Control Design (F, IT)
Computer Controlled Systems (STS, W)
2006

Instruction to the laboratory work
LQ Control of the Seesaw/Pendulum Process

**Preparation exercises:**
All exercises in Section 4.

**Reading instructions:** Glad-Ljung
Swedish version: Chapters 5.6–5.7, 6.1, 6.5–6.6, 8.5, 9.1–9.4.
English version: Chapters 5.6–5.7, 6.1, 6.3–6.4, 8.4, 9.1–9.4.

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1 Introduction

The goal of this laboratory work is to illustrate how LQG design can be used for controller synthesis for nontrivial systems. It is emphasized that the controller design is an iterative process, and that simulations and tests on the real system are of vital importance.

The system examined in this laboratory work is the seesaw/pendulum process, which is described in Section 3. The system is multivariable, nonlinear and unstable, which makes it nontrivial to control.

2 LQG: Short theoretical background

In this section the standard LQG problem is briefly described. Some approaches for incorporating integral action in LQG controllers are also discussed. A more thorough presentation of LQG is found Chapter 9 in Glad-Ljung.

2.1 The standard LQG problem

The standard LQG problem is to minimize a quadratic criterion for a linear system. That is, to find the controller that, when applied to the system, minimizes the criterion. The standard model (as in Equation (9.4) in Glad-Ljung) of the system is

\[
\begin{align*}
\dot{x} &= Ax + Bu + Nv_1 \\
z &= Mx \\
y &=Cx + v_2
\end{align*}
\]

where \( z \) is the performance signal, \( y \) is the measured output signal, and \( v_1 \) and \( v_2 \) are the white noise-signals with covariance matrices

\[
E v_1^T v_1^T = R_1, \quad E v_2^T v_2^T = R_2, \quad E v_1^T v_2^T = R_{12}
\]

For simplicity, assume that \( v_1 \) and \( v_2 \) are independent, so that \( R_{12} = 0 \).

The control error, i.e., the discrepancy between the performance signal and the reference signal, is denoted \( e = z - r \). It is of course desirable to keep \( e \) as small as possible. The standard formulation of LQG treats the regulation problem\(^1\), where \( r = 0 \). This means that \( e = z \). The criterion to be minimized then is

\[
V = E \lim_{T \to \infty} \frac{1}{T} \int_0^T (z(t)^T Q_1 z(t) + u(t)^T Q_2 u(t)) \ dt
\]

where \( Q_1 \) and \( Q_2 \) are symmetric matrices. \( Q_2 \) is positive definite and \( Q_1 \) is positive semidefinite.

---

\(^1\)For the regulation problem the control objective is to outperform disturbances that causes the performance signal to deviate from zero.
When treating the servo problem, when $r \neq 0$, the criterion should be appropriately modified. However, here $r$ will be confined to be piece-wise constant, like set point changes. In this case LQG will give a feedback gain that is the same as for the regulation problem.

The optimal control law has the familiar observer based state feedback structure (see Theorems 9.1 and 9.2 in Glad-Ljung)

$$u(t) = -L\hat{x}(t) + L_2 r(t)$$

(6)

where $\hat{x}(t)$ is the optimal estimate of $x(t)$, obtained by the Kalman filter for the system (1)–(3). The gain $L$ only depends on $Q_1$ and $Q_2$, while the Kalman filter only depends on $R_1$ and $R_2$ (and $R_{12}$ if nonzero).

Note that the LQG-controller is optimal only for the situation when the system is exactly described by (1)–(3), and when $v_1$ and $v_2$ are white Gaussian noises. This is a very idealized situation. In almost all practical cases one or more of the conditions for the LQG framework is not fulfilled. In this case study the system is not even linear. Hence, it can not be expected that the obtained controller will be optimal. Still, for many systems the LQG approach is useful due to its simplicity and to its ability to generate controllers that behave rather well, also in nonidealized situations.

In practice $R_1$ and $R_2$ are unknown and may, together with $Q_1$ and $Q_2$, be regarded as design variables. There are some guidelines in how to choose these design matrices.

For simplicity the discussion is confined to diagonal choices of $Q_1$ and $Q_2$. Each element in these matrices can be interpreted as a penalty of the corresponding signal component. The rule of thumb then is that this signal component will be small in the closed loop simulation if it is penalized considerable. That is, if the corresponding diagonal entry in the diagonal matrices are chosen large. Still, it is stressed that this is only a rule of thumb and that the design procedure should be regarded as an iterative procedure supported by closed loop simulations.

