## Atelier B

## Reusable Components

## Reference Manual

## version 3.6



ATELIER B<br>Reusable Components-Reference Manual version 3.6

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## CLEARSY

ATELIER B Maintenance
Europarc de PICHAURY
1330 Av. J.R. Guilibert Gauthier de la Lauzière - Bât C2
13856 Aix-en-Provence Cedex 3
France
Tel: +33 (0)4 42371299
Fax: +33 (0)4 42371271
Email: maintenance.atelierb@clearsy.com

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The reusable components supplied with Atelier B are basic machines and library machines.
Basic machines are the modelisation in B of modules manually coded in $\mathrm{C}, \mathrm{C}++$ or ADA . These modules are used to encapsulate the operating system functions that must be used; they must usually be performed in taking into account the specificities of the hardware that the security software will run on. This is why there are few basic machines delivered with "Atelier B".
Library machines are abstract machines written in B language. They generally model a type of mathematical object (sequence, function, etc.) and offer the operations that allow the handling of these objects.
Unlike basic machines, library machines are properly performed using the B method, i.e, using refining and implementation in B along with complete proof of the set. This proof may in principle be executed at any time in order to check its validity (warning: proving methods may depend on the demonstrator version used). Therefore, unlike basic machines, library machines may be numerous and complex while remaining secure as they are proven.

To use basic machines, simply reference them in the appropriate $B$ project, by INCLUDES, IMPORTS or any derived actions. When the final project is translated into a traditional programming language, the translation of the library machine implementations used must be redone if this was not already done at Atelier B installation.

Library machines are implemented on basic machines. As they are performed until the implementation in B language, they provide complete examples of use in the B method. They especially contain examples of proven WHILE loops. For practical advice on proving WHILE loops, refer to the "B Language User Manual".
The user may directly use library machines just like he uses basic machines. Sometimes the implementation of a library machine may use the services of a machine that it does not create an instance for (use by SEES) to avoid duplications. In this case the user will have to create the instance in question (using IMPORTS) by following the indications in the "IMPORTS REQUIRED" section of the description for each library machine.
When the final project $\mathrm{C}, \mathrm{C}++$ or ADA compilation is performed, the library compilation is automatically performed if necessary. Performing link editing will then enable incorporation into the final executable program only those object files that correspond to the library machines actually used. All this is performed in the Makefile produced by Atelier B. To integrate a software component produced by Atelier B into a traditional product, use this Makefile as a basis or refer to the "ADA Translator User Manual".
Warning:
This warning regards the use of reusable components with the Ada, C and $\mathrm{C}++$ translators
supplied with Atelier B. These translators are experimentals. Their goal is to show that it is possible to translate some B0 implementations into classical programming languages. Therefore, their use is not guaranteed. Especially the reusable components use may induce errors when compiling the code produced by the translators. The reusable components must be considered as examples. Each user can develop his own library machines according to his needs.

## 2 <br> Index of Basic Machines

BASIC_ARRAY_VAR: implanting a one dimension table VAL_ARRAY read a table element STR_ARRAY write a table element
BASIC_ARRAY_RGE: implementing a two dimensional table VAL_ARR_RGE read a table element STR_ARR_RGE write a table element COP_ARR_RGE copy a table line to another CMP_ARR_RGE compare two table lines
BASIC_IO: vt 100 style input/output
INTERVAL_READ entry by the operator of a number in mm..nn.
INT_WRITE print a number.
BOOL_READ entry by a TRUE or FALSE boolean operator
BOOL_WRITE print the TRUE or FALSE condition.
CHAR_READ entry by a character's operator.
CHAR_WRITE print a character.
STRING_WRITE print a message.

## 3 Index of Library Machines

L_ARITHMETIC1: extended integer operations: MIN, MAX, INC, DEC, EXP, SQRT, LOG
VAL_ARR_RGE read a table element
STR_ARR_RGE write a table element
COP_ARR_RGE copy a table line to another
CMP_ARR_RGE compare two table lines
BASIC_IO: vt 100 style input/output
MIN
MAX
INC
DEC
EXP
SQRTimum of two numbers.
increment a number.

LOG_BY_DEFAULT logarithm by default.
LOG_BY_EXCESS logarithm by excess.

## L_ARRAY1: one dimensional table with initialization loop

VAL_ARRAY value of an element (promoted operation)
STR_ARRAY write an element (promoted operation)
SET_ARRAY write the same value in a portion of the table

L_ARRAY3: table with non-ordered values, maximum operations

VAL_ARRAY value of an element (promoted operation).
STR_ARRAY write an element (promoted operation).
SET_ARRAY write a same value in a table portion (promoted operation).
SWAP_ARRAY exchange two elements (promoted operation).
RIGHT_SHIFT_ARRAY shift a portion to the large index (promoted operation).

LEFT_SHIFT_ARRAY shift a portion to the small index (promoted operation).
SEARCH_MAX_EQL_ARRAY search for a value in a portion of the table (promoted operation).

SEARCH_MIN_EQL_ARRAY search for a value in a portion of the table (promoted operation).

REVERSE_ARRAY invert the order of the elements in a portion of the table.

## L_ARRAY5: table with ordered values, sort operation

VAL_ARRAY value of an element (promoted operation).
STR_ARRAY write an element (promoted operation).
SET_ARRAY write the same value in a portion of the table (promoted operation).
SWAP_ARRAY exchange two elements (promoted operation).
RIGHT_SHIFT_ARRAY shift a portion to the large index (promoted operation).
LEFT_SHIFT_ARRAY shift a portion to the small index (promoted operation).
SEARCH_MAX_EQL_ARRAY search for a value in a portion of the table (promoted operation).
SEARCH_MIN_EQL_ARRAY search for a value in a portion of the table (promoted operation).
REVERSE_ARRAY invert the order of elements in a portion of the table (promoted operation).
SEARCH_MIN_GEQ_ARRAY search for the first element that exceeds a value (promoted operation).

ASCENDING_SORT_ARRAY sort of a table portion.

## L_PFNC: partial function

VAL_PFNC value of the function for an element in its domain
STR_PFNC overloads the partial function with a couple
XST_PFNC tests if an index is in the partial function domain
RMV_PFNC removes a couple from the partial function
SET_PFNC overloads a part of the function with a constant
SWAP_PFNC exchanges the images for two domain indexes
RIGHT_SHIFT_PFNC right shift of a domain part
LEFT_SHIFT_PFNC left shift of a domain part
SEARCH_MAX EQL_PFNC searches for a value in the partial function
SEARCH_MIN_EQL_PFNC searches for a value in the partial function
REVERSE_PFNC reverses the order of elements for a portion of the domain

ASCENDING_SORT_PFNC sorts in a portion of the domain

## L_SEQUENCE: building a sequence

LEN_SEQ returns the current size of the sequence.
IS_FULL_SEQ is used to determine if the sequence is full (size = LS_maxsize).
IS_INDEX_SEQ is used to determine whether ii is a valid index.
VAL_SEQ value of an element in the sequence.
FIRST_SEQ returns the first element in the sequence.
LAST_SEQ returns the last element in the sequence.
PUSH_SEQ add vv to the end of the sequence.
POP_SEQ removes the last element from the sequence (its value is lost).
STR_SEQ changes the value of an element in the sequence.
RMV_SEQ removes an element from the middle of the sequence.
INS_AFT_SEQ inserts vv right after index ii.
CLR_SEQ clears the sequence.
TAIL_SEQ removes the first element from the sequence.
KEEP_SEQ only keeps the first elements in the sequence.
CUT_SEQ cuts the nn first elements from the sequence.
PART_SEQ only retains part ii..jj in the sequence.
REV_SEQ reverses the order of elements in the sequence.
FIND_FIRST_SEQ finds vv in the sequence, from the start.
FIND_LAST_SEQ finds vv in the sequence, from the end.

## L_SET: creating a set

CARD_SET returns the cardinal for the set.
IS_FULL_SET identifies if the set is full (card = LSET_maxsize).
IS_INDEX_SET identifies if a number is a valid index.
VAL_SET value of a element in the set.
FIND_SET finds an element in the set.
RMV_SET removes an element from the set.
INS_SET inserts an element in the set.
CLR_SET clears all elements from the set.

L_ARRAY_1_RANGE: array of tables of the same size with numerical indexes VAL_ARR_RGE value of an element (promoted operation).

| STR_ARR_RGE | write an element (promoted operation). |
| :--- | :--- |
| COP_ARR_RGE | copy a table to another (promoted operation). |
| CMP_ARR_RGE | compare two tables (promoted operation). |
| DUP_ARR_RGE | duplicate the same table into a series of tables. |
| SET_ARR_RGE | copy the same value to an index set in one of the tables. <br> copy part of one of the tables to a different table to a given posi- <br> tion. |
| PCOP_ARR_RGE | find the first element that is different from two parts of two tables. <br> A Boolean element indicates if this element was found and, in this <br> case, the index of this element is returned. |

L_ARRAY $\quad 3 \_$RANGE: range of tables of the same size, with numerical in-
dexes, and values that are not ordered, maximum operations

| VAL_ARR_RGE | value of an element (promoted operation). |
| :--- | :--- |
| STR_ARR_RGE | write an element (promoted operation). |
| COP_ARR_RGE | copy a table to another (promoted operation). <br> CMP_ARR_RGE <br> compare two tables (promoted operation). |
| DUP_ARR_RGE | duplicate the same table to an array of tables (promoted opera- <br> tion). |
| SET_ARR_RGE | copy the same value to a range in one of the tables (promoted <br> operation). |
| PCOP_ARR_RGE | copy part of one of the tables to a different table, in a given position <br> (promoted operation). |
| PCMP_ARR_RGE | find the first different element from two parts in two tables. A <br> Boolean element indicates whether this element was found and, in <br> this case, the index of this element is returned (promoted opera- <br> tion). |
| SWAP_RGE | swap two elements in a table. |

RIGHT_SHIFT_RGE shift a table range to the large index.
LEFT_SHIFT_RGE shift a table range to the small index.
SEARCH_MAX_EQL_RGE find the last element that equals a value in a table range.
SEARCH_MIN_EQL_RGE find the first element that equals a value in a table range.
REVERSE_RGE reverse the order of the elements of a table part.

L_ARRAY_5_RANGE: array of tables of the same size, with numerical indexes, with ordered values, sort operations

```
VAL_ARR_RGE value of an element (promoted operation).
STR_ARR_RGE write an element (promoted operation).
```

| COP_ARR_RGE | copy a table to another (promoted operation). |
| :--- | :--- |
| CMP_ARR_RGE | compare two tables (promoted operation). |
| DUP_ARR_RGE | duplicate the same table in a range of tables (promoted operation). <br> SET_ARR_RGE <br> copy the same value to an index range in one of the arrays (pro- <br> moted operation). |
| PCOP_ARR_RGE | copy a range from one of the tables to a different table, at a given <br> position (promoted operation). |
| PCMP_ARR_RGE | find the first different element in two ranges in two tables. A <br> Boolean element indicates that this element was found and, in this <br> case, the index of this element is returned (promoted operation). |
| SWAP_RGE | swap two elements in a table (promoted operation). |

RIGHT_SHIFT_RGE shift a table range to the large index (promoted operation).
LEFT_SHIFT_RGE shift a table range to the small index (promoted operation).
SEARCH_MAX_EQL_RGE search for the last element that equals a value in a table range (promoted operation).

SEARCH_MIN_EQL_RGE search for the first element that equals a value in a table range (promoted operation).

REVERSE_RGE reverses the order of the elements of a part of a table (promoted operation).
SEARCH_MIN_GEQ_RGE search for the first element that exceeds a value in a table range.
ASCENDING_SORT_RGE sort a table range into ascending order.

## L_SEQUENCE_RANGE: sequence range

LEN_SEQ_RGE determines the length of a sequence.
IS_FULL_SEQ_RGE determines whether a sequence is full.
IS_INDEX_SEQ_RGE determines whether an integer is in a sequence range.
VAL_SEQ_RGE gives the value of a sequence for a valid index.
FIRST_SEQ_RGE gives the first element in a sequence.
LAST_SEQ_RGE gives the last element in a sequence.
PUSH_SEQ_RGE adds an element to a sequence.
POP_SEQ_RGE removes the last element from a sequence.
STR_SEQ_RGE changes the value of a sequence element.
RMV_SEQ_RGE removes an element from a sequence, with a size that decreases by 1.

INS_SEQ_RGE adds an element to a sequence, with a size that increases by 1.
CLR_SEQ_RGE clears a sequence.
TAIL_SEQ_RGE removes the first element from a sequence.

KEEP_SEQ_RGE only keeps in a sequence the N first elements.
CUT_SEQ_RGE
cuts the N first elements from a sequence.
PART_SEQ_RGE
only keeps in a sequence the indexes in a range between two limits.
REV_SEQ_RGE reverses the order of the elements in a sequence.
FIND_FIRST_SEQ_RGE finds a value in a sequence, returns a Boolean element indicating that it was found and if yes returns the smallest corresponding index.

FIND_LAST_SEQ_RGE finds a value in a sequence, returns a Boolean element indicating that it was found and if yes returns the largest corresponding index.
COP_SEQ_RGE copies from one sequence to another.
CMP_SEQ_RGE
comparison of two sequences.
PCOP_SEQ_RGE partially copies one of the sequences to another.
PCMP_SEQ_RGE partial comparison of two sequences.

## L_ARRAY_COLLECTION: collection of arrays of the same size

| CRE_ARR_COL | returns a Boolean element indicating that there is still an array <br> free in the collection and gives the index of this free array. |
| :--- | :--- |
| DEL_ARR_COL | releases the identified array. |
| VAL_ARR_COL | reads an element from one of the valid arrays. |
| STR_ARR_COL | writes an element from one of the valid arrays. |
| COP_ARR_COL | copies one of the arrays to another. |
| CMP_ARR_COL | compares two tables. |

## L_ARRAY1_COLLECTION: collection of arrays of the same size with numerical index

CRE_ARR_COL

DEL_ARR_COL
VAL_ARR_COL
STR_ARR-COL
COP_ARR_COL
CMP_ARR_COL
SET_ARR_COL
PCOP_ARR_COL
PCMP_ARR_COL
returns a Boolean element indicating that there is an array free in the collection and the index of this free array (promoted operation).
releases the listed array (promoted operation). read a element from on of the valid arrays (promoted operation). write a element from one of the valid arrays (promoted operation). copies from one of the arrays to another (promoted operation). compares two tables (promoted operation).
copies the same value to an index range in one of the arrays. copies part of one of the arrays to another, to a given position.
find the first different element between the two parts of the two different arrays. A Boolean element indicates if this element was found and in this case, the index of this element is returned.

## L_RELATION : complete binary relations

op_reset
op_isFullRelation
op_add
op_remove
op_cardinal
op_belongsTo

The relation becomes the empty relation.
Returns TRUE only if the cardinal of the relation equals max nb_2tupple.
Adds a couple to the relation.
Removes a couple to the relation.
Returns the relation cardinal $\downarrow$
Checks if a couple is present in the relation.

