Control Design (F, IT)
Computer Controlled Systems (STS, W)

Instruction to the laboratory work

MPC of the tank process

Preparation exercises:
1. Preparation exercises 1 and 2.

Reading instructions: Chapter 5, 9 and 16 in Glad–Ljung –97

<table>
<thead>
<tr>
<th>Name</th>
<th>Assistant’s comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Program</td>
<td>Year of reg.</td>
</tr>
<tr>
<td>Date</td>
<td></td>
</tr>
<tr>
<td>Passed prep. ex.</td>
<td>Sign</td>
</tr>
<tr>
<td>Passed comp. ex.</td>
<td>Sign</td>
</tr>
</tbody>
</table>
Contents

1 Introduction .................................................. 1
   1.1 Survey .................................................. 1
   1.2 Equipment .............................................. 3

2 Modeling of the process .................................. 3
   2.1 Physical modeling .................................... 3
   2.2 Experimental determination of the model parameters .... 6

3 LQ control of the tank process ............................ 8
   3.1 Theory .................................................. 8
   3.2 Controller design ..................................... 9
   3.3 Control of the tank process ......................... 9

4 Model Predictive Control of the tank process .......... 10
   4.1 Theory ................................................. 11
   4.2 Controller design .................................... 11
   4.3 Control of the tank process ....................... 13

5 State estimation with a Kalman filter .................. 14
   5.1 Kalman filter ......................................... 14
   5.2 State estimation - simulation ....................... 15
   5.3 State estimation - real tank process .............. 16
1 Introduction

This laboratory work is based on Computer Exercise 2. Therefore it is advisable to have Computer Exercise 2 fresh in mind before starting this laboratory work. The goal of this laboratory work is to illustrate how physical modeling can be used to form a mathematical model of a real process and how this model can be used for controller design.

Two different control methods, LQ control and MPC, will be compared. Both methods use state-space models in the controller design.

1.1 Survey

In this laboratory work a tank process is modeled using physical modeling. The resulting non linear model is linearized around a stationary operating point and the model is used in the design of the controllers. The water level in the lower tank is controlled using these controllers.

The laboratory work consists of the following steps:

- **Modeling** using physical relations from fluid mechanics and experimental estimation of the parameters in the model. The model is described in state-space form.

- **LQ control of the process.** Knowing the model of the system and using the experience about LQ from Computer Exercise 2 and 3, an LQ controller is designed. The water level in the lower tank is then controlled.

- **Model Predictive Control of the process.** With the knowledge of the system in forms of model and physical limitations an MPC is designed using the experience gained in Computer Exercise 2. Comparisons are then made between the LQ controller and the MPC.

- **Observer.** An observer is designed and simulated using the model of the system. It is then tested on the real system and the predicted outputs from the observer are compared to the measured outputs from the tank system.

The tank system consists of two identical water tanks placed on top of each other. Each tank has a small hole in the bottom so that the water can flow from the upper tank to the lower and from the lower tank to a container below the system. The holes are approximately equally large in both tanks. An electrical pump moves the water from the container to the upper tank. Each tank is equipped with a sensor that delivers a voltage that is approximately proportional to the water level in the tank. The process is depicted schematically in Figure 1.
The input signal to the process is the voltage $u$ (volts) to the electrical pump. The flow of water through the pump is approximately proportional to the voltage $u$:

$$q_{\text{pump}} = K_p u$$  \hspace{1cm} (1)

The output signals from the process are the voltages $y_1$ and $y_2$ (volts) from the water level sensors in the upper and lower tank respectively. Since the sensors are nearly linear the following relations between the output signals and the water levels $h_1$ and $h_2$ in the both tanks are obtained:

$$y_1 = K_1 h_1$$  \hspace{1cm} (2)

$$y_2 = K_2 h_2$$  \hspace{1cm} (3)

where $K_1$ and $K_2$ are the proportionality constants of the water level sensors.
1.2 Equipment

You need the following equipment in order to perform the laboratory work:

- A tank process.
- A PC with Matlab running under LINUX. The PC is used for the control of the process.
- A stop-watch. The stop-watch is used to determine time constants etc.
- A voltage generator. The voltage generator is used to generate a reference signal to the controller.
- A voltmeter. The voltmeter is used to determine the proportionality constants of the level sensors.

2 Modeling of the process

2.1 Physical modeling

In this section a non linear model of the tank system is derived. The model is then linearized around a stationary operating point.

