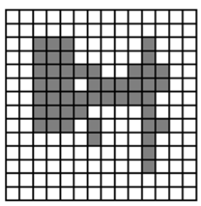
○ opening



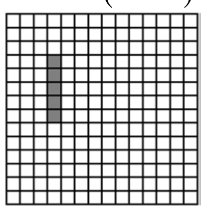
SE=B

erosion followed by dilation
eliminates protrusions, breaks necks, smoothes contours

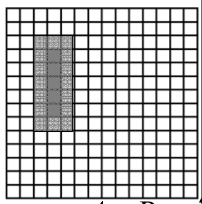
$$A \circ B = (A \Theta B) \oplus B$$



A

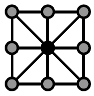


A⊖B



$A \circ B$

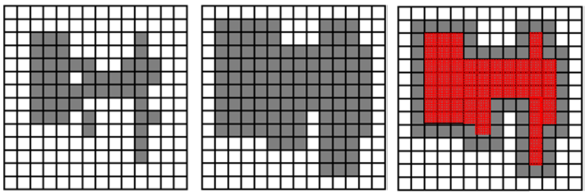
● closing



dilation followed by erosion, denoted •

Smoothes contours, fuses breaks, eliminates holes and gaps

$$A \bullet B = (A \oplus B) \ominus B$$



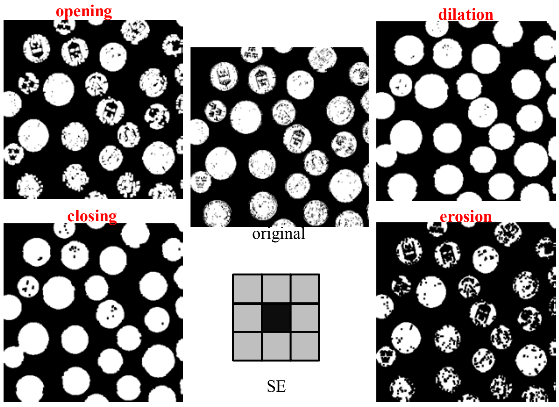
opening: roll ball(=SE) inside object

see B as a ”rolling ball”

boundary of A∘B = points in B that reaches closest to A boundary when B is rolled *inside* A

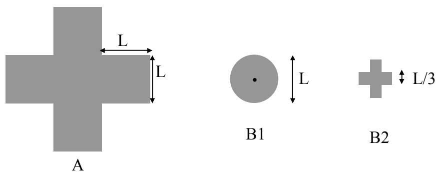
closing: roll ball(=SE) outside object

boundary of A∘B = points in B that reaches closest to A boundary when B is rolled *outside* A



Save for break exercise

- Sketch the result of A first eroded by B1 and then dilated by B2



⊗ hit-or-miss transformation (⊗,HMT)

find location of one shape among a set of shapes

”template matching”

$$A \otimes B = (A \ominus B_1) \cap (A^C \ominus B_2)$$

composite SE: object part (B_1) and background part (B_2)

does B_1 ***fit the object*** while, simultaneously, B_2 misses the object, i.e., ***fits the background***?

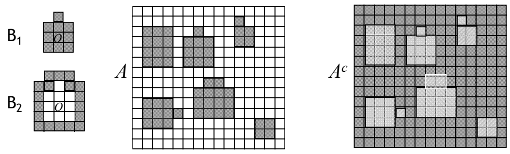
hit-or-miss transformation (⊗,HMT)

find location of one shape among a set of shapes

SE=object part B_1, and background part B_2

$$A \otimes B = (A \ominus B_1) \cap (A^C \ominus B_2)$$

$$A \otimes B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

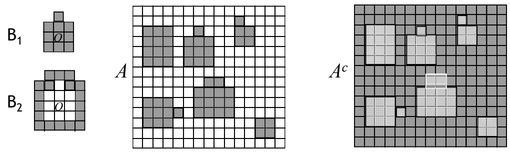


hit-or-miss transformation (⊗,HMT)

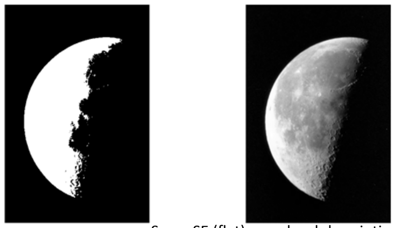
find location of one shape among a set of shapes
SE=object part B_1, and background part B_2