[^0]
## 4 Description of Basic Machines

The basic machines supplied with Atelier B allow either the creation of dynamic arrays that cannot be obtained using B0, or producing models using vt100 style inputs/outputs. "dynamics arrays" are arrays which size depends on the machine parameters. Such arrays cannot be realised directly in B0, the safety design of the ADA, C and C++ translators do not allow to treat this case. For example, the following construction is not allowed:

```
IMPLEMENTATION
    mm(xx)
CONCRETE_VARIABLES
    mytab
INVARIANT
    mytab }\in(0..xx) -> NA
END
```

Such an array would have to be realised using BASIC_ARRAY_VAR.
The atelier actual version is composed of three basic machines:
BASIC_ARRAY_VAR Arrays with dimension 1.
BASIC_ARRAY_RGE Arrays with dimension 2.
BASIC_IO Usual inputs/outputs management.
This chapter presents this three machines.
The basic machine BASIC_IO is intended to the model designing. It mustn't be considered as safe.

WARNING: The manual implementations of the basic machines BASIC_ARRAY_VAR and BASIC_ARRAY_RGE destined for the translators supplied with Atelier B are provided as demonstration. They are not safe, and are not appropriated in all the B use contexts.

### 4.1 BASIC_ARRAY_VAR: Implanting a one dimensional table

## OPERATIONS

VAL_ARRAY read a table element

STR_ARRAY write a table element

## EXAMPLE

Example of use with listed sets:

```
MACHINE
    array
SETS
    FONTS = {Times,Serif,Courier};
    FTYPE = {fixed,unfixed}
VARIABLES
    fixedsz
INVARIANT
    fixedsz }\in\mathrm{ FONTS }->\mathrm{ FTYPE
INITIALISATION
    fixedsz:={Times }\mapsto\mathrm{ unfixed,
    Serif }\mapsto\mathrm{ fixed, Courier }\mapsto\mathrm{ fixed}
END
```

```
IMPLEMENTATION
```

IMPLEMENTATION
array_1
array_1
REFINES
REFINES
array
array
IMPORTS
IMPORTS
BASIC_ARRAY_VAR(FONTS,FTYPE)
BASIC_ARRAY_VAR(FONTS,FTYPE)
INVARIANT
INVARIANT
arr_vrb = fixedsz
arr_vrb = fixedsz
INITIALISATION
INITIALISATION
STR_ARRAY(Times,unfixed);
STR_ARRAY(Times,unfixed);
STR_ARRAY(Serif,fixed);
STR_ARRAY(Serif,fixed);
STR_ARRAY(Courier,fixed)
STR_ARRAY(Courier,fixed)
END

```
END
```

arr_vrb is the name of the table encapsulated by BASIC_ARRAY_VAR

## DESCRIPTION

BASIC_ARRAY_VAR modelizes one dimensional arrays. Such arrays cannot be created directly in B0 if their size dependend on the machine parameters ("dynamic arrays"). The current design of ADA or C translators does not allow handling this case. The following construction is therefore illegal:

```
IMPLEMENTATION
    mm(xx)
VARIABLES
    mytab
INVARIANT
    mytab }\in(0..xx) -> NA
END
```

This kind of table should be generated using BASIC_ARRAY_VAR.

## MACHINE PARAMETERS

BASIC_ARRAY_VAR (BAV_INDEX,BAV_VALUE): BAV_INDEX is the set of values used to index the table, BAV_VALUE is the set of possible values for table elements.

The B language rule relating to the possible values of the BAV_VALUE parameter ensure that: if a computer variable can contain elements of MININT..MAXINT, then it can contain those of BAV_VALUE. For example, B rules forbid assigning BAV_VALUE the value of MAXINT+1,MAXINT+2

## VAL_ARRAY

```
    syntax vv \leftarrow VAL_ARRAY(ii)
    preconditions ii must be a BAV_INDEX
    outputs vv is a BAV_VALUE, the value of the array at position ii.
```


## STR ARRAY

syntax STR_ARRAY(ii,vv)
preconditions ii must be a BAV_INDEX and vv must be a BAV_VALUE
The value vv is stored in the array at ii index.

## C++ LANGUAGE

In $\mathrm{C}++$, the array is realised by an integer array. The accesses to this array are done using method that refuse the index used between 0 and the array size, guaranting an optimal memory use.
The array is dynamically reserved when launching the program. If the size indicated by the formal parameters is too big, the program stops with the following message:

Virtual memory exceede in ' $n$ new',

## C LANGUAGE

The realisation in C is based on the same principles as in $\mathrm{C}++$. The stop message on initial reservation failure is:
Fatal error: Malloc of $X$ bytes failed
Execution of current application is aborted

## ADA LANGUAGE

The use of generic packaging guarantees an optimal memory occupation. No restrictions are made on the instanciation parameters. On initial reservation failure, an exception stops the program.

## PROGRAMMING

Example of use with literal sets:

|  | IMPLEMENTATION <br> narr_1 |
| :--- | :--- |
| MACHINE | REFINES |
| narr | narr |
| VARIABLES | IMPORTS |
| myvar | BASIC_ARRAY_VAR $(0 . .2,0 . .1)$ |
| INVARIANT | INVARIANT |
| myvar $\in 0 . .2 \rightarrow 0 . .1$ | arr_vrb = myvar |
| INITIALISATION | INITIALISATION |
| myvar: $=\{0 \mapsto 0,1 \mapsto 1,2 \mapsto 1\}$ | STR_ARRAY $(0,0) ;$ |
| END | STR_ARRAY(1,1); |
|  | STR_ARRAY $(2,1)$ |
|  | END |

Another example. Only the implementation is presented. The write of a machine refined by this implementation is an exercice for the reader:

```
IMPLEMENTATION
    parr_1
REFINES
    parr
IMPORTS
    BASIC_ARRAY_VAR(FONTS,FTYPE)
VALUES
    FONTS = 5. .7;
    FTYPE = 3. .4
INVARIANT
    arr_vrb = fixedsz
INITIALISATION
    STR_ARRAY(5,3);
    STR_ARRAY(6,4);
    STR_ARRAY(7,5)
END
```

NOTE: The possible values of the BASIC_ARRAY_VAR parameters are given by the B language rules, (refer to section 12.2 page 574 of the BBOOK)

### 4.2 BASIC_ARRAY_RGE: Implementing a Two Dimensional Array

## OPERATIONS

VAL_ARR_RGE read an array element
STR_ARR_RGE write an array element
COP_ARR_RGE copy an array line to another
CMP_ARR_RGE compare two array lines

## EXAMPLE

Example of use, two lines and three columns array:

|  | IMPLEMENTATION bitab_1 |
| :---: | :---: |
| MACHINE | REFINES |
| bitab | bitab |
| SETS | IMPORTS |
| LGNS $=\{111,112\}$ | BASIC_ARRAY_RGE(1. .3,0. .255,LGNS) |
| VARIABLES mytab | INVARIANT <br> arr_rge = mytab |
| INVARIANT | INITIALISATION |
| mytab $\in$ LGNS $\rightarrow(1 . .3 \rightarrow 0 . .255)$ | STR_ARR_RGE(111,1,7); |
| INITIALISATION | STR_ARR_RGE (111,2,8); |
| mytab: $=\{111 \mapsto\{1 \mapsto 7,2 \mapsto 8,3 \mapsto 9\}$, | STR_ARR_RGE(111,3,9); |
| $112 \mapsto\{1 \mapsto 0,2 \mapsto 1,3 \mapsto 2\}\}$ | STR_ARR_RGE (112,1,0); |
| END | STR_ARR_RGE (112,2,1); |
|  | STR_ARR_RGE(112,3,2) |
|  | END |

The variable arr_rge is the name of the encapsulated array par BASIC_ARRAY_RGE

## DESCRIPTION

BASIC_ARRAY_RGE models two dimensional arrays. Such arrays cannot be created directly in B0 if their size depends on the machine parameters ("dynamic array"). The safe design of the ADA, C++ or C translators do not allow to treat this case. The following construction is forbidden:

```
IMPLEMENTATION
    mm(xx)
CONCRETE_VARIABLES
    mytab
INVARIANT
    mytab \in(0..10) }->(0..xx)\times(0..xx
...
END
```

Such an array must be implemented using BASIC_ARRAY_RGE

## MACHINE PARAMETERS

BASIC_ARRAY_RGE(BAR_INDEX,BAR_VALUE,BAR_RANGE):
BAR_INDEX represents the column indexes.
$B A R_{-} V A L U E$ is the set of the possible values for the array elements,
$B A R \_R A N G E$ represents the line indexes.
The B language rules concerning the possible values of the BAR_VALUE parameter ensure that a computing variable being able to contain the elements of MININT..MAXINT, then it can contain those of BAR_VALUE. For example, the B rules do not permit to give to BAR_VALUE the value MAXINT+1,MAXINT+2.

## VAL_ARR_RGE

$$
\text { syntax } \quad \text { vv } \leftarrow \text { VAL_ARR_RGE(rr,ii) }
$$

preconditions ii must be a BAR_INDEX, rr must be a BAR_RANGE
outputs $\quad \mathrm{vv}$ is an element of BAR_VALUE, which value is the array value at position ii, line rr.

## STR_ARR_RGE

syntax STR_ARR_RGE(rr,ii,vv)
preconditions rr must be an element of BAR_RANGE, ii an element of BAR_INDEX and vv an element of BAR_VALUE
Value vv is stored in the array line rr, index ii.

## COP_ARR_RGE

```
    syntax COP_ARR_RGE(dest,src)
    preconditions dest and src must be elements of BAR_RANGE
```

The src line is copied to the dest line.

## CMP_ARR_RGE

syntax $\quad \mathrm{bb} \leftarrow$ CMP_ARR_RGE (range1,range2)
preconditions range1 and range2 must be elements of BAR_RANGE
outputs $\quad \mathrm{bb}$ is an element of BOOL, that takes the TRUE value if the two lines are equal.

## C++ LANGUAGE

In C++, the array is realised by an array of pointers, pointing on integers arrays. The access to these arrays are done using methods that refuse the index used between 0 and the arrays size, guaranting an optimal memory occupation.
The memory is dynamically reserved when lauching the program. If the size indicated by the formal parameters is too big, the program stops with the following message:
Virtual memory exceeded in 'new'

## C LANGUAGE

The realisation in C is based on the same principles as in $\mathrm{C}++$. The stop message on the initial reservation failure is:

Execution of current application is aborted

## ADA LANGUAGE

The use of generic packages guarantees an optimal memory occupation. No restriction is made on the instancing parameters. On an initial reservation failure, an exception stops the program.

### 4.3 BASIC_IO: vt100 style inputs/outputs

## OPERATIONS

INTERVAL_READ operator input of an integer in mm..nn.
INT_WRITE print an integer.
BOOL_READ operator input of a Boolean TRUE or FALSE state
BOOL_WRITE print TRUE or FALSE.
CHAR_READ operator input of a character.
CHAR_WRITE print a character.
STRING_WRITE print a message.

## SIMPLE EXAMPLE

The following implementation displays "hello" on the terminal:

|  | IMPLEMENTATION |
| :--- | :--- |
| MACHINE | bonj_1 |
| REFINES |  |
| bonj | bonj |
| OPERATIONS | IMPORTS |
| main = skip | BASIC_IO |
| END | OPERATIONS |
|  | main = BEGIN |
|  | STRING_WRITE("hello\n") |
|  | END |

## DESCRIPTION

BASIC_IO is used for simple input/output actions on a terminal. This basic machine is used to build models. Such I/O cannot be considered as safe.
In UNIX, the system devices used are standard input and standard output (stdin and stdout), they can therefore be redirected.

## INTERVAL_READ

$$
\begin{array}{ll}
\text { syntax } & \mathrm{bb} \leftarrow \text { INTERVAL_READ }(\mathrm{mm}, \mathrm{nn}) \\
\text { preconditions } & \mathrm{mm} \text { and } \mathrm{nn} \text { must be NATs so that } \mathrm{mm} \leq \mathrm{nn} \\
\text { outputs } & \mathrm{bb} \text { integer in } \mathrm{mm} . . \mathrm{nn}
\end{array}
$$

The operator inputs an integer of the interval mm..nn. The input format forces to type a succession of number(s) followed by RETURN. The first input character must be a number. On the opposite case, the input fails " 3 " is not valid). When a character that is not the first input is not a number anymore, this character, as all the following ones, are ignored: " 3 e 2 " is a valid input of the integer 3. As long as the input is false, the message"THIS IS NOT A NUMBER IN mm..nn" is displayed and a new entry is required.

## INT_WRITE

```
syntax INT_WRITE(vv)
preconditions vv must belong to NAT
```

Output number vv, with no return.

## BOOL_READ

```
syntax bb }\leftarrow\mathrm{ BOOL_READ
outputs bb must be Boolean.
```

The operator enters Boolean TRUE or FALSE conditions, with no character before it (for example: "TRUE" is rejected because of the space before it). As long as the operator has not made a valid entry, the message "THIS IS NOT A BOOL VALUE: type TRUE or FALSE" is displayed and a new entry is required.

## BOOL_WRITE

$$
\text { syntax } \quad \text { BOOL_WRITE(bb) }
$$

preconditions bb must be Boolean
Output TRUE or FALSE, with no return.

```
CHAR READ
    syntax cc }\leftarrow\mathrm{ CHAR_READ
    outputs cc must be part of 0..255
```

Operator entry of a character that is interpreted as a number in $0 . .255$. Type in the character followed by return. If several characters has been typed, only the first one is taken into account (example: " cdef" is understood as "=32). In C, pressing Return only returns 10, ctrl-D (EOF) returns 0 . In ADA, only the 'visible' characters entries (i.e, no control characters) are accepted.

## CHAR_WRITE

syntax CHAR_WRITE(vv)
preconditions vv must belong to the range $0 . .255$
Displays the cc character on-screen (example: CHAR_WRITE(10) to produce a return). Remember, a single quote means "prime" the language's notation conventions, and B. CHAR_WRITE('A') for example, means nothing. On the contrary, the quoted strings are valid elements in a formula, they serve for STRING_WRITE below.

## STRING_WRITE

syntax STRING_WRITE(ss)
preconditions ss must be an element in the STRING set
Will display a character string on-screen. For ss use quoted strings. A "C type" formatting is used, even for a translation into ADA, i.e,:
\t produces a tab
$\backslash$ E produces Escape
$\backslash$ B produces a sound
\" produces a quote

## KNOWN PROBLEMS

STRING does not have a coherent definition. The prover proves that any character string belongs to STRING due to an ad hoc rule, that does not derive from the definition STRING $=\operatorname{seq}($ CHAR $)$. In addition, using a STRING type local variable in an implementation is not possible. To be completely rigorous, nothing ensures that the operator performs all the requested entries. Therefore the operations for entering the true data entry module (BASIC_IO.c for example) do not really implant the specifications of the corresponding $B$ operations.

PROGRAMMING
A more complete example:

| MACHINE bio OPERATIONS main $=$ skip END | ```IMPLEMENTATION bio_1 REFINES bio IMPORTS BASIC_ARITHMETIC,BASIC_IO OPERATIONS main \(=\mathrm{VAR} \mathrm{zz}, \mathrm{bb}, \mathrm{cc}\) IN \(\mathrm{zz} \longleftarrow\) INTERVAL_READ \((0,100)\); STRING_WRITE("this is the value : "); INT_WRITE(zz); CHAR_WRITE(10); bb - BOOL_READ; STRING_WRITE("this is the value : "); BOOL_WRITE(bb); CHAR_WRITE(10); cc \(\longleftarrow\) CHAR_READ; STRING_WRITE("this is the value : "); INT_WRITE(cc); STRING_WRITE(" = "); CHAR_WRITE(cc); CHAR_WRITE(10) END``` END |
| :---: | :---: |

Execution example:

```
ATELIER-B% bio
sdfsdf
THIS IS NOT A NUMBER IN 0..100
20
this is the value: 20
CRUE
THIS IS NOT A BOOL VALUE: type TRUE or FALSE
TRUE
this is the value: TRUE
cvf
```

this is the value: $99=c$
ATELIER-B\%

NOTE: To be completely rigorous, nothing ensures that the operator performs all the entries requested. The entry loops of the concrete module (BASIC_IO.c for example) do not really implant the specifications of the corresponding operations.
Possible evolutions:
It should be possible to define in the machine BASIC_IO., abstract variables modeling the inputs/outputs; it should then be possible to specify the required interactions of the external system. The abstract machine that needs to handle inputs/outputs will use BASIC_IO notions (by SEES or INCLUDES) to represent the required interactions.