The tank process can be described by the two coupled differential equations

\[ A_1 \frac{dh_1}{dt} = q_{1\text{in}} - q_{1\text{out}} \]  
(4)

and

\[ A_2 \frac{dh_2}{dt} = q_{2\text{in}} - q_{2\text{out}} \]  
(5)

where \( A \) is the area of the cross sections of the tanks. The Equations (4) and (5) simply state that the net change of volume in a tank is equal to the difference between the volume entering the tank and the volume leaving it. The flow into the upper tank, \( q_{1\text{in}} \), is equal to the flow out of the electrical pump, \( q_{\text{pump}} = K_p u \), which gives

\[ q_{1\text{in}} = K_p u. \]  
(6)

Furthermore, the flow into the lower tank is equal to the flow out of the upper tank, so

\[ q_{2\text{in}} = q_{1\text{out}}. \]  
(7)

The flows out from the two tanks, \( q_{1\text{out}} \) and \( q_{2\text{out}} \) are given by Torricelli’s principle:

\[ q_{1\text{out}} = a_1 \sqrt{2gh_1} \]  
(8)
and
\[ q_{2\text{out}} = a_2 \sqrt{2gh_2}. \] (9)

where \( a \) is the area of the holes. By putting all this together the following non-linear model for the system is obtained:
\[
\frac{A_1}{a_1} \frac{dh_1}{dt} = -\sqrt{2gh_1} + \frac{K_p}{a_1} u \quad (10)
\]
\[
\frac{A_2}{a_2} \frac{dh_2}{dt} = \sqrt{2gh_1} - \sqrt{2gh_2} \quad (11)
\]

Now introduce the stationary working point
\[
\begin{align*}
  u &= u_0 \\
  h_1 &= h_{01} \\
  h_2 &= h_{02}
\end{align*}
\]

where the last two sets of equalities should ideally be approximately the same following the fact that the areas of the holes in the two tanks are equal. Note that a stationary working point is defined by the water levels being constant so that \( dh_1/dt = dh_2/dt = dh_{01}/dt = 0 \), and from Equation (10) we obtain
\[
0 = \frac{dh_{01}}{dt} = -\sqrt{2gh_{01}} + \frac{K_p}{a_1} u_0. \quad (12)
\]

The non linear model (10)–(11) is now linearized. This is done by a Taylor series expansion around the working point. From (10)
\[
\begin{align*}
  \frac{A_1}{a_1} \frac{dh_1}{dt} &= -\sqrt{2gh_{01}} - \sqrt{\frac{g}{2h_{01}}}(h_1 - h_{01}) + \frac{K_p}{a_1} u_0 + \frac{K_p}{a_1} (u - u_0) \\
  &+ \text{Higher order terms} \\
  &= -\sqrt{\frac{g}{2h_{01}}}(h_1 - h_{01}) + \frac{K_p}{a_1} (u - u_0) \\
  &+ \text{Higher order terms}
\end{align*} \quad (13)
\]

where, in the last equality, Equation (12) has been used. From (11)
\[
\begin{align*}
  \frac{A_2}{a_2} \frac{dh_2}{dt} &= \sqrt{2gh_{01}} + \sqrt{\frac{g}{2h_{01}}}(h_1 - h_{01}) - \sqrt{2gh_{02}} - \sqrt{\frac{g}{2h_{02}}}(h_2 - h_{02}) \\
  &+ \text{Higher order terms} \\
  &= \sqrt{\frac{g}{2h_{01}}}(h_1 - h_{01}) - \sqrt{\frac{g}{2h_{02}}}(h_2 - h_{02}) \\
  &+ \text{Higher order terms}
\end{align*} \quad (15)
\]

Introducing the notations for the deviations from the operating point
\[
\begin{align*}
  \Delta h_1 &= h_1 - h_{01} \\
  \Delta h_2 &= h_2 - h_{02} \\
  \Delta u &= u - u_0
\end{align*}
\]
the linearized model (neglecting terms of second order or higher)

\[ \frac{d\Delta h_1}{dt} = -\frac{1}{T_1} \Delta h_1 + \frac{K_p}{A_1} \Delta u \]

(17)

\[ \frac{d\Delta h_2}{dt} = \frac{1}{T_1} \Delta h_1 - \frac{1}{T_2} \Delta h_2 \]

(18)

is obtained, where

\[ T_i = \frac{A_i}{a_i} \sqrt{\frac{2h_{0i}}{g}} \]

(19)