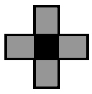
$$A \otimes B = \left(A \Theta B_1\right) \cap \left(A^C \Theta B_2\right)$$

$$A \otimes B = \left(A \Theta B_1\right) - \left(A \oplus \hat{B}_2\right)$$



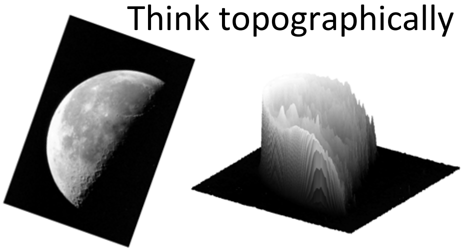
Gray-level images and SEs



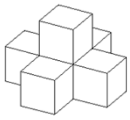


Same SE (flat), gray level description
f(0,0)=0
f(-1,0)=0
f(1,0)=0
f(0,-1)=0
f(0,1)=0 Domain(f)={(0,0),(-1,0),(1,0),(0,-1),(0,1)}

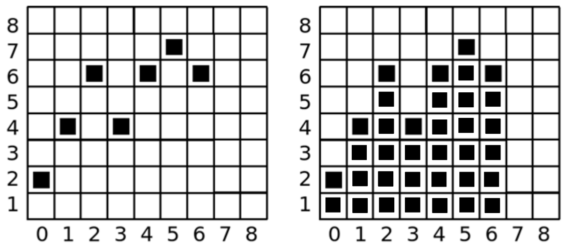
Think topographically



Gray-level SE (not flat!)
f(0,0)=1
f(-1,0)=0
f(1,0)=0
f(0,-1)=0
f(0,1)=0
Domain(f)={(0,0),(-1,0),(1,0),(0,-1),(0,1)}



Top surface & umbra



1D function f (top surface, T)

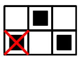
Its umbra U[f]

Umbra homeomorphism theorem

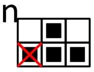
Umbra operation is a homeomorphism from grayscale morphology to binary morphology

f⊕ b=T{U[f]⊕U[b]}

Gray-scale umbra erosion

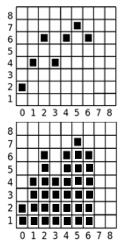


B

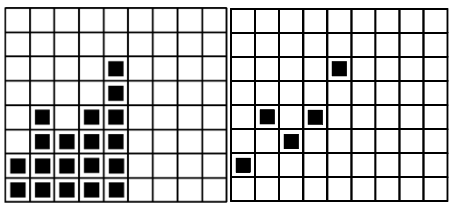


U[B]

Gray scale erosion of two functions as (binary) erosion of umbras

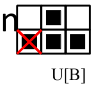


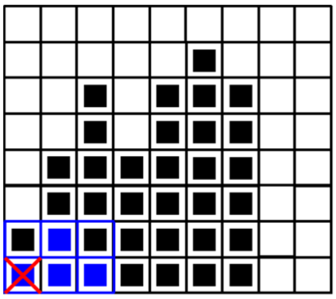
Original f and U(f)



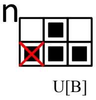
$\varepsilon_{U(B)}(U(f))$ $\varepsilon_B(f)$