## 5 Description of Library Machines

The library machines are all intended for creating mathematical objects, except machine L_ARITHMETIC1 that provides certain arithmetical functions. The modeled mathematical objects are:
total functions : these are machines contain "ARR" (array) in their name;
partial functions : machines with the "PFNC" (partial function) in their name; sets : these are machines with the "SET" (set) in their name;
sequences : these are machines with the "SEQ" (sequence) in their name.
For each mathematical object, it is possible to realize either a variable representing the object, or a variable representing several objects of this type. For each type of object, it is therefore possible to realize:

- The object itself;
- An array of objects with the same type, same size, these are machines with a name containing the "RGE" (range) radical;
- A partial function of objects with the same size and same type, these are machines with a name containing the "COL" (collection) radical;
- A partial function of objects with the same type, but with various sizes ("OBJ" radical).

The "RGE" and "COL" type machines produce objects that consume the memory necessary for the maximum number of required objects. For example, if we create a range or a collection of three sequences of at least ten elements, we will always require 30 memory spaces; but the use of a collection avoids the user program to manage the sequences available/occupied. Object machines reserve a memory space that may be freely distributed depending on the created objects and their size. Mathematical objects listed above are not all available on the different types of machines, refer to library machines table of contents for the list that corresponds to the current version.
WARNING : Most of the library machines are based on the basic machines BASIC_ARRAY_VAR and BASIC_ARRAY_RGE. The manual implementations of the basic machines BASIC_ARRAY_VAR and BASIC_ARRAY_RGE destined to the translators supplied with Atelier $B$ are provided as a demonstration. They are not safe, and not appropriate in all the B use context. In the case of a more complete use, the user would have to realize these basic machines.

### 5.1 L ARITHMETIC1: Extended Integer Operations

The "integer" term refers to the elements of NAT.NAT that is the set of the natural integers between 0 and MAXINT.

## OPERATIONS

MIN minimum of two integers.
MAX maximum of two integers.
INC increment an integer strictly inferior to MAXINT.
DEC decrement a literal integer.
EXP exponentiation.
SQRT default integer square root.
LOG_BY_DEFAULT default logarithm.
LOG_BY_EXCESS logarithm by excess.

## EXAMPLE

The example below shows a machine that uses a certain number of functionalities of the machine L_ARITHMETIC1.

| MACHINE <br> m1 | IMPLEMENTATION m1_1 |
| :---: | :---: |
| OPERATIONS | REFINES |
| $\mathrm{xx} \leftarrow \mathrm{op} 1=$ ANY tt Where | m1 |
| $\mathrm{tt} \in \mathrm{NAT} \wedge \mathrm{tt} \times \mathrm{tt}=16$ | IMPORTS |
| THEN | L_ARITHMETIC1, |
| xx :=tt | OPERATIONS |
| END; | $\mathrm{xx} \leftarrow \mathrm{op} 1=$ BEGIN |
| $\mathrm{xx} \leftarrow \mathrm{op} 2=\mathrm{ANY}$ tt WHERE | $\mathrm{xx} \leftarrow \mathrm{SQRT}(16)$ |
| $\mathrm{tt} \in \mathrm{NAT} \wedge 3^{\text {tt }}=27$ | End; |
| then xx:=tt | $\begin{aligned} & \mathrm{xx} \leftarrow \mathrm{op} 2=\mathrm{vaR} \text { rr IN } \\ & \quad \mathrm{xx}, \mathrm{rr} \leftarrow \text { LOG_BY_DEFAULT }(3,27) \end{aligned}$ |
| END | End |
| END | END |

## DESCRIPTION

L_ARITHMETIC1 offers arithmetical operations such as roots and logarithms, operations on the elements NAT and dedicated to calculatory applications. Calculus being integers values, the search operation for the logarithm and the square root return the best $\square$ approaching value in NAT. The used algorithms are optimized.

## MACHINE PARAMETERS

None.

[^1]
## MIN

$$
\begin{array}{ll}
\text { syntax } & \mathrm{uu} \leftarrow \mathrm{MIN}(\mathrm{vv}, \mathrm{ww}) \\
\text { preconditions } & \mathrm{vv} \text { and } \mathrm{ww} \text { must be in NAT. } \\
\text { outputs } & \mathrm{uu}=\min (\{\mathrm{vv}, \mathrm{ww}\})
\end{array}
$$

## MAX

$$
\begin{array}{ll}
\text { syntax } & \text { uu } \leftarrow \operatorname{MAX}(\mathrm{vv}, \mathrm{ww}) \\
\text { preconditions } & \text { vv and ww must be in NAT. } \\
\text { outputs } & \text { uu receives } \max (\{\mathrm{vv}, \mathrm{ww}\})
\end{array}
$$

## INC

$$
\begin{array}{ll}
\text { syntax } & \mathrm{uu} \leftarrow \mathrm{INC}(\mathrm{vv}) \\
\text { preconditions } & \mathrm{vv} \text { must be in 0..MAXINT-1. } \\
\text { outputs } & \mathrm{uu}=\mathrm{vv}+1
\end{array}
$$

## DEC

```
syntax \(\quad u \quad \leftarrow \operatorname{DEC}(\mathrm{vv})\)
preconditions vv must be in 1..MAXINT.
outputs \(\quad \mathrm{uu}=\mathrm{vv}-1\)
```


## EXP

syntax $\quad \mathrm{rr} \leftarrow \operatorname{EXP}(\mathrm{xx}, \mathrm{nn})$
preconditions xx and nn must be in NAT. xx and nn must not both be nil. $x x^{n n}$ must be less than or equal to MAXINT.
outputs rr receives $\mathrm{xx}^{n n}$
EXP returns xx to the power of nn . Calculating $0^{0}$ is illegal ( $0^{0}$ is not defined). The implementation uses a fast algorithm based on breaking down into base 2 of $n n\left(\log _{2}(\mathrm{nn})\right.$ iterations).

## SQRT

```
syntax nn }\leftarrow\textrm{SQRT}(\textrm{pp}
preconditions pp must be in NAT.
outputs nn so that nn }\times\textrm{nn}\leq\textrm{pp}<(\textrm{nn}+1)\times(\textrm{nn}+1
```

SQRT returns the largest nn so that $\mathrm{nn} \times \mathrm{nn} \leq \mathrm{pp}$. The implementation uses an algorithm that performs $\operatorname{SQRT}(\mathrm{nn})$ iterations, where each iteration costs two additions and a subtraction.

## LOG_BY_DEFAULT

syntax uu,rr $\leftarrow$ LOG_BY_DEFAULT(vv,ww)
preconditions ww and vv are two natural integers and vv is between 2 and MAXINT.
outputs $\quad \mathrm{uu}$ is the smallest natural so that $\mathrm{vv}^{(u u+1)}$ is strictly greater than ww. By definition, uu is a natural integer. rr takes the value vv ${ }^{u u}$.

LOG_BY_DEFAULT in base vv of ww: returns the smallest uu value so that $\mathrm{ww}<\mathrm{vv}^{(u u+1)}$. This gives $\mathrm{vv}^{u u} \leq \mathrm{ww}$, except if $\mathrm{ww}<\mathrm{vv}$ (example: $\mathrm{ww}=0$ ). Does not work for $\mathrm{vv}=0$ or 1 as $0^{i i}$ and $1^{i i}$ are constants. rr receives the value of vv ${ }^{u u}$, which easily allows judging the error made.

## LOG_BY_EXCESS

syntax uu,bb $\leftarrow$ LOG_BY_EXCESS(vv,ww)
preconditions ww belongs to NAT and vv is an element of the intervall 2..MAXINT.
outputs uu receives the smallest natural so that vv ${ }^{u u}$ is greater than or equal to ww. uu must be in NAT. bb is an element of BOOL, it indicates whether the logarithm is an exact one.
LOG_BY_EXCESS in base vv in ww: returns the smallest uu so that ww $\leq \mathrm{vv}^{u u}$. WARN-
ING: vv ${ }^{u u}$ may exceed MAXINT! Does not work for vv $=0$ or 1 as $0^{i i}$ and $1^{i i}$ are constants.
bb equals TRUE if $w w=v^{u u}$.

## IMPORTS REQUIRED

None.

### 5.2 L ARRAY1: One Dimensional Array, with Initialization Loop

## OPERATIONS

VAL_ARRAY value of an element (promoted operation)
STR_ARRAY write an element (promoted operation)
SET_ARRAY write the same value in a portion of the array

## EXAMPLE

Use SET_ARRAY to initialize an array:

|  | IMPLEMENTATION |
| :--- | :--- |
| MACHINE | m1_1 |
| m1 | REFINES |
| VARIABLES | m1 |
| vv | IMPORTS |
| INVARIANT | i1.L_ARRAY1 $(0 . .255,10)$ |
| vv $\in 0 . .10 \rightarrow 0 . .255$ | INVARIANT |
| INITIALISATION | (arr_vrb is the variable in $\left.L \_A R R A Y 1\right)$ |
| vv $:=(0 . .10) \times\{5\}$ | i1.arr_vrb = vv |
| END | INITIALISATION |
|  | i1.SET_ARRAY $(0,10,5)$ |
|  | END |

## DESCRIPTION

As it is possible, L_ARRAY1 is used instead of BASIC_ARRAY_VAR. L_ARRAY1 realises, using an array, an abstract variable representing a function. It is then possible to have an initialization operation of the entire function or of a part of it (initialization loop). The starting part of the function performed is an interval: if not, it would not be possible to indicate a portion of this set without mentioning all elements involved.

## MACHINE PARAMETERS

L_ARRAY1(LAU_VALUE, LAU_maxidx): LAU_VALUE is the set of possible values for the array elements, 0..LAU_maxidx is the set of array indexes.

## VAL_ARRAY

syntax $\quad$ vv $\leftarrow$ VAL_ARRAY (ii)
preconditions ii must be in 0..LAU_maxidx
outputs $\quad$ vv is an element of LAU_VALUE, the array value at position ii.

## STR_ARRAY

syntax STR_ARRAY(ii,vv)
preconditions ii and vv must belong to the 0..LAU_maxidx and LAU_VALUE respectively.
vv value is stored in the array at ii index. SET_ARRAY
syntax SET_ARRAY (ii,jj,vv)
 For implementation reasons [1, jj and MAXINT must be different.
The value vv is stored in the array for all the indexes between ii to jj . If $\mathrm{ii}>\mathrm{jj}$, the array does not change.
Note that it would not have been advisable to set $\mathrm{ii} \leq \mathrm{jj}$ as a precondition of this operation, as this would have limited its use. Let us consider the case of a call to SET_ARRAY in a loop. The last iteration fo the loop contain s a call with the form SET_ARRAY (ii, jj , vv) with $\mathrm{i}=\mathrm{jj}+1$. The presence of a precondition in the definition of the operation SET_ARRAY would force us to "guard" all the calls to SET_ARRAY by an IF. More generally, the precondition must be selected as minimal to protect us fromm a B code of "defensive" aspect.

## IMPORTS REQUIRED

None.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_VAR machine (IMPORTS BASIC_ARRAY_VAR(...)). The addition of an instance of the machine BASIC_ARRAY_VAR requires choosing a new instance name, as, for example: i1.BASIC_ARRAY_VAR).

[^2]
### 5.3 L ARRAY3: Array with Non Ordered Values, Maximum Operations

## OPERATIONS

| VAL_ARRAY | value of an element (promoted operation). |
| :--- | :--- |
| STR_ARRAY | write an element (promoted operation). |
| SET_ARRAY | write the same value in an array portion (promoted operation). |
| SWAP_ARRAY | exchange two elements (promoted operation). |
| RIGHT_SHIFT_ARRAY shift a portion to the main index (promoted operation). |  |
| LEFT_SHIFT_ARRAY shift a portion to the small index (promoted operation). |  |
| SEARCH_MAX_EQL_ARRAY search for a value in an array (promoted operation). |  |
| SEARCH_MIN_EQL_ARRAY search for a value in an array portion (promoted opera- |  |
| tion). |  |

REVERSE_ARRAY reverse the order of elements in an array portion.

## EXAMPLE

The example below is a machine that represents the color assigned to 101 points, this color may be red, green or blue for each point. An operation is used to find a red dot.

```
MACHINE
    m1
SETS
    COLOR = {red, green, blue}
VARIABLES
    color
INVARIANT
    color }\in0..100 -> COLOR
INITIALISATION
    color :=(0..100) }\times{\mathrm{ red }
OPERATIONS
    ii,bb}\leftarrow\mathrm{ trouve_red = PRE
    rouge }\in\operatorname{ran}(\mathrm{ color)
THEN
    ii :\in color}\mp@subsup{}{}{-1}[{\mathrm{ red}] |
    bb :\in BOOL
END
END
```

```
IMPLEMENTATION
    m1_1
REFINES
    m1
IMPORTS
    i1.L_ARRAY3(COLOR,100)
INVARIANT
    i1.arr_vrb = color
INITIALISATION
    i1.SET_ARRAY(0,100,red)
OPERATIONS
    ii,bb}\leftarrow\mathrm{ trouve_red =
    vaR bb IN
        ii,bb}
        i1.SEARCH_MAX_EQL_ARRAY(0,100,red)
    END
END
```


## DESCRIPTION

L_ARRAY3 is the most complete of the one dimensional array machines that do not require that the output set be part of an interval. L_ARRAY5 has been constrained. It is therefore possible to create arrays with values that are elements of a listed set while having access to complete operations such as element order reversal. The operation that
is not available is the one that would require an order relationship on the array elements: sort.

## MACHINE PARAMETERS

L_ARRAY3(LAT_VALUE,LAT_maxidx): LAT_VALUE is the set of possible values for array elements, $0 . . L A T \_m a x i d x$ is the set of array indexes.

## VAL_ARRAY

syntax $\quad$ vv $\leftarrow$ VAL_ARRAY(ii)
preconditions ii must be in 0..LAT_maxidx
outputs $\quad v v$ is a LAT_VALUE, it is the value of the array at position ii.

## STR ARRAY

syntax STR_ARRAY(ii,vv)
preconditions ii must be in 0..LAT_maxidx and vv must belong to LAT_VALUE
The vv value is stored in the array at index ii.

## SET_ARRAY

syntax SET_ARRAY(ii,jj,vv)
preconditions ii..jj must be a subset of 0..LAT_maxidx and vv belong to LAT_VALUE. For implementation reasons it is also necessary that jj be different from MAXINT.

The vv value is stored in the array for all indexes between ii and jj . If $\mathrm{ii}>\mathrm{jj}$, the array will not change.

```
SWAP_ARRAY
    syntax SWAP_ARRAY(ii,jj)
preconditions ii,jj must be in 0..LAT_maxidx.
```

The ii and jj elements in the array are exchanged.