Using Equation (10) with \( u \) equal to zero, or Equation (11) with \( h_1 \) equal to zero, gives

\[ \frac{dh_i}{\sqrt{h_i}} = -\frac{a_i}{A_i} \sqrt{2g} dt \]

(20)

Integrating the left hand side from the height \( h_{1i} \) to the height \( h_{2i} \) and the right hand side from time \( t_1 \) to \( t_2 \) gives an expression for \( \frac{A_i}{a_i} \) as

\[ \frac{a_i}{A_i} = \frac{2(\sqrt{h_{1i}} - \sqrt{h_{2i}})}{\sqrt{2g} \Delta t_i} \]

(21)

Hence

\[ T_i = \frac{\Delta t_i \sqrt{h_{0i}}}{\sqrt{h_{1i}} - \sqrt{h_{2i}}} \]

(22)

The Equations (2) and (3) now give the final model:

\[ \frac{d\Delta y_1}{dt} = -\frac{1}{T_1} \Delta y_1 + \frac{K_p K_1}{A_1} \Delta u \]

(23)

\[ \frac{d\Delta y_2}{dt} = \frac{1}{T_1} K_2 \Delta y_1 - \frac{1}{T_2} \Delta y_2 \]

(24)

Note that the model is linearized around a certain working point \( (h_{0i}, u_0) \). The linearized system will thus not behave exactly as the original system, but the approximation is reasonable for small deviations from the working point.

**Preparation exercise 1:**

Give a model of the system in state-space form. Let the states be the (voltage) deviation of the water levels from the working point and let the output be the level (voltage) deviation in the lower tank.

**ANSWER:**
2.2 Experimental determination of the model parameters

In this section we determine the the parameters.

\[
K_1 = \frac{y_1}{h_{01}} \quad (25)
\]

\[
K_2 = \frac{y_2}{h_{02}} \quad (26)
\]

\[
T_i = \frac{\Delta t_i \sqrt{h_{01}}}{\sqrt{h_{1i}} - \sqrt{h_{2i}}} \quad (27)
\]

\[
\frac{K_p}{A_1} = \frac{\Delta h}{u_0 \Delta t} \quad (28)
\]

- Determine the steady-state level around 10 cm in both the tank (use the command \texttt{proc\_da}(0, u) where \(0 \leq u \leq 9\)). Note that you also must measure the working point levels for the tanks in Volts.

\[
\begin{align*}
  u_0 &= \text{...........} \ [V] & h_{01} &= \text{...........} \ [cm] & h_{02} &= \text{...........} \ [cm] \\
  y_{01} &= \text{...........} \ [V] & y_{02} &= \text{...........} \ [V] \\
  K_1 &= \text{...........} \ [V/cm] & K_2 &= \text{...........} \ [V/cm]
\end{align*}
\]

- Determine \(T_i\) for the tanks by measuring the time, \(\Delta t_i\), it takes for the water to sink from the level \(h_{1i} = 14\) cm to \(h_{2i} = 6\) cm. Note that \(h_0\) may be different for the two tanks. No water should of course enter the tank while you measure these parameters.

\[
\begin{align*}
  \Delta t_1 &= \text{...........} \ [s] & T_1 &= \text{...........} \ [s] & \Delta t_2 &= \text{...........} \ [s] & T_2 &= \text{...........} \ [s]
\end{align*}
\]

- \(\frac{K_p}{A_1}\) is determined by putting a finger at the bottom of the upper tank and measuring the time, \(\Delta t\), it takes for the pump to increase the level \(\Delta h\) cm when the input voltage of the pump is held constant at \(u_0\) Volts (determined above). Use \texttt{proc\_da}(0, u_0). \textbf{Note that \(\Delta h\) should be given in centimeters.}

\[
\begin{align*}
  \Delta t &= \text{...........} \ [s] & \frac{K_p}{A_1} &= \text{...........} \ [cm/Vs]
\end{align*}
\]

\textbf{Exercise:}

Give the state space model for the system with the numerical values for the
parameters.

**ANSWER:**

**Exercise:**
Are there any physical limitations on the system?
If yes, what are they?

**ANSWER:**

We now have the continuous-time model of the process. In Matlab linear systems can be represented as LTI objects (LTI is an abbreviation for Linear Time Invariant). Thus, a linear system may be treated as a single object rather than being described by the system matrices. In order for your model to be used for control design, it is necessary to convert the model into this form. The controllers will then convert the model into discrete time using the sampling time chosen by you.