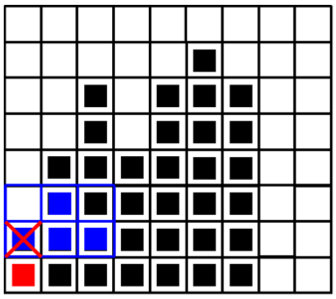
Gray-scale umbra erosion



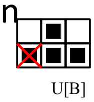


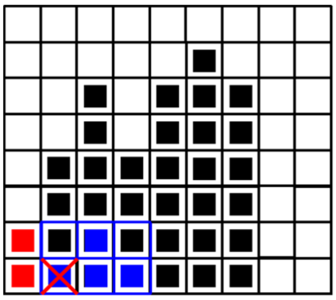
Gray-scale umbra erosion



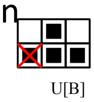


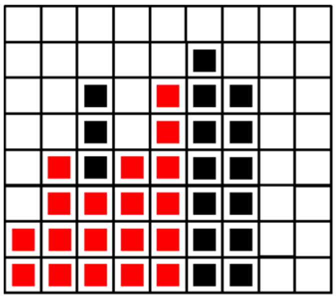
Gray-scale umbra erosion





Gray-scale umbra erosion





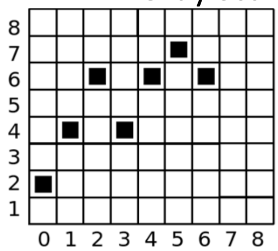
Gray-scale Morphological erosion

B

$$[\varepsilon_B(f)]\,(x) = \min_{b \in \mathcal{D}_B} \{f(x+b) - B(b)\}$$

domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

Gray scale erosion



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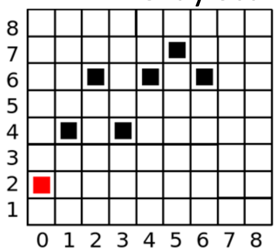
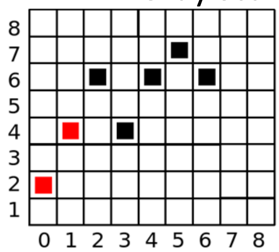


domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

$$[\varepsilon_B(f)](x) = \min_{b \in \mathcal{D}_B} \{f(x+b) - B(b)\}$$

$[\varepsilon_B(f)]$ (0) = min(2-0,4-1,6-0)=2

Gray scale erosion

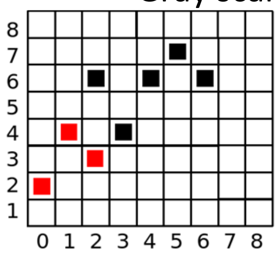


domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

$$[\varepsilon_B(f)](x) = \min_{b \in \mathcal{D}_B} \{f(x+b) - B(b)\}$$

$[\varepsilon_B(f)]$ (1) = min(4-0,6-1,4-0)=4

Gray scale erosion

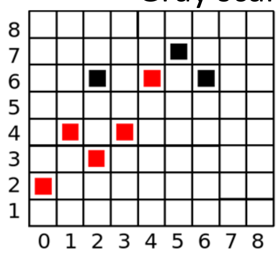


domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

$$[\varepsilon_B(f)](x) = \min_{b \in \mathcal{D}_B} \{f(x+b) - B(b)\}$$

$[\varepsilon_B(f)]$ (2) = min(6-0,4-1,6-0)=3

Gray scale erosion

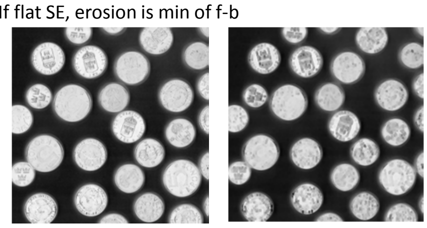


domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

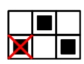
$$[\varepsilon_B(f)](x) = \min_{b \in \mathcal{D}_B} \{f(x+b) - B(b)\}$$

Example, gray-scale erosion flat SE, square 3x3

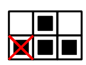
- b with positive elements → darker output
- bright details are reduced
- If flat SE, erosion is min of f-b



Gray scale dilation

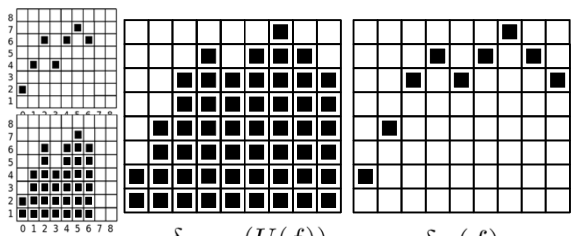


B



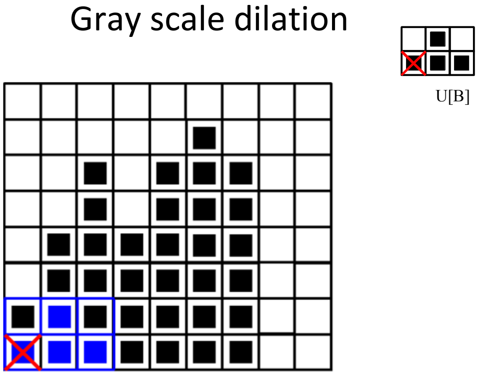
U[B]