## RIGHT_SHIFT_ARRAY <br> syntax RIGHT_SHIFT_ARRAY(ii,jj,nn)

preconditions ii,jj,nn must be in $0 . . L A T \_m a x i d x$, with $\mathrm{ii} \leq \mathrm{jj}$ and $\mathrm{jj}+\mathrm{nn} \leq$ LAT_maxidx to make possible the possible the shift to the right by nn spaces.

Part ii $+\mathrm{nn} . . \mathrm{jj}+\mathrm{nn}$ receives a copy of part ii..jj of the array (shift nn spaces to the right).

## LEFT_SHIFT_ARRAY

syntax LEFT_SHIFT_ARRAY(iii,jj,nn)
preconditions ii, jj must be in 0 ..LAT_maxidx, with $\mathrm{ii} \leq \mathrm{jj}$. nn must be NAT with $\mathrm{nn} \leq \mathrm{ii}$ to make possible the shift to the left by nn places. For implementation reasons, jj must be not equal MAXINT.
The ii-nn..jj-nn part receives a copy of part ii..jj from the array (shift nn spaces to the left).

## SEARCH_MAX_EQL_ARRAY

syntax $\quad \mathrm{rr}, \mathrm{bb} \leftarrow$ SEARCH_MAX_EQL_ARRAY(ii,jj,vv)
preconditions ii and jj must be in 0 ..LAT_maxidx, $\mathrm{i} \mathrm{i} \leq \mathrm{jj}$ and vv belong to LAT_VALUE.
outputs TRUE if vv was found, FALSE if not rr is a NAT, if $\mathrm{bb}=$ TRUE then rr is the largest index in the array worth vv.
Search for an array element equal to vv, by scanning the ii..jj part starting from jj .

## SEARCH_MIN_EQL_ARRAY

```
syntax rr,bb }\leftarrow\mathrm{ SEARCH_MIN_EQL_ARRAY(ii,jj,vv)
```

preconditions ii and jj must be in 0..LAT_maxidx, $\mathrm{ii} \leq \mathrm{jj}$ and vv belong to LAT_VALUE.
outputs TRUE if vv was found, FALSE if not. rr is a NAT, if $\mathrm{bb}=$ TRUE, then rr is the smallest index in the array worth vv.
Search for an array element that equals vv, by scanning the ii..jj part starting from ii.

## REVERSE_ARRAY

syntax REVERSE_ARRAY(ii,jj)
preconditions ii and jj must be in 0..LAT_maxidx.
Reverse the order of elements in the ii..jj portion of the array.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES)

BASIC_ARITHMETIC; BASIC_BOOL.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_VAR machine (clause IMPORTS BASIC_ARRAY_VAR(...)). Therefore if another instance is necessary, it must be given a different instance name (for example: i1.BASIC_ARRAY_VAR).

### 5.4 L ARRAY5: Array with Ordered Values, Sort Operation

## OPERATIONS

VAL_ARRAY value of an element (promoted operation).
STR_ARRAY
SET_ARRAY

SWAP_ARRAY exchange two elements (promoted operation).
RIGHT_SHIFT_ARRAY
LEFT_SHIFT_ARRAY
SEARCH_MAX_EQL_ARRAY search for a value in a portion of the array (promoted operation).

SEARCH_MIN_EQL_ARRAY search for a value in a portion of the array (promoted operation).

REVERSE_ARRAY reverse the order of the elements in a portion of the array (promoted operation).
SEARCH_MIN_GEQ_ARRAY search for the first element that exceeds a value (promoted operation).
ASCENDING_SORT_ARRAY sort a portion of the array.

## EXAMPLE

| MACHINE | IMPLEMENTATION m1_1 |
| :---: | :---: |
| m1 | REFINES |
| VARIABLES | m1 |
| vv | IMPORTS |
| INVARIANT | L_ARRAY5 $(0,255,4)$ |
| $\begin{aligned} & \text { vv } \in 0 . .4 \rightarrow 0 . .255 \wedge \\ & \quad \forall \mathrm{xx} .(\mathrm{xx} \in 0 . .3 \Rightarrow \end{aligned}$ | INVARIANT <br> arr_vrb = vv |
| $\operatorname{vv}(\mathrm{xx}) \geq \mathrm{vv}(\mathrm{xx}+1))$ | INITIALISATION |
| INITIALISATION | SET_ARRAY $(0,4,50)$; |
| vv : (vv $\in 0 . .4 \rightarrow 0 . .255 \wedge$ | STR_ARRAY $(2,10)$; |
| $\forall x \mathrm{x} .(\mathrm{xx} \in 0.3 \Rightarrow$ | STR_ARRAY $(4,30)$; |
| vv( xx$) \geq \mathrm{vv}(\mathrm{xx}+1))$ ) | ASCENDING_SORT_ARRAY(0,4); |
| END | REVERSE_ARRAY $(0,4)$ |
|  | END |

## DESCRIPTION

L_ARRAY5 is the most complete of the one dimensional array machines. It especially comprises a sort operation implanted using a shift sort (fast algorithm).

## MACHINE PARAMETERS

L_ARRAY5(LAC_minval,LAC_maxval,LAC_maxidx): LAC_minval..LAC_maxval is the set of possible values for the elements in the array, 0..LAC_maxidx is the set of index values for the array. LAC_minval, LAC_maxval, LAC_maxidx must be NATs: this machine does not allow negative values. It is also necessary for LAC_minval $\leq$ LAC_maxval and $1 \leq$ LAC_maxidx.

```
VAL_ARRAY
    syntax vv }\leftarrow\mathrm{ VAL_ARRAY(ii)
    preconditions ii must be in 0..LAC_maxidx
    outputs vv is in LAC_minval..LAC_maxval, is the array value at position ii.
```

```
STR_ARRAY
    syntax STR_ARRAY(ii,vv)
preconditions ii must be in 0..LAC_maxidx and vv in LAC_minval..LAC_maxval and
    LAC_VALUE.
```

The vv value is stored in the array at index ii.

## SET_ARRAY

syntax SET_ARRAY(ii,jj,vv)
preconditions ii..jj must be included in 0..LAC_maxidx and vv must be in LAC_VALUE. For implementation, it is also necessary that jj be different from the MAXINT constant.
The vv value is stored in the array for all indexes from ii to jj . If $\mathrm{ii}>\mathrm{jj}$, the array does not change.

## SWAP_ARRAY

```
    syntax SWAP_ARRAY(ii,jj)
    preconditions ii,jj must be in 0..LAC_maxidx.
```

The ii and jj elements in the array are exchanged.

## RIGHT_SHIFT_ARRAY

syntax RIGHT_SHIFT_ARRAY(ii,jj,nn)
preconditions ii $\mathrm{j} \mathrm{j}, \mathrm{nn}$ must be in 0 ..LAC_maxidx, with $\mathrm{ii} \leq \mathrm{jj}$ and $\mathrm{j} \mathrm{j}+\mathrm{nn} \leq$ LAC_maxidx to make possible the right shift by nn spaces.
The ii $+\mathrm{nn} . . \mathrm{jj}+\mathrm{nn}$ part receives a copy of the ii..jj part of the array (shift right by nn spaces).

## LEFT_SHIFT_ARRAY

syntax LEFT_SHIFT_ARRAY(ii,jj,nn)
preconditions ii, jj must be in 0..LAC_maxidx, with ii $\leq \mathrm{jj}$. nn must be a NAT with nn $\leq \mathrm{ii}$ to allow the left shift by nn spaces. For implementation reasons, jj cannot equal MAXINT.
The ii-nn..jj-nn part receives a copy of the ii..jj part of the array (shift left by nn spaces).

## SEARCH_MAX_EQL_ARRAY

```
syntax rr,bb }\leftarrow\mathrm{ SEARCH_MAX_EQL_ARRAY(ii,jj,vv)
preconditions ii and jj must be in 0..LAC_maxidx, ii\leqjj and vv be in LAC_VALUE.
outputs TRUE if vv was found, FALSE if not. rr is a NAT, if bb = TRUE, then
    rr is the highest index in the array worth vv.
```

Search for an array element equal to vv, by scanning the ii..jj part starting from jj .

```
SEARCH_MIN_EQL_ARRAY
syntax rr,bb }\leftarrow\mathrm{ SEARCH_MIN_EQL_ARRAY(ii,jj,vv)
preconditions ii and jj must be in 0..LAC_maxidx, ii }\leq\textrm{jj}\mathrm{ and vv be in LAC_VALUE.
outputs TRUE if vv was found, FALSE if not. rr is a NAT, if bb = TRUE then
    rr is the smallest index in the array worth vv.
```

Search for an array element equal to vv, by scanning the ii..jj part starting from ii.

## REVERSE_ARRAY

syntax REVERSE_ARRAY(ii,jj)
preconditions ii and jj must be in 0..LAC_maxidx.
Reverse the order of the elements in the ii..jj portion of the array.

```
SEARCH_MIN_GEQ_ARRAY
    syntax \(\quad\) ii,bb \(\leftarrow\) SEARCH_MIN_GEQ_ARRAY \((j j, k k, v v)\)
    preconditions jj and kk must be in \(0 .\). LAC_maxidx, \(\mathrm{j} \mathrm{j} \leq \mathrm{kk}\) and vv be in LAC_minval..
        LAC_maxval. For implementation location reasons, kk must not equal
        the MAXINT constant.
outputs TRUE if an element that is greater or equal to vv was found, FALSE if
        not. ii is a NAT, if \(\mathrm{bb}=\) TRUE, then ii is the smallest index in the image
        array that is greater than or equal to vv.
```

Search for an element that is greater than or equal to vv in $\mathrm{jj} . \mathrm{kk}$ starting from jj .

## ASCENDING_SORT_ARRAY

syntax ASCENDING_SORT_ARRAY(ii,jj)
preconditions ii and jj must be in 0..LAC_maxidx. For implementation reasons, ii and jj must not equal MAXINT.
Shift sort, in ascending order (the smallest first) on the ii..jj portion.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES)
BASIC_ARITHMETIC; BASIC_BOOL.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_VAR machine (clause IMPORTS BASIC_ARRAY_VAR(...)). Therefore if another instance is required it must be given a different instance name (for example: i1.BASIC_ARRAY_VAR).

### 5.5 L PFNC: Partial Function

## OPERATIONS

VAL_PFNC value of the function for an element in its domain
STR_PFNC overloads the partial function with a pair
XST_PFNC test that an index is in the partial function domain
RMV_PFNC removes a pair from the partial function
SET_PFNC overloads a part of the function with a constant
SWAP_PFNC exchanges the images for two domain indexes
RIGHT_SHIFT_PFNC right shift part of the domain
LEFT_SHIFT_PFNC left shift part of the domain
SEARCH_MAX EQL_PFNC search for a value in the partial function
SEARCH_MIN_EQL_PFNC search for a value in the partial function
REVERSE_PFNC reverse the order of elements in a portion of the domain
ASCENDING_SORT_PFNC sort in a portion of the domain

## EXAMPLE

|  | IMPLEMENTATION |
| :--- | :--- |
| MACHINE | m1_1 |
| m1 | REFINES |
| VARIABLES | m1 |
| pf | IMPORTS |
| INVARIANT | L_PFNC(0,255,10) |
| pf $\in 0 . .10 \rightarrow 0 . .255$ | INVARIANT |
| INITIALISATION | pfnc_vrb $=$ pf |
| pf $:=\{4 \mapsto 6\}$ | INITIALISATION |
| END | STR_PFNC $(4,6)$ |
|  | END |

## DESCRIPTION

L_PFNC implements a partial function with almost all of the operations available in L_ARRAY5 (In fact only SEARCH_MIN_GEQ is not used). The practical usefulness of partial functions is that they dispense with the need to add a "non existent" or "unused" element in the input sets in order to implant them as total functions. The implementation of L_PFNC performs these elements by using the seldom used MAXINT value.

## MACHINE PARAMETERS

L_PFNC(LPF_minval,LPF_maxval,LPF_maxidx): LPF_minval..LPF_maxval is the input set of the function, 0..LPF_maxidx is the source set. LPF_minval, LPF_maxval,

LPF_maxidx must be NATs: this machine does not allow negative values. Moreover, LPF_minval $\leq L P F \_$maxval and $1 \leq L P F \_$maxidx; as well as LPF_maxval<MAXINT: This is because MAXINT is used to indicate that the corresponding index is not part of the partial function. Again to simplify implementation, it is also illegal to have LPF_maxidx $=$ MAXINT.

VAL_PFNC
syntax $\quad$ vv $\leftarrow$ VAL_PFNC(ii)
preconditions ii must be in the partial function domain
outputs $\quad$ vv is in LPF_minval..LPF_maxval, it is the value of the array at position ii.

## STR_PFNC

syntax STR_PFNC(ii,vv)
preconditions ii must be in 0..LPF_maxidx and vv be in LPF_minval..LPF_maxval.
The partial function is overloaded by $\{\mathrm{ii} \mapsto \mathrm{vv}\}$.

## XST_PFNC

$$
\text { syntax } \quad \mathrm{bb} \leftarrow \mathrm{XST} \text {-PFNC(ii) }
$$

outputs bb is TRUE if ii is in the domain of the function, FALSE if not.

## RMV_PFNC

syntax RMV_PFNC(ii)
preconditions ii must be in the domain of the partial function.
The $\left\{\mathrm{ii} \mapsto \mathrm{pfnc} \_\right.$vrb(ii) $\}$pair is removed from the partial function pfnc_vrb.

## SET_PFNC

$$
\text { syntax } \quad \text { SET_PFNC(ii,jj,vv) }
$$

preconditions ii..jj must be included in 0..LPF_maxidx and vv be in LPF_minval..LPF_maxval. ii and jj must be NATs.
The partial function is overloaded by (ii..jj) $\times \mathrm{vv}$. If ii $>\mathrm{jj}$,
ii..jj is blank and the partial function is not modified, but it is still necessary for ii and jj to be NATs.

## SWAP_PFNC

syntax SWAP_PFNC(ii,jj)
preconditions ii,jj must be in the domain of the partial function.
The ii and jj elements in the array are exchanged.

## RIGHT_SHIFT_PFNC

syntax RIGHT_SHIFT_PFNC(ii,jj,nn)
preconditions ii, $\mathrm{j}, \mathrm{nn}$ must be in $0 . . \mathrm{LPF} \_$maxidx, with $\mathrm{ii} \leq \mathrm{jj}$ and $\mathrm{j}+\mathrm{nn} \leq$ LPF_maxidx to allow the right shift by nn spaces. It is also necessary for ii..jj to be included in the domain of the partial function.
The $\mathrm{ii}+\mathrm{nn} . . \mathrm{jj}+\mathrm{nn}$ part is overloaded by a copy of the ii..jj part in the partial function (shift by nn spaces to the right).

## LEFT_SHIFT_PFNC

syntax LEFT_SHIFT_PFNC(ii,jj,nn)
preconditions ii, jj must be in 0 ..LPF_maxidx, with $\mathrm{ii} \leq \mathrm{jj}$. nn must be a NAT with $\mathrm{nn} \leq$ ii to allow the left shift by nn spaces. In addition it is necessary for ii...jj to be included in the domain of the partial function.
The ii-nn..jj-nn part is overloaded by a copy of the ii..jj part in the partial function (shift left by $n n$ spaces).