- Form the LTI object `contsys` using the `ss` command. *Note* that it is possible to get help in Matlab for the different functions by typing `help` frame.
- Define the parameters $\mathbf{K}_1$, $\mathbf{K}_2$, $\mathbf{h}_0$, $\mathbf{u}_0$ numerically in Matlab. Note that the parameter $\mathbf{h}_0$ has to be a two dimensional vector composed of $h_{01}$ and $h_{02}$.
- Save the model of the tank system and the parameters for future use in this laboratory work, i.e. type `save /usr/tmp/tankdata contsys K1 K2 h0 u0`. The model is then saved in a temporary directory.
3 LQ control of the tank process

3.1 Theory

An LQ controller as described in Computer Exercise 2 and in Glad–Ljung Chapter 9 will be used. We will use the LQ controller to find a suitable desired closed-loop system. The system is described in the form of a block diagram in Figure 2. The main idea for the LQ controller is to minimize a quadratic criterion.

Consider a system in state space form

\[
\begin{align*}
x(k + 1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k)
\end{align*}
\]  

where all the states are measured. The system can be controlled with a state-feedback:

\[
u(k) = -Lx(k) + mr(k)
\]  

where \(m\) is chosen so that the closed loop system has unit static gain. In the LQ design, the gain matrix, \(L\), is determined so that the following cost function is minimized:

\[
V = \min \sum_{k=1}^{\infty} [r(k) - y(k)]^2 Q_1 + [u(k)]^2 Q_2
\]  

where \(y\) and \(u\) are scalars. Eq (31) can be compared with eq (9.5) in Glad–Ljung.

The closed loop system in state space form is then

\[
\begin{align*}
x(k + 1) &= (A - BL)x(k) + Bmr(k) \\
y(k) &= Cx(k)
\end{align*}
\]  

The gain matrix, \(L\), hence depends on the weights \(Q_1\) and \(Q_2\) in eq. (31).
Preparation exercise 2:
Consider the criterion in eq. (31).
Explain how $Q_1$ and $Q_2$ affect the closed loop system. How can $Q_1$ and $Q_2$
affect the fastness of the closed loop system and why?

**ANSWER:**

3.2 Controller design

The controller is designed in the same way as the LQ controller in Computer
Exercise 2. The Matlab function `lq_reg`

- loads your continuous state-space model of the system, `contsys` and the
  parameters $K_1$, $K_2$, $u_0$, $h_0$ in `tankdata.mat`.
- converts the model into discrete time using a sampling interval $tsamp$.
- calculates $L$ from the LQ criteria.
- calculates $m$ so that the static gain $(r \rightarrow y = x_2)$ equals 1.
- controls the tank system given an external reference signal, $r(t)$.

The LQ control of the tank process is activated with the function:

$$lq\_reg(Q_1, Q_2, Tscale, tsamp)$$

where

$Q_1$ is the weight on the output signal and must be given. Note that only the
ratio $Q_1/Q_2$ determines the characteristics of the controller.
$Q_2$ is the weight on the manipulated signal and must be given.
$Tscale$ is the time scale for the plot. Default value is [360] seconds.
$tsamp$ is the sampling time. Default is [2] seconds.

3.3 Control of the tank process

Before starting a control sequence make sure the water levels are approximately
$h_{0i}$, i.e. close to the working point. This is done by filling the tanks using the command `proc_da(0,u)`.

The reference signal is the voltage that you set with the voltage generator. The step changes in the reference signal should be around the working point, for
example 4 → 6 and 6 → 4.

**OBS** The control will not start unless there is a reference signal!

To stop the control, turn the reference signal to zero or press `ctrl+c` and then give the command **stop** in the Matlab window.

The LQ control will start with the command:

```matlab
lq_reg(Q1, Q2)
```

**Exercise**

How does the ratio $Q_1/Q_2$ (for example 1, 100 and the result you had in Computer Exercise 2) affect the response of the system? Are there any benefits or drawbacks with big values of $Q_1/Q_2$?

**ANSWER:**

**Exercise**

How can overflow be prevented?

**ANSWER:**

4 Model Predictive Control of the tank process

As seen in the LQ control of the process the tanks can easily overflow which might be very expensive and/or dangerous and not at all optimal. The most desirable feature in MPC is the way it allows and implements constraints into the control design. Here you will try some different MPC designs that were simulated in Computer Exercise 2.
4.1 Theory

Theory was presented in Computer Exercise 2 and will therefore only be mentioned briefly.