### 2.2 Integral action in LQG

A drawback with the basic LQG formulation is that no inherent integral action can be obtained. If integral action is desired (this is, according to the basic course in automatic control, often desired) it must be enforced into the problem formulation. There are several ways of doing this, and a few approaches will be presented here.

The first approach is to pretend that there is a constant disturbance on the input signal. This would mean that the input signal recognized by the system is $\tilde{u} = u + d_a$. Here $u$ is the actual input signal, and $d_a$ is the constant disturbance. To be able to handle this within the LQG framework, $d_a$ must be modelled in an appropriate way (since it is not white noise). The analogy between white
noise and impulses (see Chapter 5.3 in Glad-Ljung) can be exploited here; \( d_u \) can be modelled as (almost) integrated white noise, like

\[
d_u(t) = \frac{1}{p + \delta} v_u(t), \quad E v_u^T v_u = R_{v_u}
\]

where \( p \) is the differential operator and \( v_u(t) \) is the white noise (or an impulse). For pure integration the parameter \( \delta \) should be zero. However, this would violate the condition of stabilizability in Theorem 9.1 in Glad-Ljung. Therefore \( \delta \) should be chosen small and positive.

Now the disturbance model can be included into the state space description, as explained in Chapter 5.6 in Glad-Ljung. An augmented state space description of the system plus disturbances would be

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} A & B \\ 0 & -\delta I \end{bmatrix} x + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} N & 0 \\ 0 & I \end{bmatrix} v_1 \\
z &= \begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d_u \end{bmatrix} \\
y &= \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d_u \end{bmatrix} + v_2
\end{align*}
\]  

(7)

To eliminate the effect of \( d_u \) on the control error, the input signal should (in general) be chosen such that \( \ddot{u} = u + d_u = 0 \), at least statically (as \( t \to \infty \)). Therefore it seems reasonable to substitute the second term of the criterion (5) for \( \ddot{u}^T Q_2 \ddot{u} = (u + d_u)^T Q_2 (u + d_u) \). Remember that \( d_u \) is a part of the state vector in the augmented state space description. This means that there will be a cross-coupled term between the input signal and the states in the criterion. However, it can be shown that LQG will give the control law \( u = -L \dot{x} - d_u + L_r r \) in this case, where \( L \) is the feedback gain matrix LQG would yield for the original system (1)-(3) and the original criterion (5) with the same \( Q_1 \) and \( Q_2 \). Thus, only the Kalman filter is affected by the disturbance in this approach, and the covariance matrix \( R_{v_2} \) is the only additional information needed.

A more direct, and perhaps more ad hoc approach to introduce integral action into the control loop, is to add a term to the criterion. This term should penalize the integrated control error. Let \( \epsilon(t) = \int_0^t \dot{r} \, dt \), then the added term would be \( \epsilon(t)^T Q \epsilon(t) \). To deal with this within the LQG framework, \( \epsilon \) is introduced as an extra, fictitious output signal of the system. This implies that \( \epsilon \) also must be included as additional states in the state space description:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} A & 0 \\ M & 0 \end{bmatrix} \begin{bmatrix} x \\ \epsilon \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} N & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} v_1 \\ r \end{bmatrix} \\
z &= \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ \epsilon \end{bmatrix} \\
y &= \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \epsilon \end{bmatrix} + v_2
\end{align*}
\]  

(10)

In (10), the reference signal is entering the system together with the process noise \( v_1 \). This is only to emphasize that from the controllers point of view, \( r \) may
be regarded as a disturbance. Since the extra states are fictitious, the standard Kalman filter, based on (1)–(3), applies. Equations (10)–(12) only need to be considered when the state feedback gain matrix \( L \) is evaluated. However the integrated control error must be included into the controller.

3 The seesaw/pendulum process

3.1 System description

The modelling of the seesaw/pendulum process is divided into four movable objects: The seesaw system (gungbräda), two carts (vagnar) and the pendulum. The seesaw system consists of two seesaws rigidly coupled in parallel. The carts move along tracks on top of these seesaws. The pendulum is mounted to one of the carts via an articulated shaft (ledad axel). The other cart has an extra weight attached to it. The carts will therefore be referred to as the pendulum cart and the weight cart, respectively. Figure 1 shows a sketch of the arrangement. Each

![Figure 1: The seesaw/pendulum process.](image)

cart is actuated by a DC-motor, whose shaft is connected to a gear. The tracks have toothed racks, which the DC-motor gears mesh with. Four optical sensors are used to measure the positions of the carts, the angle of the seesaw and the angle of the pendulum (compared to the normal of the seesaw, the angle \( \alpha \) in Figure 1). Only the deviations from the initial positions/angles are measured.