## SEARCH_MAX_EQL_PFNC

```
syntax rr,bb }\leftarrow\mathrm{ SEARCH_MAX_EQL_PFNC(ii,jj,vv)
preconditions ii and jj must be in 0..LPF_maxidx, ii\leqjj and vv be in LPF_minval..LPF-
    _maxval.
outputs TRUE if vv was found, FALSE if not, rr is a NAT, if bb = TRUE, then rr is the largest index, the image of which by the partial function is vv .
```

Search for an array element that equals vv, by scanning the ii..jj part, starting from jj .

## SEARCH_MIN_EQL_PFNC

$$
\text { syntax } \quad \text { rr, } \mathrm{bb} \leftarrow \mathrm{SEARCH} \_M I N \_E Q L \_P F N C(\mathrm{ii}, \mathrm{jj}, \mathrm{vv})
$$

preconditions ii and jj must be in $0 . . \mathrm{LPF}$ _maxidx, $\mathrm{i} \leq \mathrm{jj}$ and vv be in LPF_minval..LPF_maxval.
outputs TRUE if vv was found, FALSE if not, rr is a NAT, if $\mathrm{bb}=$ TRUE, then rr is the smallest index, the image of which by the partial function is vv.
Search for an array element that equals vv, by scanning the ii..jj part starting from ii.

## REVERSE_PFNC

syntax REVERSE_PFNC(iii,jj)
preconditions ii and jj must be in 0..LPF_maxidx, and ii..jj must be included in the domain of the partial function.

Reverse the order of the elements in the ii..jj portion of the partial function.

## ASCENDING_SORT_PFNC

syntax ASCENDING_SORT_PFNC(ii,jj)
preconditions ii and jj must be in 0..LPF_maxidx, and ii..jj must be included in the domain of the partial function.
Shift sort, in ascending order (the smallest first) in the ii..jj portion.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES) BASIC_ARITHMETIC; BASIC_BOOL.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_VAR machine (clause IMPORTS BASIC_ARRAY_VAR(...)). Therefore if another instance is necessary, it must be given a different instance name (for example: i1.BASIC_ARRAY_VAR).

### 5.6 L SEQUENCE: Creating a Sequence

## OPERATIONS

LEN_SEQ returns the current size of the sequence.
IS_FULL_SEQ shows whether the sequence is full (size = LS_maxsize).
IS_INDEX_SEQ shows whether ii is a valid index.
VAL_SEQ value of an element in the sequence.
FIRST_SEQ returns the first element in the sequence.
LAST_SEQ returns the last element in the sequence.
PUSH_SEQ adds vv to the end of the sequence.
POP_SEQ removes the last element from the sequence (its value is lost).
STR_SEQ changes the value of an element in the sequence.
RMV_SEQ removes an element from the middle of the sequence.
INS_AFT_SEQ inserts vv right after index ii.
CLR_SEQ clears the sequence.
TAIL_SEQ removes the first element from the sequence.
KEEP_SEQ only keeps the nn first elements in the sequence.
CUT_SEQ cuts the nn first elements from the sequence.
PART_SEQ only keeps the ii..jj portion in the sequence.
REV_SEQ reverses the order of the elements in the sequence.
FIND_FIRST_SEQ searches for vv in the sequence, starting from the beginning.
FIND_LAST_SEQ searches for vv in the sequence, starting from the end.

## EXAMPLE

The example below shows the use of L_SEQUENCE for a listed set.

|  | IMPLEMENTATION |
| :--- | :--- |
| MACHINE | m1_1 |
| m1 | REFINES |
| SETS | m1 |
| ST $=$ \{classic,baroque\} | IMPORTS |
| VARIABLES | L_SEQUENCE(10,ST) |
| vv | INVARIANT (seq_vrb is the variable in L_SEQUENCE) |
| INVARIANT | seq_vrb = vv |
| vv $\in$ seq(ST) $\wedge$ | INITIALISATION |
| size(vv) $\leq 10$ | PUSH_SEQ(baroque) (L_SEQUENCE guarantees |
| INITIALISATION | PUSH_SEQ(baroque) that the sequence is empty at the start) |
| vv $:=[$ baroque,baroque] | END |
|  |  |

## DESCRIPTION

L_SEQUENCE provides a sequence type variable, the maximum size of which is a machine parameter. Conventional search and shift functions are provided for the practical use of this sequence. This answers the frequent problem in programming applications which is to maintain a list with no blanks.

## MACHINE PARAMETERS

L_SEQUENCE(LS_maxsize,LS_VALUE): the variable is a sequence of LS_VALUE elements, with a maximum size that is LS_maxsize.

## LEN_SEQ

| syntax | $\mathrm{nn} \leftarrow$ LEN_SEQ |
| :--- | :--- |
| outputs | $0 .$. LS_maxsize |

Returns the current size of the sequence.

## IS_FULL_SEQ

```
syntax bb }\leftarrow\mathrm{ IS_FULL_SEQ
outputs bb is TRUE if the sequence is full, FALSE if not.
```

Specifies whether the sequence is full (size = LS_maxsize).

```
IS_INDEX_SEQ
syntax bb \leftarrow IS_INDEX_SEQ(ii)
preconditions ii must be a NAT.
outputs bb is TRUE if ii is an index in the sequence, FALSE if not.
Specifies whether ii is a valid index.
```


## VAL_SEQ

syntax $\quad$ vv $\leftarrow$ VAL_SEQ(ii)
preconditions ii must be an index in the sequence (ii $\in 1$..size(seq_vrb)).
outputs $\quad \mathrm{vv}$ is the value of the ii-ith element (vv $\in$ VALUE).
Value of an element in the sequence.

## FIRST_SEQ

syntax $\quad$ vv $\leftarrow$ FIRST_SEQ
preconditions the sequence must not be empty.
outputs $\quad \mathrm{vv}$ is the value of the first element ( $\mathrm{vv} \in$ VALUE).
Returns the first element in the sequence.

## LAST_SEQ

syntax $\quad \mathrm{vv} \leftarrow$ LAST_SEQ
preconditions the sequence must not be empty.
outputs $\quad \mathrm{vv}$ is the value of the last element ( $\mathrm{vv} \in$ VALUE).
Returns the last element in the sequence.

## PUSH_SEQ

syntax PUSH_SEQ(vv)
preconditions vv must be in VALUE and the sequence must not be full.
Add vv at the end of the sequence.

## POP_SEQ

syntax POP_SEQ
preconditions the sequence must not be empty.
Removes the last element from the sequence (its value is lost).

## STR_SEQ

```
    syntax STR_SEQ(ii,vv)
```

preconditions vv must be in VALUE and ii must be a valid index for the sequence.
Changes the value of an existing element in the sequence.

## RMV_SEQ

syntax RMV_SEQ(ii)
preconditions ii must be a valid index in the sequence.
Removes an element from the middle of the sequence.

## INS_AFT_SEQ

syntax INS_AFT_SEQ(ii,vv)
preconditions vv must be in VALUE and ii must be a valid index for the sequence. The sequence must not be full.
Inserts vv right after index ii.

## CLR_SEQ

$$
\text { syntax } \quad \text { CLR_SEQ }
$$

Clears the sequence.

## TAIL_SEQ

syntax TAIL_SEQ
preconditions the sequence must not be empty.
Removes the first element from the sequence.
KEEP_SEQ
syntax KEEP_SEQ(nn)
preconditions nn must be a NAT.
Only retains the nn first elements in the sequence. For $\mathrm{nn}=$ size(seq_vrb), this operation does not take action.

## CUT_SEQ

```
syntax CUT_SEQ(nn)
```

preconditions nn must be a NAT.
Deletes the nn first elements from the sequence. For $\mathrm{nn}=$ size(seq_vrb), this operation is equivalent to CLR_SEQ.

## PART_SEQ

syntax PART_SEQ(ii,jj)
preconditions ii and jj must be non null NATs, with $\mathrm{ii} \leq \mathrm{jj}$.
Only retains the ii..jj portion in the sequence. ii..jj may not be included in the domain of the sequence.

## REV_SEQ

```
syntax REV_SEQ
```

Reverses the order of the elements in the sequence. Applies even for sequences that are empty or of size 1 .

## FIND_FIRST_SEQ

```
syntax bb,ii }\leftarrow\mathrm{ FIND_FIRST_SEQ(vv)
preconditions vv must be in VALUE.
outputs bb is TRUE if vv is in the sequence, FALSE if not. ii belongs to the range
    1..LS_maxsize, if bb = TRUE, then it indicates the first position equal
    to vv.
```

Search for vv in the sequence, starting from the start.
FIND_LAST_SEQ
syntax bb,ii $\leftarrow$ FIND_LAST_SEQ(vv)
preconditions vv must be in VALUE.
outputs bb is TRUE if vv is in the sequence, FALSE if not. If $\mathrm{bb}=$ TRUE, ii belongs to the range 1..LS_maxsize and indicates the last position equal to vo.

Search for vv in the sequence, starting from the end.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES) BASIC_ARITHMETIC; BASIC_BOOL.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_VAR machine (clause IMPORTS BASIC_ARRAY_VAR(...)). Therefore if another instance is required, it must be given a non blank instance name (for example: i1.BASIC_ARRAY_VAR).

### 5.7 L SET: Creating a Set

## OPERATIONS

CARD_SET returns the cardinal for the set.
IS_FULL_SET identifies whether the set is full (card = LSET_maxsize).
FIND_SET finds an element in the set.
RMV_SET removes an element from the set.
INS_SET inserts an element in the set.
CLR_SET removes all of the elements from the set.
IS_INDEX_SET identifies whether a number is a valid index.
VAL_SET value of an element in the set.

## EXAMPLE

The example below shows the use of L_SET on a listed set.

|  | IMPLEMENTATION |
| :--- | :--- |
| MACHINE | m1_1 |
| m1 | REFINES |
| SETS | m1 |
| ST $=\{$ cat, dog, bird $\}$ | IMPORTS |
| VARIABLES | L_SET(3,ST) |
| vv | INVARIANT |
| INVARIANT | (set_vrb is the variable in L_SET) |
| vv $\subseteq$ ST | ran (set_vrb) = vv |
| INITIALISATION | INITIALISATION |
| vv $:=\{$ cat,bird $\}$ | (L_SET ensures that the set is empty at the start) |
| END | INS_SET(cat); |
|  | INS_SET(bird) |
|  | END |
|  |  |

## DESCRIPTION

L_SET creates a set that is modeled by an injective sequence type variable, set_vrb the maximum size of which is a machine parameter. It offers functions to search for, add and delete elements.

The use of an injective sequence type variable enables easy access to each element of the set via an index. The user can therefore create loops by using the CARD_SET and VAL_SET functions. This would not have been possible if the variable directly represented the set.
WARNING: The user must add the gluing invariant ran(set_vrb) = var_locale to his machine in order to link his set variable with the L_SET machine state.

## MACHINE PARAMETERS

L_SET (LSET_maxsize, LSET_VALUE): the variable is an injective sequence of elements from LSET_VALUE, with a maximum size LSET_maxsize.

## CARD_SET

```
syntax nn \leftarrow CARD_SET
output nn is the size of the set (the cardinal of ran (set_vrb)). Therefore, nn
    belongs to 0.. LSET_maxsize
```

Returns the size of the set.

## IS_FULL_SET

```
syntax bb }\leftarrow\mathrm{ IS_FULL_SET
output bb is TRUE if the set is full, FALSE if not.
```

States whether the set is full (size = LSET_maxsize).

## IS_INDEX_SET

```
syntax bb \leftarrow IS_INDEX_SET(ii)
preconditions ii must be a NAT.
outputs bb is TRUE if ii is an index of the set, FALSE if not.
```

States whether ii is a valid index.

## VAL_SET

syntax $\quad$ vv $\leftarrow$ VAL_SET(ii)
preconditions ii must be an index of the set (ii $\in$ 1..size(seq_vrb)).
outputs $\quad v v$ is the value of the ii-the element ( $\mathrm{vv} \in \operatorname{LSET}$ VALUE).
Value of an element of the set.

## FIND_SET

syntax bb, ii $\leftarrow$ FIND_SET(vv)
preconditions vv must be in LSET_VALUE.
outputs bb is TRUE if vv is in the set, FALSE if not. ii is a NAT, if $\mathrm{bb}=$ TRUE, then it indicates the position of element vv.
Search for vv in the set.

## RMV_SET

syntax RMV_SET(vv)
preconditions vv must be in the set.
Removes an element from the set.

## INS_SET

syntax INS_SET(vv)
preconditions vv must be in LSET_VALUE.
Adds an element to the end of the set, if it is not already in it, if not it does nothing.

## CLR_SET

syntax CLR_SET
Clears the set.

### 5.8 L ARRAY1_ RANGE: A Range of Arrays of the Same Size, with Numerical Indexes

## OPERATIONS

VAL_ARR_RGE
STR_ARR_RGE
COP_ARR_RGE
CMP_ARR_RGE
DUP_ARR_RGE
SET_ARR_RGE
PCOP_ARR_RGE
PCMP_ARR_RGE
value of an element (promoted operation). write an element (promoted operation). copy an array to another (promoted operation). compare two arrays (promoted operation) duplicate the same array to a series of arrays. copy the same value to an index interval in one of the arrays. copy part of one array to a different array, to a given position. search for the first element that is different between two parts of two arrays. A Boolean element indicates whether this element was found and, in this case, the index of this element in returned.

## EXAMPLE

Using SET_ARR_RGE and DUP_ARR_RGE to initialize a set of arrays:

|  | IMPLEMENTATION <br> MACHINE <br> m1 |
| :--- | :--- |
| VARIABLES | m1_1 |
| REFINES |  |
| vv | m1 |
| INVARIANT | IMPORTS |
| vv $\in 0 . .20 \rightarrow(0 . .10 \rightarrow 0 . .255)$ | i1.L_ARRAY1_RANGE $(0,20,10,0 . .255)$ |
| INITIALISATION | INARIANT |
| vv $:=(0 . .20) \times\{(0 . .10) \times\{5\}\}$ | INITr_rge $=$ vv |
| END | i1.SET_ARR_RGE $(0,0,10,5) ;$ |
|  | i1.DUP_ARR_RGE $(1,20,0)$ |
|  | END |

## DESCRIPTION

L_ARRAY1_RANGE is used in place of BASIC_ARRAY_RANGE, so that a range of arrays may create a set of function type abstract variables when operations are required to perform complete array initialization.
It also allows performing operations that use parts of two different arrays.
The index and range sets are intervals so that it is possible to indicate only portions of these sets without listing all elements involved.

## MACHINE PARAMETERS

L_ARRAY1_RANGE (LAUR_minrge, LAUR_maxrge, LAUR_maxidx, LAUR_VALUE): The range interval is the LAUR_minrge..LAUR maxrge interval, the index interval is $0 . . L A U R \_m a x i d x$ and LAUR_VALUE is the set of possible values.

```
VAL_ARR_RGE
    syntax vv }\leftarrow\mathrm{ VAL_ARR_RGE (range, index)
    preconditions range must belong to LAUR_minrge..LAUR_maxrge and index belong to
    0..LAUR_maxidx.
    outputs vv is a LAUR_VALUE, it is the value of the array range at the index
    position.
```


## STR_ARR_RGE

syntax STR_ARR_RGE (range, index, value)
preconditions range must belong to LAUR_minrge..LAUR maxrge, index belong to 0 ..LAUR_maxidx and value belong to LAUR_VALUE.

The value data value is stored in the indexed array range.