Gray scale dilation of two functions as (binary) dilation of umbras

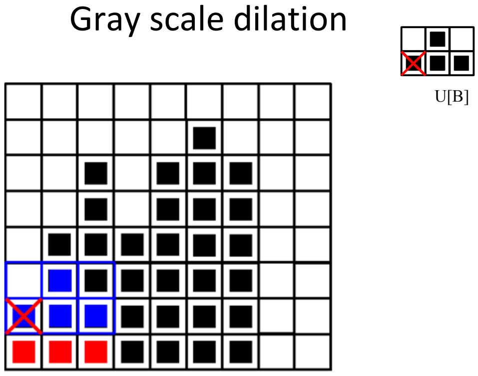


Original f and U(f) $\delta_{U(B)}(U(f))$ $\delta_B(f)$

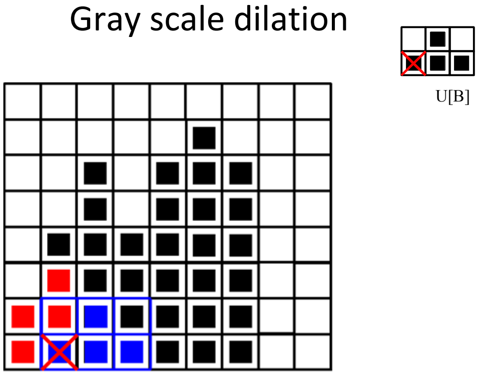
Gray scale dilation



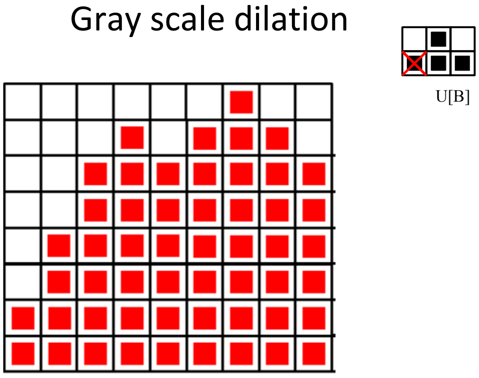
Gray scale dilation



Gray scale dilation

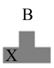


Gray scale dilation



Gray-scale morphological dilation

$$[\delta_B(f)]\,(x) = \max_{b \in \mathcal{D}_B} \{f(x-b) + B(b)\}$$



domain(B)={(0,1,2)}
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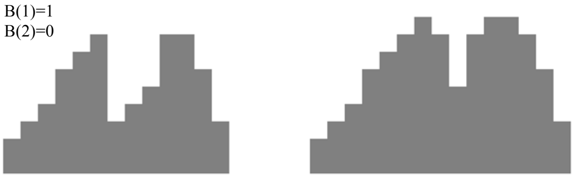
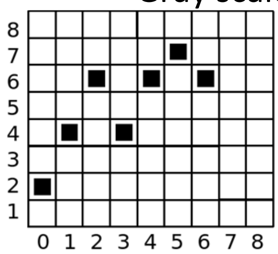


Gray scale dilation



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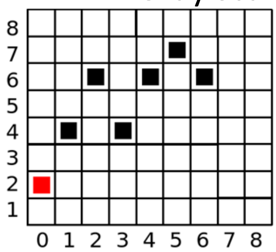
B(1)=1
B(2)=0

$$[\delta_B(f)]\,(x) = \max_{b \in \mathcal{D}_B} \{f(x-b) + B(b)\}$$

Gray scale dilation

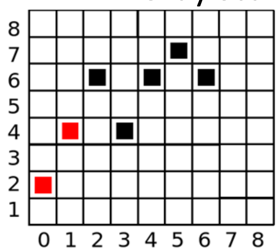


domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

$$[\delta_B(f)](x) = \max_{b \in \mathcal{D}_B} \{f(x-b) + B(b)\}$$

$[\delta_B(f)]$ (0) = max(2+0,0+1,0+0)=2

Gray scale dilation

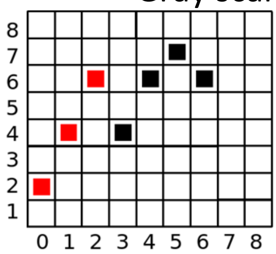


domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

$$[\delta_B(f)](x) = \max_{b \in \mathcal{D}_B} \{f(x-b) + B(b)\}$$

$[\delta_B(f)]$ (1) = max(4+0,2+1,0+0)=4

Gray scale dilation

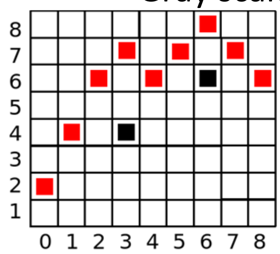


domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

$$[\delta_B(f)](x) = \max_{b \in \mathcal{D}_B} \{f(x-b) + B(b)\}$$

$[\delta_B(f)]$ (2) = max(6+0,4+1,2+0)=6

Gray scale dilation

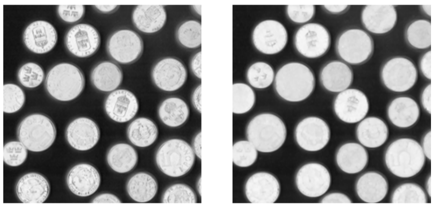


domain(B)={(0,1,2)}
B(0)=0
B(1)=1
B(2)=0

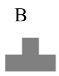
$$[\delta_B(f)](x) = \max_{b \in \mathcal{D}_B} \{f(x-b) + B(b)\}$$