3.2 Physical modelling

To facilitate any kind of analysis of the system, a model is needed. The model should describe the behaviour of the system in an acceptable way. Preferably it should be a mathematical model. Such a model can be achieved in many different ways. For instance one can get an empirical model by performing
experiments on the system. Another way is to use a priori knowledge about the
system. For a mechanical system, like the seesaw/pendulum process, the laws
of physics serve as a priori knowledge. For this system classical mechanics will
be exploited to obtain a mathematical model. The derivation of this model will
not be shown here, since it involves some rather tedious algebraic manoeuvres.
Instead the presentation is confined to the principal elements of the derivation.

To begin with, the degrees of freedom of the system should be determined.
Then some generalized coordinates, reflecting the degrees of freedom, should be
chosen. The seesaw/pendulum process has four obvious degrees of freedom; the
carts can move linearly along the tracks and the seesaw and the pendulum can
rotate around their points of attachment. Thus, a natural choice of generalized
coordinates is the positions of the carts and the angles of the seesaw and the
pendulum, i.e., $\xi_w$, $\xi_p$, $\theta$ and $\alpha$ in Figure 1. Let $q = \begin{bmatrix} \xi_w & \xi_p & \theta & \alpha \end{bmatrix}^T$ denote
the generalized coordinates. To simplify the modelling, the friction is assumed
to be negligible, and is thus omitted.

The mathematical model is derived with the aid of Lagrange’s equation. The
Lagrangian for the system is

$$L(q, \dot{q}) = T - V$$

where $T$ and $V$ are the total kinetic and potential energies of the system, re-
respectively, expressed in the generalized coordinates $q$ and their time derivatives
$\dot{q}$. Lagrange’s equation then is

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = F$$

(13)

where $F$ is the generalized external force acting on the system. Despite its
very compact form in (13), Lagrange’s equation is a system of second order
nonlinear differential equations — one differential equation for each generalized
coordinate. For the seesaw/pendulum process there are hence four equations.
Both sides in (13) are thus vector valued functions. The entries in $F$ re-
prest the generalized external forces acting in the directions of the generalized
coordinates. The generalized forces acting in the $\xi_w$- and $\xi_p$-directions are the
external forces acting on the carts. These are mainly caused by the DC-motors
on the carts. The generalized forces acting in the $\theta$- and $\alpha$-directions are the
external torques acting on the seesaw and the pendulum. These may be caused
by disturbances, for instance if the pendulum is hit on its top.

Now a mathematical model is achieved, but for the purpose here, it would be
more suitable to have the model as a system of first order differential equations,
i.e., a state space description. A natural choice of state vector is the generalized
coordinates and their time derivatives. The state vector is thus

\[
x = \begin{bmatrix} \dot{q} \\ \dot{\dot{q}} \end{bmatrix} = \begin{bmatrix} \xi_w \\ \xi_p \\ \theta \\ \alpha \\ \xi_w \\ \xi_p \\ \theta \\ \dot{\alpha} \end{bmatrix}
\]

(14)

In order to get the state space description, an expression for \( \dot{x} = \frac{dx}{dt} \) must be found, an expression that only involves \( x \) and \( F \). The simple part is \( \frac{\partial}{\partial q} \dot{q} = \dot{q} \), which is a part of the state vector. However, \( \frac{\partial}{\partial q} \dot{q} = \ddot{q} \) must be solved for in (13). This yields the state space description

\[
\dot{x} = \dot{f}(x, F)
\]

which has the generalized external forces as input signal. It would be preferable, though, to have the voltage over the DC-motors as input signals, rather than the forces they cause. This must somehow be included in the model. Fortunately, this can be interpreted as if state feedback is applied. Thus, no new state variables are needed.

The obtained nonlinear state space model is then

\[
\dot{x} = f(x, u, \ddot{v}_1)
\]

(15)

where \( x \) is the state vector as in (14), \( u = \begin{bmatrix} u_w \\ u_p \end{bmatrix} \) is the input signal available for control, and \( \ddot{v}_1 = \begin{bmatrix} v_{w1} \\ v_{1a} \end{bmatrix} \) is the external torques on the seesaw and the pendulum. \( u_w \) is the input voltage to the weight cart, and \( u_p \) is the input voltage to the pendulum cart.