## COP_ARR_RGE

syntax COP_ARR_RGE (dest, src)
preconditions dest and src are in LAUR_minrge..LAUR_maxrge
The src array is copied to the dest array.

## CMP_ARR_RGE

syntax $\quad \mathrm{bb} \leftarrow \mathrm{CMP} \_$ARR_RGE (range1, range2)
preconditions range1 and range2 are in LAUR_minrge..LAUR_maxrge
outputs $\quad b b$ is a BOOL element that is TRUE if the two arrays are equal and FALSE if not.

## SET_ARR_RGE

syntax SET_ARR_RGE (range,iii,jj,vv)
preconditions range must belong to LAUR_minrge..LAUR maxrge, ii..jj be included in 0. .LAUR maxidx and vv belong to LAUR_VALUE. For implementation reasons, jj must also be different from MAXINT.
The vv value is stored in the array range for all index values between ii and jj . If $\mathrm{ii}>\mathrm{jj}$, the array remains unchanged.

## DUP_ARR_RGE

syntax DUP_ARR_RGE (dest1, dest2, src)
preconditions dest1, dest2, src are in LAUR_minrge..LAUR_maxrge. For implementation reasons, dest2 must also be different from MAXINT.
The src array is duplicated in all of the arrays of the dest1..dest2 interval.

PCOP_ARR_RGE
syntax PCOP_ARR_RGE (dest, idx_dst, src,ii,jj)
preconditions dest and src must be different elements of LAUR_minrge..LAUR_maxrge, ii..jj be a non empty interval of 0..LAUR_maxidx, idx_dst belong to 0..LAUR_maxidx, jj be different from MAXINT and idx_dst + jj - ii belong to 0..LAUR_maxidx (condition necessary to ensure that the copy does not overflow).
The ii..jj part of the src array is copied to the dest array, from the idx_dst index.

## PCMP_ARR_RGE

$$
\text { syntax } \quad \mathrm{idx}, \mathrm{bb} \leftarrow \mathrm{PCMP} \_A R R \_R G E(\text { rng2,idx2,rng1,ii,jj })
$$

preconditions rng1 and rng2 must belong to LAUR_minrge..LAUR_maxrge, ii..jj be a non empty interval of 0..LAUR_maxidx, idx2 and idx $2+\mathrm{jj}$-ii are in 0..LAUR_maxidx.

The ii..jj part of array rng1 is compared to the part with the same size in the rng2 array. The idx $2+\mathrm{jj}$-ii $\in 0 . . L A U R \_m a x i d x$ condition guarantees that this comparison is possible. bb is a Boolean element that is FALSE if the two parts are equal and TRUE if they are different. In the latter case, idx and index are the first element that is different from ii..jj.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES) BASIC_ARITHMETIC, BASIC_BOOL.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_RANGE machine (IMPORTS BASIC_ARRAY_RANGE(...) clause). Therefore if another instance is necessary, it must be given the name of a non empty instance (for example: i1.BASIC_ARRAY_RANGE).

### 5.9 L ARRAY3_RANGE: A Range of Arrays of the Same Size, with Non Ordered Values, Maximum Operations

## OPERATIONS

VAL_ARR_RGE
STR_ARR_RGE
COP_ARR_RGE
CMP_ARR_RGE
DUP ARR RGE
SET_ARR_RGE

PCOP_ARR_RGE

PCMP_ARR_RGE

SWAP_RGE exchange two array elements.
RIGHT_SHIFT_RGE shift part of an array to the large index.
LEFT_SHIFT_RGE shift part of an array to the small index.
SEARCH_MAX_EQL_RGE search for the last element that equals a value in part of an array.

SEARCH_MIN_EQL_RGE search for the first element that equals a value in part of an array.
REVERSE_RGE reverse the order of the elements in part of an array.

## EXAMPLE

The following example is a machine that represents the color assigned to 101 dots for each array in a range; this color may be red, green or blue for each dot. A operation enables finding a red dot in an array.

```
MACHINE
    m3
SETS
    COLOR = {red,green,blue }
VARIABLES
    color
INVARIANT
    color }\in0..10->(0..100 -> COLOR
INITIALISATION
color:=(0..10) }\times{(0..100)\times{red}
OPERATIONS
    ii,bb \longleftarrow find_red(rng) = PRE
    rng \in 0..10 ^
    rouge }\in\operatorname{ran(color(rng))
THEN
    ii:\in color(rng)-1 [{red}] ||
    bb:\in BOOL
END
END
```

```
IMPLEMENTATION
    m3_1
REFINES
    m3
IMPORTS
    i1.L_ARRAY3_RANGE(0,10,100,COLOR)
INVARIANT
    i1.arr_rge = color
INITIALISATION
    il.SET_ARR_RGE(0,0,100,red);
    il.DUP_ARR_RGE(1,10,0)
OPERATIONS
    ii,bb \longleftarrow find_red(rng) = var bb IN
        ii,bb}
            i1.SEARCH_MAX_EQL_RGE(rng,0,100,red)
    END
END
```


## DESCRIPTION

L_ARRAY3_RANGE is the most complete of the two dimensional array machines with no constraint [3] This makes it possible to create arrays with values that are the elements of an enumerated set, while retaining access to complete operations such are reversing the order of elements.

The operation that is not available is the one that would require an order relation on the elements in the array: sort.

## MACHINE PARAMETERS

L_ARRAY3_RANGE (LATR_minrge, LATR_maxrge, LATR_maxidx, LATR_VALUE):
The range interval is LATR minrge..LATR maxrge, the index interval 0..LATR_maxidx and LATR_VALUE is the set of possible values.

## VAL_ARR_RGE

```
syntax vv }\leftarrow\mathrm{ VAL_ARR_RGE (range, index)
preconditions range must belong to LATR_minrge..LATR_maxrge, index belong to
    0..LATR_maxidx
outputs vv is a LATR_VALUE, it is the value of the array range at the index
    position.
```


## STR_ARR_RGE

syntax STR_ARR_RGE (range, index, value)
preconditions range must belong to LATR_minrge..LATR_maxrge, index belong to 0..LATR_maxidx and value belong to LATR_VALUE.

The LATR_VALUE value is stored in the array range in the index.

[^3]
## COP_ARR_RGE

```
    syntax COP_ARR_RGE (dest, src)
    preconditions dest and src are in LATR_minrge..LATR_maxrge
```

The src array is copied to the dest array.

```
CMP_ARR_RGE
    syntax bb \leftarrow CMP_ARR_RGE (range1, range2)
preconditions range1 and range2 are in LATR_minrge..LATR_maxrge
outputs bb is an BOOL that equals TRUE if the two arrays are equal and FALSE
    if not.
```


## SET_ARR_RGE

```
syntax SET_ARR_RGE (range,ii,jj,vv)
preconditions range must belong to LATR_minrge..LATR_maxrge, ii..jj be included in 0..LATR_maxidx and vv belong to LATR_VALUE. For implementation reasons, jj must also be different to MAXINT.
```

Value vv is stored in the array range for all indexes in the range from ii to jj . If $\mathrm{ii}>\mathrm{jj}$, the array remains unchanged.

## DUP_ARR_RGE

```
syntax DUP_ARR_RGE (dest1, dest2, src)
```

preconditions dest1, dest2, src are in LATR_minrge..LATR_maxrge. For implementation reasons, dest2 must also be different to MAXINT.
The src array is duplicated in all of the arrays of the dest1..dest2 interval.

## PCOP_ARR_RGE

syntax PCOP_ARR_RGE (dest, idx_dst, src,ii,jj)
preconditions dest and src must belong to LATR_minrge..LATR_maxrge and be different, ii...jj be a non empty interval of 0..LATR_maxidx, idx_dst belong to 0..LATR_maxidx, jj be different from MAXINT and idx_dst +jj - ii belong to 0 ..LATR_maxidx (necessary condition to avoid copy overflow).
The ii..jj part in the src array is copied to the dest array, from the idx_dst index.

## PCMP_ARR_RGE

syntax idx, bb $\leftarrow$ PCMP_ARR_RGE (rng2,idx2,rng1,ii,jj)
preconditions rng1 and rng2 are in LATR_minrge..LATR_maxrge, ii..jj is a non empty interval of 0 ..LATR_maxidx idx 2 and idx $2+\mathrm{jj}$-ii are in 0 ..LATR_maxidx.

The ii..jj part of array rng1 is compared with the part with the same size in array rng2. The idx $2+\mathrm{jj}$-ii $\in 0$..LATR_maxidx condition guarantees that this comparison is possible. bb is a Boolean element that is FALSE if the two parts are equal and TRUE if they are different. In the latter case, idx is the index of the first element that is different to ii..jj.

## SWAP_RGE

```
syntax SWAP_RGE (rng,ii,jj)
```

preconditions rng is in LATR_minrge..LATR_maxrge, ii and jj in 0..LATR_maxidx.
The ii and jj elements in the array are exchanged.

## RIGHT_SHIFT_RGE <br> syntax RIGHT_SHIFT_RGE (rng,iii,j,nn) <br> preconditions rng must belong to LATR_minrge..LATR_maxrge, $\mathrm{ii}, \mathrm{jj}$ and nn belong to 0 ..LATR_maxidx, with $\mathrm{ii} \leq \mathrm{jj}$ and $\mathrm{j} \mathrm{j}+\mathrm{nn} \leq$ LATR_maxidx to allow a right shift by nn spaces.

The $\mathrm{ii}+\mathrm{nn} . . \mathrm{jj}+\mathrm{nn}$ part in the rng array receives a copy of the ii..jj part of this same array (shift right by nn spaces).

## LEFT_SHIFT_RGE

syntax LEFT_SHIFT_RGE (rng,ii,jj,nn)
preconditions rng is in LATR_minrge..LATR_maxrge; ii,jj must be in 0..LATR_maxidx, with $\mathrm{ii} \leq \mathrm{jj}$. nn must be a NAT with $\mathrm{nn} \leq \mathrm{ii}$ to allow the left shift by nn spaces. For implementation reasons, jj must be equal to MAXINT.
The ii-nn..jj-nn part of the rng array receives a copy of the ii..jj part of this same array (shift left by nn spaces).

## SEARCH_MAX_EQL_RGE

$$
\text { syntax } \quad \text { rr,bb } \leftarrow \text { SEARCH_MAX_EQL_RGE }(\mathrm{rng}, \mathrm{ii}, \mathrm{jj}, \mathrm{vv})
$$

preconditions rng must be in LATR_minrge..LATR_maxrge. ii and jj must be in $0 .$. LATR maxidx, $\mathrm{ii} \leq \mathrm{jj}$ and vv must belong to LATR_VALUE.
outputs $\quad$ TRUE if vv was found, FALSE if not. rr is a NAT, if $\mathrm{bb}=$ TRUE then rr is the largest index in the rng array equal to vv.
Search for an element in an array equal to vv, by scanning the ii..jj part starting from jj .

## SEARCH_MIN_EQL_RGE

$$
\begin{array}{ll}
\text { syntax } & \mathrm{rr}, \mathrm{bb} \leftarrow \mathrm{SEARCH} \_ \text {MIN_EQL_RGE }(\mathrm{rng}, \mathrm{ii}, \mathrm{jj}, \mathrm{vv}) \\
\text { preconditions } & \text { rng must belong to LATR_minrge..LATR_maxrge, ii and } \mathrm{jj} \text { belong to } \\
& \text { 0..LATR_maxidx, ii } \leq \mathrm{jj} \text { and vv belong to LATR_VALUE. } \\
\text { outputs } & \begin{array}{l}
\text { TRUE if vv was found, FALSE if not. rr is a NAT, if bb = TRUE, then } \\
\text { rr is the smallest index in the rng array equal to vv. }
\end{array}
\end{array}
$$

Search for an element in an array that is equal to vv, by scanning the ii..jj part starting from ii.

## REVERSE_RGE

```
syntax REVERSE_RGE(rng,ii,jj)
preconditions rng must belong to LATR_minrge..LATR_maxrge, ii and jj belong to 0 ..LATR_maxidx.
```

Reversing the order of elements in the ii..jj part of the rng array.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES) BASIC_ARITHMETIC; BASIC_BOOL.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_RANGE machine (IMPORTS BASIC_ARRAY_RANGE(...) clause). Therefore if another instance is necessary, it must be given a non empty instance name (for example: i1.BASIC_ARRAY_RANGE)

### 5.10 L ARRAY5_RANGE: Range of Arrays of the Same Size, with Ordered Value Numerical Indexes, Sort Operation

## OPERATIONS

VAL_ARR_RGE
STR_ARR_RGE
COP_ARR_RGE
CMP_ARR_RGE
DUP_ARR_RGE
SET_ARR_RGE

PCOP_ARR_RGE

PCMP_ARR_RGE
search for the first different element between two parts of two arrays. A Boolean element indicates whether this element was found and, in this case, the index of this element is returned (promoted operation).

SWAP_RGE exchange two elements in an array (promoted operation).
RIGHT_SHIFT_RGE shift a part of an array to the large index (promoted operation).
LEFT_SHIFT_RGE shift a part of an array to the small index (promoted operation).
SEARCH_MAX_EQL_RGE search for the last element that is equal to a value in an array range (promoted operation).

SEARCH_MIN_EQL_RGE search for the fist element that equals a value in an array range (promoted operation).
REVERSE_RGE reverse the order of the elements of a part of an array (promoted operation).
SEARCH_MIN_GEQ_RGE search for the first element that exceeds a value in an array range.
ASCENDING_SORT_RGE sort part of an array and arrange in ascending order.

## EXAMPLE

```
\
MACHINE
    m5
VARIABLES
    vv
INVARIANT
    vv \in 0..20 -> (0..4 -> 0..255) ^
    \forall(xx,yy).(xx < 0..20 ^ yy \in 0..3
    => vv(yy)(xx)>=vv(yy)(xx+1))
INITIALISATION
    vv:(vv \in 0..20 -> (0..4 -> 0..255) ^
    \forall(xx,yy).(xx \in 0..20 ^ yy \in 0..3 =
        vv(yy)(xx)>=vv(yy)(xx+1)))
END
```

```
    m5_1
```

    m5_1
    REFINES
REFINES
m5
m5
IMPORTS
IMPORTS
L_ARRAY5_RANGE(0,20,4,0,255)
L_ARRAY5_RANGE(0,20,4,0,255)
INVARIANT
INVARIANT
arr_rge = vv
arr_rge = vv
INITIALISATION
INITIALISATION
SET_ARR_RGE(0,0,4,50);
SET_ARR_RGE(0,0,4,50);
STR_ARR_RGE (0,2,10);
STR_ARR_RGE (0,2,10);
STR_ARR_RGE(0,4,30);
STR_ARR_RGE(0,4,30);
ASCENDING_SORT_RGE(0,0,4);
ASCENDING_SORT_RGE(0,0,4);
REVERSE_RGE(0,0,4);
REVERSE_RGE(0,0,4);
DUP_ARR_RGE(1,20,0)
DUP_ARR_RGE(1,20,0)
END

```
END
```


## DESCRIPTION

L_ARRAY5_RANGE is the most complete two dimensional array machines. It especially contains a sort operation, implanted by a shift sort (fast algorithm).

## MACHINE PARAMETERS

L_ARRAY5_RANGE (LACR_minrge, LACR_maxrge, LACR_maxidx, LACR_minval, LACR_maxval):
LACR_minrge..LACR_maxrge is the set of ranges, 0..LACR_maxidx is the set of indexes and LACR_minval..LACR_maxval, the set of possible values. All of the parameters must be NATs: this machine does not allow negative values.
In addition, LACR minrge $\leq$ LACR maxrge, $1 \leq$ LACR_maxidx and LACR_minval $\leq$ LACR maxval.