The tank system is given in state space form:

\[
\begin{align*}
    x(k+1) &= Ax(k) + Bu(k) \\
    y(k) &= Cx(k)
\end{align*}
\]  

(33)

where both the states \( x_1 \) and \( x_2 \) are assumed measurable. The basic criterion to be minimized, with respect to the input signal, is given by:

\[
V(k) = \sum_{j=1}^{P} (Q_1(j)(r(k+j) - \hat{y}(k+j)))^2 \\
+ \sum_{j=1}^{M} (Q_2(j)\Delta \hat{u}(k+j))^2
\]

(34)

where

\( Q_1 \) = the weight on the deviation of the output from the reference.
\( Q_2 \) = the weights on the change in the input signal.
\( r \) = the reference signal (target).
\( \hat{y} \) = the predicted output, see example 16.1 in Glad–Ljung.
\( \Delta \hat{u} \) = the predicted changes in the input.

For a schematic view of the prediction horizon (P) and the control horizon (M), see Figure 3. As in the LQ controller the ratio of the weights \( Q_1/Q_2 \) is of importance for the minimization.

When the constraints are added to the controller the minimization problem becomes nonlinear and numerical iteration methods are needed to solve the minimization problem.

4.2 Controller design

The design is very similar to the MPC design in Computer Exercise 2. The Matlab function \texttt{mpc_reg}

\begin{itemize}
  \item loads your continuous state-space model of the system, \texttt{contsys in tankdata.mat}.
  \item converts the model into discrete time using a sampling interval \( tsamp \).
  \item designs the controller according to the parameters you choose.
  \item controls the tank system given an external reference signal, \( r(t) \).
\end{itemize}

The Model Predictive Controller of the tank process is activated with the Matlab function:
\texttt{mpc\_reg( ylim, ulim, Q1, Q2, M, P, Tscale, tsamp)}

where

\texttt{ylim} are the constraints on the two output signals, the tank levels: \([C_{y_{min}} \quad C_{y_{min}} \quad C_{y_{max}} \quad C_{y_{max}}]\). The default values are [-inf -inf inf inf], i.e. no constraints.

\texttt{ulim} are the constraints on the manipulated signal, the pump voltage: \([C_{umin} \quad C_{umin} \quad C_{umax} \quad C_{urate}]\). The default values are [-inf inf inf], i.e. no constraints.

\texttt{Q1} is the weight on the output signal, the water level in the lower tank. Default value for \texttt{Q1} is [1].

\texttt{Q2} is the weight on the manipulated signal. Default value for \texttt{Q2} is [0.01].

\texttt{M} is the control horizon (called \texttt{N} in Glad-Ljung). It is usually much smaller than \texttt{P}. Can be from 1 up to \texttt{P}. The size of the optimization problem is determined by the size of \texttt{M}. Default value is [3].

\texttt{P} is the prediction horizon (called \texttt{M} in Glad-Ljung) and should cover the rise time of the system. Default value is [30].

\texttt{Ts} is the duration of the simulation. Default value is [360] seconds.

\texttt{tsamp} is the sampling time. Default is [2] seconds.
4.3 Control of the tank process

Before starting a control sequence make sure the water levels are approximately $b_0$, i.e., close to the working point. The reference signal is given in the same way as earlier.

To stop the control, set the reference signal to zero or press `ctrl+c` and then give the command `stop` in the Matlab window.

Exercise

Control the system without any constraints in the controller design. Try $Q_1/Q_2 = 100$. Start the control by running the command:

```matlab
mpc_reg( ylim, ulim, Q1, Q2, M, P, Tscale, tsamp)
```

Are there any major differences between this controller and the LQ controller? Can the system overflow?

\[ \text{ANSWER:} \]

Implement the constraints into the controller design. Observe that since the model is an approximation, the constraints on the water levels should be harder than the real physical limitations to be on the safe side.

Choose `ylim=[0 0 9.2 9.2]` Volts and `ulim=[0 9 9]` Volts. Try $Q_1/Q_2 \geq 100$ and make some step changes.

Exercise

How do the constraints affect the control of the system? Are there any major differences between this control and the LQ control (consider approximate rise times, sensitivity for measurement noise and risk of overflow)?

\[ \text{ANSWER:} \]

Try different horizons and sampling time. For example try some of the following combinations:

- $M = 3 \quad P = 30 \quad tsamp = 2$ (default)
- $M = 3 \quad P = 10 \quad tsamp = 2$
- $M = 3 \quad P = 10 \quad tsamp = 0.5$
- $M = 3 \quad P = 15 \quad tsamp = 5$
Of course you can try other values instead/as well. Try different values for all the parameters. Make some step changes and evaluate the results!