Example, gray-scale dilation flat SE, square 3x3

- SE with positive elements → brighter output
- dark details are reduced or eliminated
- If flat SE, dilation is max of f+b



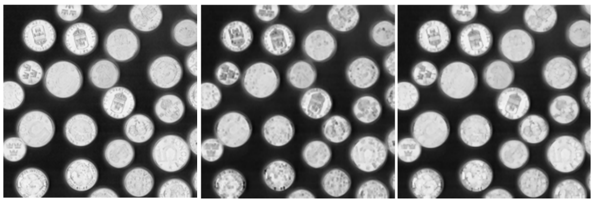
Morphological opening



$$\gamma_B(f) = \delta_B[\varepsilon_B(f)]$$

Example, gray-scale opening, flat SE, square 3x3

- remove small bright details
- leave overall gray-levels
- leave larger bright features



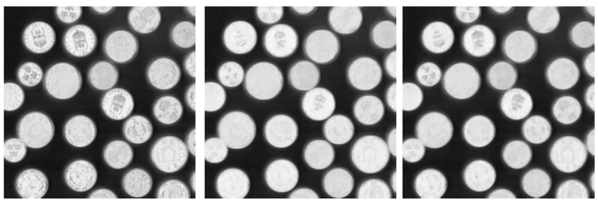
Morphological closing

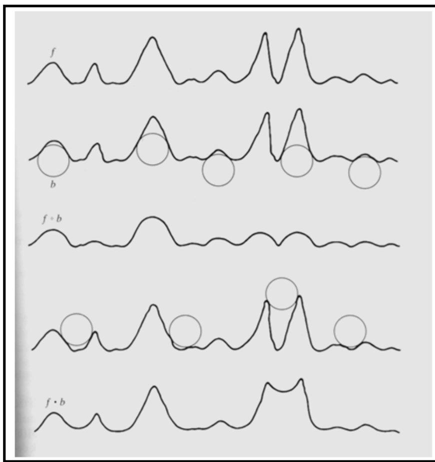
B

$$\phi_B(f) = \varepsilon_B\left[\delta_B(f)\right]$$

Example, gray-scale closing, flat SE, square 3x3

- remove dark details
- Leave overall gray-levels
- leave bright features

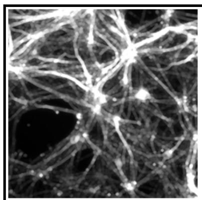




geometrical 1D interpretation **Gray-level opening and closing (from GW)**

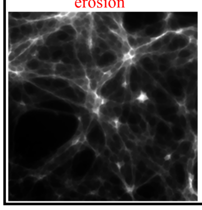
opening

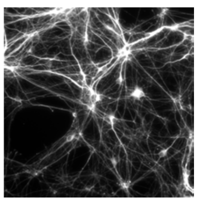
closing



dilation

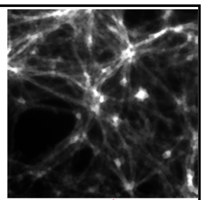
erosion



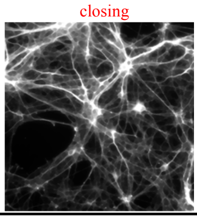


Which operation (erosion, dilation, opening, closing) is applied to the image above? SE circle, radius=4

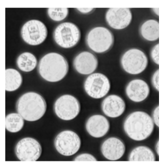
opening



closing

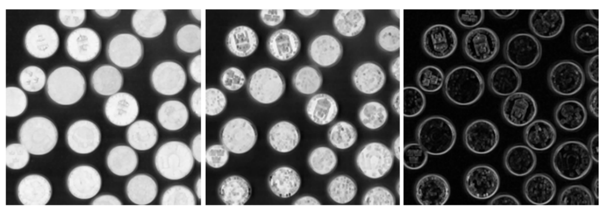


Morphological gradient



$(X \oplus B \setminus X \ominus B)$
dilated image –eroded image

Other ways?