VAL_ARR_RGE
syntax $\quad$ vv $\leftarrow$ VAL_ARR_RGE (range, index)
preconditions range must belong to LACR_minrge..LACR_maxrge, index belong to $0 .$. LACR maxidx.
outputs $\quad$ vv is a LACR_VALUE, it is the value of the array range at the index position.

## STR_ARR_RGE

syntax STR_ARR_RGE (range, index, value)
preconditions range must be in LACR_minrge..LACR_maxrge index must be in 0 .. LACR_maxidx value must belong to LACR_VALUE.

The value of the value element is stored in the array range as an index.

## COP_ARR_RGE

```
    syntax COP_ARR_RGE (dest, src)
    preconditions dest and src are in LACR_minrge..LACR_maxrge
```

The src array is copied to the dest array.

```
CMP_ARR_RGE
    syntax bb }\leftarrow\mathrm{ CMP_ARR_RGE (range1, range2)
preconditions range1 and range2 are in LACR_minrge..LACR_maxrge
outputs bb is a BOOL element that is TRUE if the two arrays are equal and
    FALSE if not.
```


## SET_ARR_RGE

```
syntax SET_ARR_RGE (range,ii,jj,vv)
preconditions range must belong to LACR_minrge..LACR_maxrge, ii..jj be included in
    0..LACR_maxidx and vv belong to LACR_VALUE. For implementation
    reasons, it is also necessary that jj be different from MAXINT.
```

The vv value is stored in the array range for all indexes between ii and jj . If $\mathrm{ii}>\mathrm{jj}$, the array remains unchanged.

## DUP_ARR_RGE

```
syntax DUP_ARR_RGE (dest1, dest2, src)
```

preconditions dest1, dest2, src are in LACR_minrge..LACR_maxrge. For implementation reasons, it is also necessary for dest2 to be different from MAXINT.

The src array is duplicated to all arrays for the dest1..dest2 range.

## PCOP_ARR_RGE

syntax PCOP_ARR_RGE (dest, idx_dst, src,ii,jj)
preconditions dest and src must be different elements of LACR_minrge..LACR_maxrge, ii..jj be a non empty subset of 0..LACR_maxidx and idx_dst belong to 0..LACR_maxidx; jj is different from MAXINT and idx_dst + jj - ii belong to 0 ..LACR_maxidx (condition to avoid copy overflow).
The ii..jj range in the src array is copied to the dest array, for the idx_dst index.

## PCMP_ARR_RGE

```
syntax idx, bb \leftarrow PCMP_ARR_RGE (rng2,idx2,rng1,ii,jj)
```

preconditions rng1 and rng2 are in LACR_minrge..LACR_maxrge, ii..jj is a non empty range 0 ..LACR_maxidx, idx 2 and $\mathrm{idx} 2+\mathrm{jj}$-ii are in 0 ..LACR_maxidx.

The ii..jj part of the rng1 array is compared with the part of the same size in the rng2 array. The $\mathrm{idx} 2+\mathrm{jj}$-ii $\in 0$..LACR_maxidx condition guarantees that this comparison is possible. bb is a Boolean element that is FALSE if the two parts are equal and TRUE if they are different. In the latter case, idx is the index of the first element that is different from ii..jj.

## SWAP_RGE

syntax SWAP_RGE (rng,ii,jj)
preconditions rng is in LACR_minrge..LACR_maxrge, ii and jj in 0..LACR_maxidx.
The ii and jj elements in the array are exchanged.

## RIGHT_SHIFT_RGE

syntax RIGHT_SHIFT_RGE (rng,ii,jj,nn)
preconditions rng must belong to LACR_minrge..LACR maxrge. ii, jj and nn belong to 0. .LACR maxidx, with $\mathrm{ii} \leq \mathrm{jj}$ and $\mathrm{j} \mathrm{j}+\mathrm{nn} \leq \mathrm{LACR}$ maxidx to allow the shift right by nn spaces.
The $\mathrm{ii}+\mathrm{nn} . . \mathrm{jj}+\mathrm{nn}$ part of the rng array receives a copy of the ii..jj part from this same array (shift nn spaces to the right).

## LEFT_SHIFT_RGE

```
syntax LEFT_SHIFT_RGE (rng,ii,jj,nn)
```

preconditions rng must belong to LACR_minrge..LACR_maxrge, ii and jj belong to 0 ..LACR_maxidx, with $\mathrm{ii} \leq \mathrm{jj}$. nn must belong to NAT with nn $\leq$ ii to make possible the left shift by nn spaces. For implementation reasons, jj cannot equal MAXINT.
The ii-nn..jj-nn part of the rng array receives a copy of the ii..jj part of this same array (shift nn spaces to the left).

## SEARCH_MAX_EQL_RGE

$$
\text { syntax } \quad \text { rr,bb } \leftarrow \text { SEARCH_MAX_EQL_RGE }(\mathrm{rng}, \mathrm{ii}, \mathrm{jj}, \mathrm{vv})
$$

preconditions rng must belong to LACR_minrge..LACR_maxrge. ii and jj belong to $0 . . L A C R$ maxidx, $\mathrm{ii} \leq \mathrm{jj}$ and vv must belong to LACR_VALUE.
outputs TRUE if vv was found, FALSE if not. rr is a NAT, if bb = TRUE then rr the largest index in the array that equals vv.
Search for an array element that equals vv, by scanning the ii..jj part starting from jj .

## SEARCH_MIN_EQL_RGE

$$
\begin{array}{ll}
\text { syntax } & \mathrm{rr}, \mathrm{bb} \leftarrow \mathrm{SEARCH} \_ \text {MIN_EQL_RGE }(\mathrm{rng}, \mathrm{ii}, \mathrm{jj}, \mathrm{vv}) \\
\text { preconditions } & \text { rng must belong to LACR_minrge..LACR_maxrge, ii and } \mathrm{jj} \text { belong to } \\
& 0 . . \mathrm{LACR} \text { _maxidx, ii } \leq \mathrm{jj} \text { and vv must belong to VALUE. } \\
\text { outputs } & \text { TRUE if vv was found, FALSE if not. rr is a NAT, if bb = TRUE, then } \\
& \text { rr is the smallest index in the rng array equal to vv. }
\end{array}
$$

Search for an element in an array equal to vv, by scanning the ii..jj part starting from ii.

## REVERSE_RGE

```
syntax REVERSE_RGE(rng,ii,jj)
preconditions rng must belong to LACR_minrge..LACR_maxrge, ii and jj belong to 0 ..LACR_maxidx.
```

Reverse the order of elements in the ii..jj range of the rng array.

# SEARCH_MIN_GEQ_RGE <br> syntax $\quad \mathrm{i}, \mathrm{bb} \leftarrow$ SEARCH_MIN_GEQ_RGE(rng,jj,kk,vv) <br> preconditions rng must belong to LACR_minrge..LACR_maxrge. jj and kk belong to 0 ..LACR_maxidx, $\mathrm{j} \mathrm{j} \leq \mathrm{kk}$ and vv belong to LACR_minval..LACR_maxval. For implementation reasons, kk must be different from MAXINT. <br> outputs $\quad \mathrm{bb}$ is a Boolean element, TRUE is an element that exceeds or is equal to the vv value found, FALSE if not. ii is a NAT, if bb = TRUE, then ii is the smallest index in the image array that exceeds or is equal to vv. <br> Search for an element that exceeds or is equal to vv in the $\mathrm{jj} . . \mathrm{kk}$ range, starting from jj . 

## ASCENDING_SORT_RGE

syntax ASCENDING_SORT_RGE (rng,iii,j)
preconditions rng must belong to LACR_minrge..LACR_maxrge, ii and jj belong to 0. LACR_maxidx. For implementation reasons, ii and jj must not be different from MAXINT.
Shift sort, in ascending order (starting with the smallest) on the ii..jj range in an array.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine
sees them with SEES)
BASIC_ARITHMETIC; BASIC_BOOL.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_RANGE machine (IMPORTS BASIC_ARRAY_RANGE(...) clause). Therefore if another instance is necessary, it must be given a non empty instance name (for example: i1.BASIC_ARRAY_RANGE)

### 5.11 L SEQUENCE_RANGE: Range of Sequences

## OPERATIONS

LEN_SEQ_RGE gives the size of a sequence.
IS_FULL_SEQ_RGE indicates whether a sequence is full.
IS_INDEX_SEQ_RGE indicates whether an integer is in the sequence domain.
VAL_SEQ_RGE gives the value of a sequence for a valid index.
FIRST_SEQ_RGE gives the first element of a sequence.
LAST_SEQ_RGE gives the last element of a sequence.
PUSH_SEQ_RGE adds an element to a sequence.
POP_SEQ_RGE removes the last element from a sequence.
STR_SEQ_RGE changes the value of an element in a sequence.
RMV_SEQ_RGE removes an element from a sequence, the size of which is reduced by 1 .
INS_SEQ_RGE adds an element to a sequence, the size of which increases by 1.
CLR_SEQ_RGE empties a sequence.
TAIL_SEQ_RGE removes the first element from a sequence.
KEEP_SEQ_RGE only retains the first N in a sequence elements.
CUT_SEQ_RGE cuts the N first elements from a sequence.
PART_SEQ_RGE only retains in a sequence the indexes between the two limit values.
REV_SEQ_RGE reverses the order of the elements in a sequence.
FIND_FIRST_SEQ_RGE searches for a value in a sequence, returns a Boolean element indicating whether it was found, and if yes, returns the smallest corresponding index.
FIND_LAST_SEQ_RGE searches for a value in a sequence, returns a Boolean element indicating whether it was found and if yes, returns the largest corresponding index.
COP_SEQ_RGE
copies one of the sequences to another.
CMP_SEQ_RGE compares two sequences.

PCOP_SEQ_RGE partial copy from one sequence to another.

PCMP_SEQ_RGE partial comparison of two sequences.

## EXAMPLE

The example below shows the use of L_SEQUENCE_RANGE on a numbered set.

|  | IMPLEMENTATION |
| :---: | :---: |
| MACHINE | REFINES |
| SETS | sr |
| ST $=$ \{classical,baroque,rock,rap,funk $\}$ | IMPORTS <br> s1.L_SEQUENCE_RANGE $(1,5,10, S T)$ |
| VARIABLES | INVARIANT <br> s1.seq_rge $=$ vv |
| INVARIANT $\begin{aligned} & \mathrm{vv} \in 1 . .5 \rightarrow \operatorname{seq}(\mathrm{ST}) \wedge \\ & \forall \mathrm{rr} .(\mathrm{rr} \in 1 . .5 \Rightarrow \operatorname{size}(\mathrm{vv}(\mathrm{rr})) \leq 10) \end{aligned}$ | INITIALISATION <br> s1.CLR_SEQ_RGE(1); |
| INITIALISATION $\text { vv:=(1..5) } \times\{[\text { baroque,rock,rap }]\}$ | s1.PUSH_SEQ_RGE(1,baroque); <br> s1.PUSH_SEQ_RGE(1,rock); <br> s1.PUSH_SEQ_RGE(1,rap); |
| OPERATIONS $\begin{aligned} & \mathrm{ii}, \mathrm{bb} \longleftarrow \text { trouve_rap(rng) } \longleftarrow \text { PRE } \\ & \text { rng } \in 1 . .5 \end{aligned}$ | s1.COP_SEQ_RGE $(2,1)$; <br> s1.COP_SEQ_RGE $(3,1)$; |
| THEN <br> $\mathrm{i}: \in \operatorname{vv}(\mathrm{rng})^{-1}[\{\operatorname{rap}\}] \\|$ | s1.COP_SEQ_RGE(4,1); <br> s1.COP_SEQ_RGE(5,1) |
| $\mathrm{bb}: \in \mathrm{BOOL}$ | OPERATIONS |
| $\begin{array}{r} \text { END } \\ \text { END } \end{array}$ | $\begin{aligned} & \mathrm{ii}, \mathrm{bb} \longleftarrow \text { trouve_rap(rng) }=\text { BEGIN } \\ & \quad \mathrm{bb}, \mathrm{ii} \longleftarrow \text { s1.FIND_FIRST_SEQ_RGE(rng,rap) } \end{aligned}$ |
|  | $\begin{array}{r} \text { END } \\ \text { END } \end{array}$ |

## DESCRIPTION

L_SEQUENCE_RANGE enables implementing and using a set number of sequences with a fixed maximum size. The sequence number evolves in a range that is a machine parameter, the maximum size of all of the sequences is also a machine parameter. The purpose is to be able to make comparisons and copies between these sequences directly, using an additional operation to the traditional operations on each of the sequences.

## MACHINE PARAMETERS

L_SEQUENCE_RANGE (LSR_minrge, LSR_maxrge, LSR_maxsize, LSR_VALUE): the variable is a total function of LSR_minrge..LSR_maxrge in the set of VALUE sequences with a maximum size of LSR maxisize.

## LEN_SEQ_RGE

syntax $\quad \mathrm{nn} \leftarrow$ LEN_SEQ_RGE (range)
preconditions range must belong to the LSR_minrge..LSR_maxrge range.
outputs $\quad \mathrm{nn}$ is the size of the range position, $\mathrm{nn} \in 0 . . \mathrm{LSR} \_$maxsize.
Gives the size of a sequence.

## IS_FULL_SEQ_RGE

$$
\begin{array}{ll}
\text { syntax } & \mathrm{bb} \leftarrow \mathrm{IS} \_ \text {FULL_SEQ_RGE }(\text { range }) \\
\text { preconditions } & \text { range must belong to the range LSR_minrge..LSR_maxrge. } \\
\text { outputs } & \mathrm{bb} \text { is TRUE if the range position sequence is full, FALSE if not. }
\end{array}
$$

Indicates whether a sequence is full.

## IS_INDEX_SEQ_RGE

```
syntax bb }\leftarrow\mathrm{ IS_INDEX_SEQ_RGE (range, ii)
```

preconditions range must belong to the LSR minrge..LSR maxrge range, ii must be a NAT.
outputs bb is TRUE if ii is an index in the range position sequence, FALSE if not.
Identifies whether an integer is in a sequence domain.

```
VAL_SEQ_RGE
syntax vv \leftarrow VAL_SEQ_RGE (range, ii)
preconditions range must belong to the LSR_minrge..LSR_maxrge range, ii must be an
    index in the range position sequence (ii \in 1..size (seq_rge (range))).
outputs vv is the value of the ii-th element in the range position sequence (vv \in
    VALUE).
```

Gives the value of a sequence for a valid index.

## FIRST_SEQ_RGE

```
syntax vv }\leftarrow\mathrm{ FIRST_SEQ_RGE (range)
```

preconditions range must belong to the LSR_minrge..LSR_maxrge range, the range position sequence must not be empty.
outputs $\quad \mathrm{vv}$ is the value of the first element in the range position sequence ( $\mathrm{vv} \in$ VALUE).
Gives the first element in a sequence.

## LAST_SEQ_RGE

```
syntax vv }\leftarrow\mathrm{ LAST_SEQ_RGE (range)
preconditions range must be in the LSR_minrge..LSR_maxrge range, the range position
    sequence must not be empty.
outputs vv is the value of the last element in the range position sequence (vv }
    VALUE).
```

Gives the last element of a sequence.

```
PUSH_SEQ_RGE
    syntax PUSH_SEQ_RGE (range, vv)
    preconditions range must belong to the LSR_minrge..LSR maxrge range, vv must be
    in LSR_VALUE and the range position sequence cannot be full.
```

Adds an element to a sequence.