Exercise
How do the different tuning parameters affect the controller and thereby the closed loop system?

- **a**, The effect of different control horizons, \( M \)?
- **b**, The effect of different prediction horizons, \( P \)?
- **a**, The effect of different sampling times, \( tsamp \)?

---

**ANSWER:**

---

5 State estimation with a Kalman filter

Often it is not possible to measure all the states are. However the states can often (if the system is observable) be estimated from measurable signals with an observer. Two typical applications of an observer are:

- Control design. The estimated states are used for state feedback control.
- Parameter estimation. With an observer it is possible to find crucial process parameters/states from measurement data. This is a key application of the Kalman filter which has been applied in numerous applications.

In this exercise we will assume that the water level in the upper tank is not measured. Instead we will try to estimate it with a Kalman filter, using measurements from the lower tank.

5.1 Kalman filter

In the design of a Kalman filter the noise of the system plays a crucial part. By extending the model with process noise, \( v_1 \), and measurement noise, \( v_2 \), the tank system is described by the following state-space model:

\[
\begin{align*}
x(k+1) &= Ax(k) + Bu(k) + Nv_1(k) \\
y(k) &= Cx(k) + v_2(k)
\end{align*}
\]  

(35)
Assume the following linear observer:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K(y(k) - C\hat{x}(k))$$

(36)

where $\hat{x}$ is the prediction of the states $x$.

The observer is designed using a Kalman filter. The Kalman filter is an optimal reconstruction of the states if the covariance matrices

$$Ev_1v_1^T = R_1 = \begin{pmatrix} R_{11} & 0 \\ 0 & R_{22} \end{pmatrix}$$

$$Ev_2v_2^T = R_2$$

$$Ev_1v_2^T = R_{12}$$

are known. Most often they are not known and $R_1$ and $R_2$ are used as design parameters and $R_{12}$ is assumed to be zero, i.e. the process noise and measurement noise are not correlated.

$K$ is calculated from

$$K = (APA^T + NR_{12})(CPC^T + R_2)^{-1}$$

(37)

where

$$P = APA^T + NR_1N^T$$

$$- (APC^T + NR_{12})(CPC^T + R_2)^{-1}(APC^T + NR_{12})^T$$

(38)

Equation (38) is the Riccati equation.

5.2 State estimation - simulation

In this part of the laboratory work we study how the choice of the covariance matrices, $R_1$ and $R_2$, influences the behavior of the observer.

- Use the function `obs_sim(R_{11}, R_{22}, R_2, pn, mn)` to simulate the system with observed states. The parameters `pn` and `mn` are used to simulate the process noises and measurement noise respectively.

- Try some of the combinations below.

  $$R_{11} = 1 - 1000$$
  $$R_{22} = 1 - 1000$$
  $$R_2 = 1 - 1000$$
  $$pn = 0 - 0.1$$
  $$mn = 0 - 0.1$$

You can also try some parameters of your own choice.
Exercise
How should the covariance matrices be chosen if there is
a, a high level of process noise compared to the measurement noise?
b, a high level of measurement noise compared to the process noise?
c, both process and measurement noise on the system?

ANSWER:

Exercise
What relation do you find between the speed of the observer and its sensitivity
to measurement noise?

ANSWER:

5.3 State estimation - real tank process

We will now study the observer on the real tank process. Remember that \( R_{11} \),
\( R_{22} \) and \( R_2 \) are design parameters and in this case not known.

Remember to fill the tanks up to about the working point. The initial values
for the observer are the working points from section 2.2. The input signal, \( u \),
will now be generated by you on the voltage generator (used to be the reference
signal!).

The observer can be activated with the function:

\[
\text{obs\_tank}(R_{11}, R_{22}, R_2, \text{Tscale}, \text{tsamp})
\]

where

- \( R_{11} \) and \( R_{22} \) are the diagonal elements in the covariance matrix of the process
  noise.
- \( R_2 \) is the variance of the measurement noise. Note that it is the ratio \( R_1/R_2 \)
  that determines the characteristics of the observer.
- \( \text{Tscale} \) is the time scale for the plot. Default value is [360] seconds.
- \( \text{tsamp} \) is the sampling time. Default is [1] second.
Exercise
Try some different values on the covariance matrices and design the observers so that the estimate is as accurate as possible. Generate input signals similar to those in the simulations.

How did you choose $R_1$ and $R_2$?

**ANSWER:**

Exercise
When can it be advantageous to use an observer?

**ANSWER:**