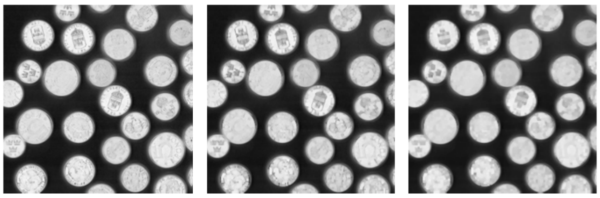


Morphological smoothing

removal or attenuation of bright & dark artifacts/noise

$(X \circ B) \bullet B$

Opening followed by closing



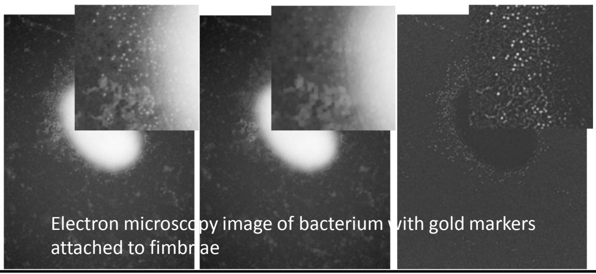
Top hat transformation

$X \setminus (X \circ B)$

original image –opened image

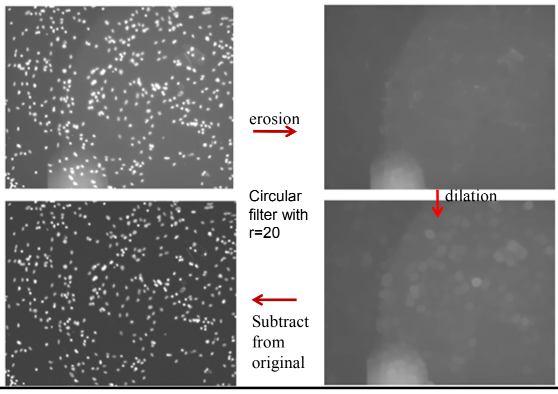
Highlight/segment features of certain size & shape, correct for uneven background

Use SE slightly larger than objects you want to highlight



Top hat for background correction

Fluorescence microscopy image with stained cell nuclei



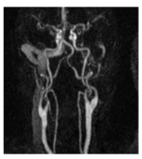
Gray scale hit-or-miss

$$[\mathrm{UHMT}_{\mathbf{B}}(f)](\mathbf{x}) = \begin{cases} [\varepsilon_{B_{FG}}(f)](\mathbf{x}) - [\delta_{B_{BG}}(f)](\mathbf{x}), & \text{if } [\delta_{B_{BG}}(f)](\mathbf{x}) < [\varepsilon_{B_{FG}}(f)](\mathbf{x}) \\ 0 & \text{otherwise} \end{cases}$$

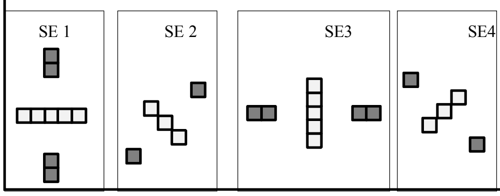
where B_{FG} is the SE for the object (foreground) and B_{BG} is the SE for the background.
Basically an erosion minus a dilation.

Gray scale hit-or-miss

2D example



Problem:
Find vessels in the 2D
mip image.
(The 3D image is acquired by MR.)

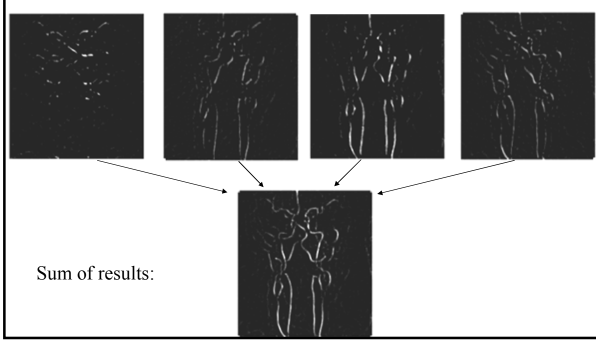


Light gray: B_{FG}

Dark gray: B_{BG}

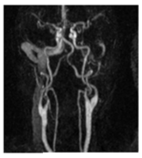
Gray scale hit-or-miss

Result with



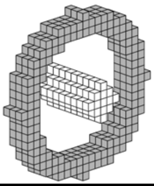
Gray scale hit-or-miss

3D example



Problem:
Find vessels in the 3D MR image.