3.3 Linearization

In order to be able to control the system with an LQ-controller, the dynamics are linearized around the origin (an equilibrium point). The origin is defined as is shown in Figure 1. That is, the seesaw is horizontal, the pendulum is erected vertical, and both carts are on the middle of their tracks.

The linearized model should be in the form of the standard model:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Nv_1 \\
z &= Mx \\
y &= Cx + v_2
\end{align*}
\]
The state vector \( x \) and the input signal \( u \) are the same as in the nonlinear model (15). Apart from \( \bar{v}_1 \), disturbances acting on the input signal will be considered. The process noise will therefore be \( v_1 = [v_{1w} \ v_{1p} \ v_{1b} \ v_{1a}]^T \), and thus \( \tilde{N} = [B \ \tilde{N}] \). The matrices \( A, B \) and \( \tilde{N} \) are given by

\[
A = \frac{\partial f}{\partial x}, \quad B = \frac{\partial f}{\partial u}, \quad \tilde{N} = \frac{\partial f}{\partial \bar{v}_1}
\]
evaluated at \( x = 0, u = 0 \) and \( \bar{v}_1 = 0 \). Numerically these will be

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-1.4 & -1.1 & 9.2 & 0.0 & -9.0 & -0.17 & 0.0 & 0.0 \\
-1.4 & -1.1 & 9.2 & -2.7 & -0.17 & -14.0 & 0.0 & 0.0 \\
9.7 & 7.7 & 4.3 & 0.0 & 1.2 & 1.2 & 0.0 & 0.0 \\
-9.7 & -7.7 & -4.3 & 29.0 & -1.2 & 32.0 & 0.0 & 0.0
\end{bmatrix}
\]

and

\[
B = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
2.0 & 0.038 \\
0.038 & 3.2 \\
-0.27 & -0.27 \\
0.27 & -7.2
\end{bmatrix}, \quad \tilde{N} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
-0.16 & 0.16 \\
-0.16 & -4.2 \\
1.1 & -1.1 \\
-1.1 & 48
\end{bmatrix}, \quad N = [B \ \tilde{N}]
\]

The performance signal \( z \) and the measured output \( y \) are the generalized co-ordinates. Thus \( z = q = [I \ 0] x \) and \( y = q + v_2 = [I \ 0] x + v_2 \), and hence \( M = C \), which is

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

When working with linearized models, it is very important to remember that the linear model is an approximation. It may be viewed as a first order Taylor expansion of the nonlinear model, around some operation point. The further away from this point, the origin in this case, the less accurate the linear description is. Thus the linear model should be regarded as a "local model".

### 3.4 Implementation

The controllers for the seesaw/pendulum process are implemented in Matlab. For this reason the system is connected to a PC. The four measured output
signals are directly connected to the parallel port of the PC. The input signals to the system, the voltages over the DC-motors on the carts, are fed to the system from the PC via a D/A-converter and power amplifiers.

A computer works in discrete time, and therefore effects caused by this must be incorporated into the synthesis procedure. This problem will be disregarded here — the controller synthesis in the laboratory work is performed in continuous time, yielding time-continuous controllers. For the implementation, these are approximated by sampling controllers\(^2\), obtained with the Matlab function \texttt{c2d}. The sampling period is \(T = 0.01\) seconds.

4 Preparation exercises

The feedback gain \(L\) in LQG control only depends on the matrices \(Q_1\) and \(Q_2\) in the criterion (5). These matrices are design parameters in the LQG design. If they are chosen as \(Q_1 = kQ_1\) and \(Q_2 = kQ_2\), for some constant matrices \(Q_1\) and \(Q_2\), and some positive scalar \(k\), the feedback gain \(L\) will be the same, regardless of how \(k\) is chosen.

Question 4.1: Use the criterion (5) to motivate why this is the case.

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\textbf{Answer:} \\
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\end{tabular}
\end{center}

The sensitivity function \(S(s)\) is the transfer function, from an additive disturbance on the output, to the output, for the closed loop system.

\(^2\)See Chapter 4 in Glad-Ljung for more thorough comments on sampling and sampled systems.
**Question 4.2:** How can you tell if there is integral action in the control loop by studying $S(s)$?