## POP_SEQ_RGE

```
    syntax POP_SEQ_RGE (range)
    preconditions range must belong to the LSR_minrge..LSR_maxrge range, the range
        position sequence must not be empty.
```

Removes the last element in a sequence.

## STR_SEQ_RGE

syntax STR_SEQ_RGE (range, ii, vv)
preconditions range must belong to LSR minrge..LSR_maxrge, ii be a valid index in the range position sequence and vv belong to LSR_VALUE.

Change the value of an element in a sequence.

## RMV_SEQ_RGE

syntax RMV_SEQ_RGE (range, ii)
preconditions range must belong to the LSR minrge..LSR maxrge range, ii must be a valid index in the range sequence.
Removes an element from a sequence, the size of which decreases by 1 .

## INS_AFT_SEQ_RGE

```
syntax INS_AFT_SEQ_RGE (range, ii, vv)
preconditions range must belong to the LSR_minrge..LSR_maxrge range, ii must be a
    valid index in the range position sequence, vv must be in LSR_VALUE,
    the range position sequence must not be full.
```

Adds an element to a sequence, the size of which increases by 1.

## CLR_SEQ_RANGE

```
syntax CLR_SEQ_RANGE (range)
preconditions range must belong to the LSR_minrge..LSR_maxrge range.
```

Clears a sequence.
TAIL_SEQ_RGE
syntax TAIL_SEQ_RGE (range)
preconditions range must belong to the LSR_minrge..LSR_maxrge range and the range position sequence cannot be empty.
Removes the first element in a sequence.

## KEEP_SEQ_RGE

syntax KEEP_SEQ_RGE (range, nn)
preconditions range must belong to the LSR_minrge..LSR_maxrge range, nn must be a NAT.

Only retains the nn first elements in a sequence. For nn $=$ size (seq_rge(range)); this operation has no effect.

## CUT_SEQ_RGE

```
syntax CUT_SEQ_RGE (range, nn)
preconditions range must belong to the LSR_minrge..LSR_maxrge range, nn must be
    in NAT.
```

Clears the sequence of its first $n n$ elements. For $n n=$ size(seq_rge(range)), this operation is equivalent to CLR_SEQ_RGE.

## PART_SEQ_RGE

syntax PART_SEQ_RGE (range, ii, jj)
preconditions range must belong to the LSR_minrge..LSR_maxrge range, ii and jj must be NATs that are not null, with ii $\leq \mathrm{jj}$.

In a sequence, only retains the indexes between two limits. ii..jj may not be in the sequence domain.

## REV_SEQ_RGE

```
    syntax REV_SEQ_RGE (range)
    preconditions range must belong to the LSR_minrge..LSR_maxrge range.
```

Reverses the order of the elements in a sequence.

## FIND_FIRST_SEQ_RGE

```
syntax bb, ii \leftarrow FIND_FIRST_SEQ_RGE (range, vv)
preconditions range must belong t the LSR_minrge..LSR_maxrge range, vv must be in
                    LSR_VALUE.
outputs bb is TRUE if vv is in the range position sequence, FALSE if not. ii is a
    NAT, if bb = TRUE, it indicates the first position that equals vv in the
    sequence.
```

Searches for a value in a sequence starting from the beginning.

## FIND_LAST_SEQ_RGE

syntax $\quad \mathrm{bb}, \mathrm{ii} \leftarrow$ FIND_LAST_SEQ_RGE (range, vv)
preconditions range must belong to the LSR_minrge..LSR_maxrge range, vv must be in LSR_VALUE.
outputs $\quad \mathrm{bb}$ is TRUE if vv is in the range position sequence, FALSE if not. ii is a NAT; if $\mathrm{bb}=$ TRUE, this indicates the last position that equals vv in the sequence.

Searches for a value in a sequence, starting from the end.

## COP_SEQ_RGE

syntax COP_SEQ_RGE (dst, src)
preconditions dst and src must belong to the LSR minrge..LSR_maxrge range.
Copy the seq_rge(src) sequence to the seq_rge(dst) sequence.

## CMP_SEQ_RGE

$$
\text { syntax } \quad \text { bb } \leftarrow \text { CMP_SEQ_RGE }(\text { rng1, rng2 })
$$

preconditions rng1 and rng2 must belong to the LSR_minrge..LSR_maxrge range.
outputs bb is TRUE if the two rng1 and rng2 position sequences are equal, FALSE if not.

Compare two sequences.

## PCOP_SEQ_RGE

syntax PCOP_SEQ_RGE (dst, idx, src, ii, jj)
preconditions dst and src must belong to the LSR minrge..LSR _maxrge range, dst must be different from src, ii and jj must be valid indexes in the src position sequence, with ii $\leq \mathrm{jj}$ and $\mathrm{jj} \leq$ MAXINT- 1 idx must be a valid index for the dst sequence or where the size of this sequence +1 , idx +jj - ii belongs to the $1 . . L S R \_$maxsize range.
Copy the ii..jj part of the src position sequence to the dst position from the idx index.

## PCMP_SEQ_RGE

```
syntax idx, bb \leftarrow PCMP_SEQ_RGE (rng1, ii, jj, rng2, kk)
```

preconditions rng1 and rng2 must be in the LSR_minrge..LSR_maxrge range, ii and jj must be valid indexes in the rng1 and $\mathrm{ii} \leq \mathrm{jj}$ position sequences, kk must be a valid index in the rng2 position sequence, ( $\mathrm{kk}+\mathrm{jj}-\mathrm{ii}$ ) must be a valid index in the rng2 position sequence.
output bb is TRUE if there is an element of the ii...jj part in the seq_rge (rng1) sequence that is different to the $\mathrm{kk} . . \quad(\mathrm{kk}+\mathrm{jj}-\mathrm{ii})$ part of the seq_rge (rng2) sequence, FALSE if not. idx is a NAT if bb is TRUE, the idx represents the index of the first element that is different in the seq_rge (rng1 $\in$ ii..jj) sequence.
Partial comparison of two sequences.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES)
BASIC_ARITHMETIC ;
BASIC_BOOL.
WARNING: The implementation of this machine creates the default instance for the BASIC_ARRAY_RANGE and BASIC_ARRAY_VAR machines. Therefore, if other instances are required they must be given a name that is not blank.
(example: i1.BASIC_ARRAY_RANGE).

### 5.12 L ARRAY_COLLECTION: collection of arrays of the same size

## OPERATIONS

| CRE_ARR_COL | returns a Boolean element that indicates that there remains an <br> array available in the collection and gives the index of this available <br> array. |
| :--- | :--- |
| DEL_ARR_COL | releases the specified array. |
| VAL_ARR_COL | read an element from one of the valid arrays. |
| STR_ARR-COL | write an element from one of the valid arrays. |
| COP_ARR_COL | copy one of the arrays to another. |
| CMP_ARR_COL | compare two arrays. |

## EXAMPLE



## DESCRIPTION

L_ARRAY_COLLECTION is used to handle identically sized one dimensional arrays. It contains basic operations (create, delete, read, write, compare).

## MACHINE PARAMETERS

L_ARRAY_COLLECTION (LACOLL_maxobj, LACOLL_INDEX, LACOLL_VALUE): LACOLL_maxobj is the maximum number of arrays in the collection. LACOLL_INDEX is the set of array indexes, LACOLL_VALUE is the set of possible values of array elements.

## CRE_ARR_COL

Syntax ii, bb $\leftarrow$ CRE_ARR_COL
Outputs bb is a Boolean element indicating whether any available arrays are left in the collection, ii is the index of this available array.
Assigning an array in the collection.

## DER_ARR_COL

Syntax DEL_ARR_COL (ii)
Preconditions ii must belong to 1..LACOLL_maxobj
The array of index ii in the collection is released. It may once again be assigned using CRE_ARR_COL.

## VAL_ARR_COL

Syntax $\quad$ vv $\leftarrow$ VAL_ARR_COL (ii, jj)
Preconditions ii must belong to 1..LACOLL_maxobj and jj belong to LACOLL_INDEX. Output vv contains the jj number value of array ii.

Use vv to store the value of number jj in array ii.

## STR_ARR_COL

Syntax STR_ARR_COL (ii, jj, vv)
Preconditions ii must belong to 1..LACOLL_maxobj, jj belong to LACOLL_INDEX and vv belong to LACOLL_VALUE.
Write the value of vv to cell number jj in array ii.

## COP_ARR_COL

Syntax COP_ARR_COL (dest, src)
Preconditions dest and sre must belong to 1..LACOLL_maxobj.
Copy the contents of the src array to the dest array.

## CMP_ARR_COL

Syntax $\quad \mathrm{bb} \leftarrow$ CMP_ARR_COL (range 1, range 2)
Preconditions range 1 and range 2 must belong to 1..LACOLL_maxobj.
Output bb is a Boolean element indicating whether array range 1 and range 2 are identical.

Comparison between the two 2 arrays.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES).
BASIC_ARITHMETIC BASIC_BOOL

### 5.13 L ARRAY1_COLLECTION: array of the same size, with numerical indexes

## OPERATIONS

CRE_ARR_COL returns a Boolean element indicating whether an array remains available in the collection and the index of this available array (promoted operation)

DEL_ARR_COL releases the array mentioned (promoted operation).

VAL_ARR_COL read an element from one of the valid arrays (promoted operation).

STR_ARR-COL write an element from one of the valid arrays (promoted operation).

COP_ARR_COL
copy one of the arrays to another (promoted operation).
CMP_ARR_COL compare two arrays (promoted operation).

SET_ARR-COL
PCOP_ARR_COL
copy part of one of the arrays to another in a given position.
search for the first different element between two parts of two different arrays. A Boolean element indicates whether the element was found and in this case, the index of this element is returned.

## EXAMPLE

Using SET_ARR_COL to fill-in two arrays and PCOP_ARR_COL to define a third one. Note the need to test the Boolean output elements from CRE_ARR_COL in order to use the arrays created.
The example is as follows:

```
MACHINE
    M1
OPERATIONS
    op = skip
END
```

```
IMPLEMENTATION
    M1_1
REFINES
    M1
IMPORTS
    L_ARRAY1_COLLECTION(3,3,1,10)
OPERATIONS
    op = VAR i1,i2,i3,b1,b2,b3 IN
        i1,b1 \longleftarrow CRE_ARR_COL;
        i2,b2\longleftarrow CRE_ARR_COL;
        i3,b3 \longleftarrow CRE_ARR_COL;
        IF b1 = TRUE ^
        b2 = TRUE ^
        b3 = TRUE
        THEN
        SET_ARR_COL(i1,0,3,1);
        SET_ARR_COL(i2,0,3,2);
        PCOP_ARR_COL(i3,0,i1,0,1);
        PCOP_ARR_COL(i3,2,i2,2,3)
        END
    END
END
```


## DESCRIPTION

L_ARRAY1_COLLECTION enables the use of a collection of arrays without the need to code loops to position a set of elements or arrays. This was not possible with the previous machine L_ARRAY_COLLECTION where index sets are normally unordered.

## MACHINE PARAMETERS

L_ARRAY1_COLLECTION (LAUC_maxobj, LAUC_maxidx, LAUC_minval,
LAUC_maxval): The variable is a partial function of 1..LAUC_maxobj in the set of total functions of 0..LAUC_maxidx to LAUC_minval..LAUC_maxval. LAUC_maxobj is a NAT1 that is different from MAXINT. LAUC_maxidx, LAUC_minval and LAUC_maxval are NATs and LAUC_minval $\leq$ LAUC_maxval.

## CRE_ARR_COL

## Syntax ii, bb $\leftarrow$ CRE_ARR_COL

Outputs bb is a Boolean element indicating whether any available arrays remain in the collection, ii is the index of this available array.
Allocate an array in the collection.

## DEL_ARR_COL

Syntax DEL_ARR_COL (ii)
Preconditions ii must belong to 1..LAUC_maxobj
Array ii in the collection is released. It may once again be assigned using CRE_ARR_COL.

## VAL_ARR_COL

Syntax vv $\leftarrow$ VAL_ARR_COL (ii, jj)
Preconditions ii must belong to 1..LAUC_maxobj jj must belong to 1..LAUC_maxidx.
Output vv contains the value of number jj in array ii.
Store in vv the value of number jj in array ii.

## STR ARR_COL

Syntax STR_ARR_COL (ii, jj, vv)
Preconditions ii must belong to 1..LAUC_maxobj; jj must belong to 1..LAUC_maxidx. vv must belong to LAUC_VALUE.
Write value vv to the jjth cell in array ii.

## COP_ARR_COL

Syntax COP_ARR_COL (dest, src)
Preconditions dest and src must belong to 1..LAUC_maxobj.
Copy the contents of the src array to the dest array.

## CMP_ARR_COL

Syntax $\quad \mathrm{bb} \leftarrow$ CMP_ARR_COL (range 1, range 2)
Preconditions range 1 and range 2 must belong to 1..LAUC_maxobj.
Output bb is a Boolean element that indicates whether array ranges 1 and 2 are identical.

Comparison between the two arrays.

## SET_ARR_COL

Syntax SET_ARR_COL (range, ii, jj, vv)
Preconditions range belonging to dom(arr_col), i.e. it corresponds to the index of a previously created array. ii and jj are in 1..LAUC_maxidx, jj must be different from MAXINT. vv is in LAUC_minval..LAUC_maxval.
The value vv is copied to the range array for all indexes between ii and jj . If ii $>\mathrm{j}$, the array remains unchanged.

## PCOP_ARR_COL

Syntax
PCOP_ARR_COL (dest, idx_dst, src, ii, jj)
Preconditions dest and sre are elements that are different from 1..LAUC_maxobj, corresponding to arrays already created. ii..jj is a non blank interval of 0..LAUC_maxidx and $\mathrm{jj} /=$ MAXINT. idx_dst..idx_dst $+\mathrm{jj}-\mathrm{ii}$ is an interval of 0..LAUC_maxidx.
The ii..jj part in the src array is copied to the idx_dst..idx_dst + jj - ii part of the dst array.

## PCMP_ARR_COL

Syntax idx, bb $\leftarrow$ PCMP_ARR_COL (nn2, idx2, nn1, ii, jj)
Preconditions nn 1 and nn 2 are elements that are different from 1..LAUC_maxobj and correspond to arrays already created. ii..jj is a non blank interval of 0..LAUC_maxidx. idx2..idx2 + jj - ii is an interval of 0..LAUC_maxidx.

Outputs bb is a BOOL. idx is in ii..jj.
The ii..jj part in array nn1 is compared to part idx2..idx2 +jj - ii in array nn 2 . bb is FALSE if the two parts are identical, TRUE if not. In this case, idx is the index of the first element that is different from ii..jj.

## IMPORTS REQUIRED

(instances to import as the implementation tree for this library machine sees them with SEES) BASIC_ARITHMETIC, BASIC_BOOL.


[^0]:    ${ }^{1}$ i.e. the number of couple present in the relation.

[^1]:    ${ }^{1}$ The NAT element immediatly inferior or superior wether the calcul is performed by inferior value or superior value

[^2]:    ${ }^{2}$ Indeed, the loops used make a pre-incrementation, that does not produce literal excedent)

[^3]:    ${ }^{3}$ L_ARRAY5_RANGE can only have a finite integer set as range.