SEs: Rotations of this SE.



Result

Granulometry

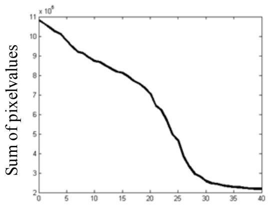
"Measurement of grain sizes of sedimentary rock"

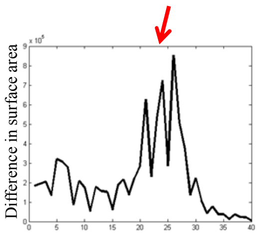
- Measuring particle size distribution indirectly
- Shape information without
  - segmentation
  - separated particles
- Apply morphological openings of increasing size
- Compute the sum of all pixel values in the opening of the image

Example, coin image

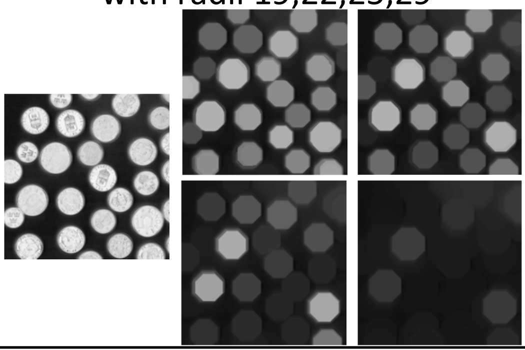
```
sumpixels=zeros(1,maxsize+1);
for k=0:maxsize
  se=strel('disk',k);
  fo=imopen(f,se);
  sumpixels(k+1)=sum(fo(:));
end
```

The peaks correspond to the size of the elements!





Original image, openings of discs with radii 19,22,25,29



Geodesic transformations

Geodesic dilation

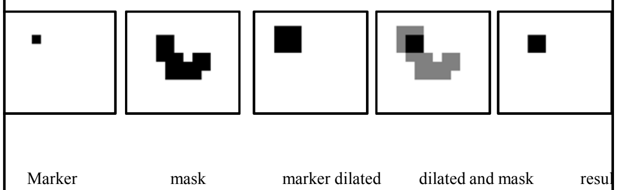
Input: marker image *f* and mask image *g*.

- Dilate the marker image *f* with the unit ball.
- Output the minimum value of the dilation of *f* and the mask image *g*

$$\delta_g^{(1)}(f) = \delta(f) \wedge g$$

Geodesic transformations

Geodesic dilation
example



Geodesic transformations

Geodesic erosion

Input: marker image *f* and mask image *g*.

- Erode the marker image *f* with the unit ball.
- Output the maximum value of the erosion of *f* and the mask image *g*

$$\varepsilon_g^{(1)}(f) = \varepsilon(f) \vee g$$

Morpological reconstruction

- X is set of connected components $X_1,\ldots,X_n$. Y is markers in X.

- Reconstruction by dilation: Geodesic dilations until stability.
- Reconstruction by erosion: Geodesic erosions until stability.

Morpological reconstruction

- Reconstruction by dilation: Geodesic dilations until stability.

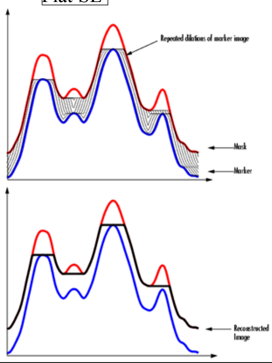
$$\delta_g^{(n)}(f) = \underbrace{\delta_g^{(1)}(\delta_g^{(1)}(\ldots\delta_g^{(1)}(f)))}_{n \text{ times}}$$

- Reconstruction by erosion: Geodesic erosions until stability.

$$\varepsilon_g^{(n)}(f) = \underbrace{\varepsilon_g^{(1)}(\varepsilon_g^{(1)}(\ldots\varepsilon_g^{(1)}(f)))}_{n \text{ times}}$$

Morphological reconstruction by dilation

Flat SE



Reconstruction by erosion: Minima imposition

Mask image *g*

Marker image *f*

Minima imposition: Reconstruction by erosion.

Reconstruction by erosion: Minima imposition

First iteration: Erode marker image with elementary SE.

Pointwise max of

and

=

Reconstruction by erosion: Minima imposition

Second iteration: Erode result from first iteration with elementary SE.