**Answer:**

A continuous signal $y(t)$ is sampled at $t = kT$ for $k = \ldots, -1, 0, 1, 2, \ldots$, where $T$ is the sampling period. A simple way to approximate the time derivative $\frac{dy}{dt}(t)$, is to use the backward difference

$$\frac{d}{dt}y(kT) \approx \frac{y(kT) - y(kT - T)}{T}$$

For the seesaw/pendulum process, the state vector is $x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$ (see (14)), where $q = [\xi_w \ \xi_\rho \ \theta \ \alpha]^T$ is measured, while $\dot{q}$ is not. In the first laboratory task, pure state feedback is applied, i.e., $u = -Lx$. Since $\dot{q}$ is not measured, it is computed with a backward difference like above. Let $\nu$ be the backward difference approximation of $\dot{q}$. The control law then is implemented as $u(kT) = -L \begin{bmatrix} q(kT) \\ \nu(kT) \end{bmatrix}$.

The backward difference can be represented in state space form as

$$\zeta(kT + T) = F\zeta(kT) + Gq(kT)$$
$$\nu(kT) = H\zeta(kT) + Jq(kT)$$


**Question 4.3:** Give expressions for the matrices $F, G, H$ and $J$ above.

**Answer:**
5 Laboratory work

In the laboratory work LQG will be used to design controllers for the seesaw/pendulum process. The controllers will be analysed, simulated and tested on the real system. The laboratory work starts in Section 5.2. In Section 5.1 the Matlab functions, specific for the laboratory work, are described.

5.1 Introduction to the laboratory work

Here the Matlab functions used in this laboratory work are described.

Controller generating functions

These functions generate controllers that can be used as input parameters for other functions. The input parameters to these functions represent the weight (or penalty) matrices in the criterion (5), and the noise intensities (4), respectively. \( Q_1 \) represents \( Q_1 \) (4 \( \times \) 4 matrix), \( Q_2 \) represents \( Q_2 \) (2 \( \times \) 2 matrix), \( R_1 \) represents \( R_1 \) (4 \( \times \) 4 matrix), and \( R_2 \) represents \( R_2 \) (4 \( \times \) 4 matrix). For the integrating controllers, \( r \) represents \( R_w \) (2 \( \times \) 2 matrix) in the approach with constant disturbances on the input signal, and \( q \) represents \( Q_e \) (2 \( \times \) 2 matrix) in the approach with integrated control error. If a matrix is chosen to be diagonal (which is recommended), it is sufficient to represent it with a vector consisting of the diagonal entries only. For instance, if

\[
Q_2 = \begin{bmatrix}
10 & 0 \\
0 & 1
\end{bmatrix}
\]

it can be represented either as the matrix \( Q_2 = [10 \ 0; \ 0 \ 1] \), or as the vector \( Q_2 = [10 \ 1] \).

The controller generating functions follow below.

**lqseeaw:**

Syntax: \( \text{L}=\text{lqseeaw}(Q1,Q2) \);

Generates the feedback gain matrix \( L \) corresponding to \( Q_1 \) and \( Q_2 \).

**lqgseeaw:**

Syntax: \( \text{reg}=\text{lqgseeaw}(Q1,Q2,R1,R2) \);

Generates an LTI object, \( \text{reg} \), representing the controller corresponding to \( Q_1 \), \( Q_2 \), \( R_1 \) and \( R_2 \).

**lqgintin:**

Syntax: \( \text{reg}=\text{lqgintin}(Q1,Q2,R1,R2,r) \);

Generates an LTI object, \( \text{reg} \), representing the integrating controller corresponding to \( Q_1 \), \( Q_2 \), \( R_1 \), \( R_2 \) and \( r \), for the approach with constant disturbances on the input signals.
lqgintout:
Syntax: reg=lqgintout(Q1,Q2,R1,R2,q);
Generates an LTI object, reg, representing the integrating controller corresponding to Q1, Q2, R1, R2 and q, for the approach with integrated control error.

Remark: It is a good idea to give the controllers different names (at least the controllers that work on the real system), for instance like reg1, reg2, reg3intin and so on. In this way the different controllers are saved in Matlab’s work space so that they can be compared to each other.

Functions for analysis

These functions help to analyse what will happen to the closed loop system for the linear model for the different controllers.

tfseesaw:
Syntax: tfseesaw(reg) or tfseesaw(reg1,reg2,...,reg6)
Plots the singular values for a number of transfer functions for the closed loop system (these depend on the controller). For instance S(s), T(s) and G_{ry}(s).
The input parameters should be LTI objects that represent the controllers. Up to six different controllers can be compared.
sensfunc:
Syntax: [S,Gry]=sensfunc(reg);
Generates LTI objects representing the sensitivity function, S(s), and the transfer function from the disturbances (the process noise) to the output, G_{ry}(s), corresponding to the controller reg, which should be an LTI object.