Pointwise max of

and

=

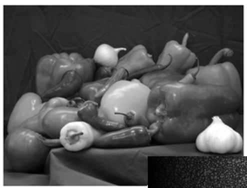
Reconstruction by erosion: Minima imposition

When stability is reached:

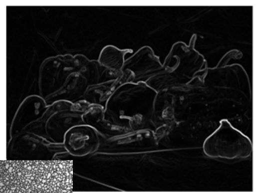
All local minima except for the marked minimum are removed!

This can be used for seeded watershed!

Application - Seeded watershed by Minima imposition

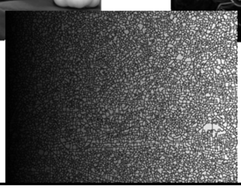


Input image



Edge image by morphological gradient

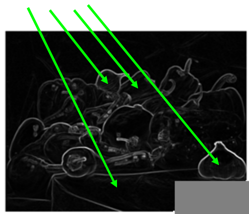
$\delta(f) - \varepsilon(f)$

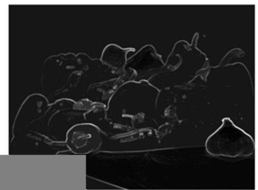


Watershed on edge image. Oversegmentation!

Seeded watershed by Minima imposition

Seeds





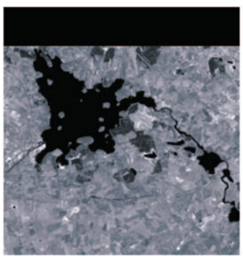
Minima imposition using the seeds as markers

Watershed on the minima imposition

Application - Image compositing

Two images should be merged.

Decide where the “seam” should be.



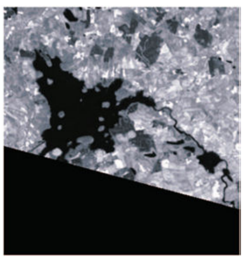


Image compositing

Compute gradient.

Do seeded watershed with minima imposition. (Seeds on the border of the image.)

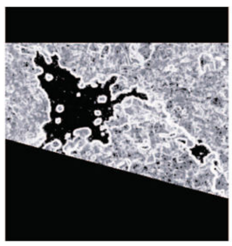
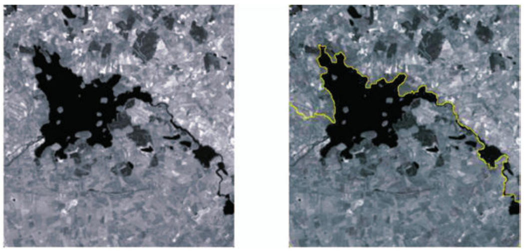
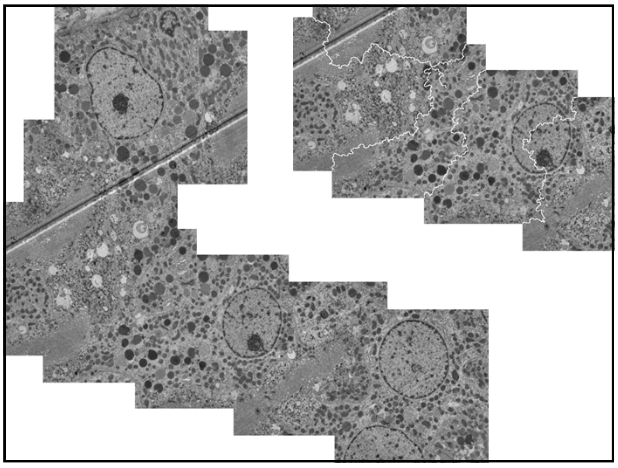


Image compositing

Result

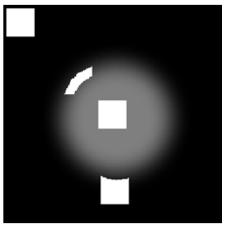




Adaptive SE

The appropriate shape and size of a structuring element strongly depends on the image data and objects of interest

⇒ Let the SE adapt to the local surroundings



Research from CBA:
Vladimir Curic, Cris Luengo

Adaptive SE

- G(eneral)A(daptive)N(eighbourhood): based only on intensity range = connected component
- Amoebas: Pathbased - combines distance and intensity (grow until or on the edge)
- S(alience)A(daptive)SE: Pathbased - uses an attribute (e.g., grad mag.) -weigthed distance transform from segmented objects (e.g., edges). No spatial weight. Only intensity sum
- Ellipse: based on structure tensor (local intensity direction)