Functions for testing

With these functions the obtained controllers can be simulated and tested on the real system.
simseesaw:
Syntax: simseesaw(L) or simseesaw(reg)
Simulates the system when controlled with L or reg. When the input parameter is a matrix (L) pure state feedback is simulated, and when the input parameter is an LTI object (reg) output feedback is simulated. The default situation simulated is that the system starts at rest. Then, after 0.5 seconds, the pendulum is hit at its top. Other situations can be simulates as well, which require more (optional) input parameters — use help to see how this is done.
regulate:
Syntax: regulate(L) or regulate(reg) or regulate(reg1,...,reg4)
Applies the controllers on the real system. When the input parameter is a matrix (L) state feedback is applied. The state variables not measured are approximated
with a backward difference (see Section 4). When the input parameter is an LTI object (\texttt{reg}) output feedback is applied.

5.2 Stabilizing the system

The first laboratory task is to stabilize the system, i.e., to find a controller that manages to balance the pendulum and the seesaw simultaneously. Throughout this section the reference signal, or the set point, is identically zero. Thus, this is mainly the regulation problem.

The functions described in Section 5.1 require that a number of global variables are set. This is done with \texttt{initseesaw}. Thus, before anything else is done, type \texttt{initseesaw}.

\textbf{Task 5.1} First regard pure state feedback control, \( u = -Lx \). Use \texttt{kgseesaw} to generate the feedback gain matrix \( L \). Try to find matrices \( Q_1 \) and \( Q_2 \) for which the system behaves reasonable. Use \texttt{simseesaw} to simulate the system. Specially, consider the limitations of the system. The control signal must lie within the interval \([-6, 6]\) Volts, and the carts cannot move more than 0.45 meters to any side. Also, the angles of the pendulum and the seesaw should not be too large. With \texttt{regulate} the controllers can be tested on the real system. See in Section 5.1 how the functions should be used.

Initially it may be benificial to focus on one parameter in the tuning procedure. Therefore, set \( Q_1 = I \) and \( Q_2 = \rho I \), where \( \rho > 0 \) is varied. The objective is to achieve as fast response as possible, without saturating the input signal.

Start with \( \rho = 1 \). To see any essential effect in the behavior of the system, \( \rho \) should be changed at least by a factor of ten.

When a \( \rho \) is found, that works well with the real system, some fine tuning of the controller can be done, by changing the entries in \( Q_1 \). Again, to see any effect, the changes should be by a factor of ten.

Contact the instructor when a working controller has been found.

The state feedback controller implemented on the real system does not apply true state feedback, since not all state variables are measured. The state variables not measured are approximated by a backward difference, as is explained in Section 4. This implementation may be viewed as an observer (with a direct term). Next, a more sophisticated observer design will be considered.

\textbf{Task 5.2} Use state feedback from estimated states, \( u = -L\dot{x} \), and use a Kalman filter to estimate the states. Regard \( R_1 \) and \( R_2 \) as design parameters. Use the following iterative scheme:

1. Use the \( Q_1 \) and the \( Q_2 \) obtained in the previous task.

2. Set \( R_1 = I \) and \( R_2 = rI \), where \( r > 0 \). Choose/adjust \( r \), and use \texttt{kgseesaw} to generate an LTI object representing the corresponding controller.
To see any essential effects in the behavior of the system, $r$ should be changed by a factor of ten or hundred.

3. Use \texttt{tfseesaw} to study the relevant transfer functions. If these are not satisfactory, go back to point 2.

4. Simulate the system with \texttt{simseesaw}. If the result is not satisfactory, go back to point 2.

5. Try the controller on the real system with \texttt{regulate}. If the result is not satisfactory, go back to point 2.

Contact the instructor when a working controller has been found. This controller will be the basis for the controller design in the laboratory tasks below.

\section*{5.3 Set point changes}

So far the system has been stabilized, i.e., the obtained controller manages to balance the system. However, the controller should also keep the control error as small as possible. This means that the closed loop system should be able to follow the reference signal, in spite of disturbances and model errors. In particular the static control error should vanish.

The seesaw/pendulum process have two input signals and four measurable output signals. With two input signals it seems reasonable that only two of the output signals can be “controlled” in the sense that they can follow a reference signal. The question is, which two output signals are controllable in this sense? With no further theoretical investigation, the following physical reasoning may give a hint: Consider constant reference signals, set points. This means that the system should be at rest. Hence the angle of the pendulum can be ruled out as a controllable output signal, since the system can be at rest only if the pendulum is vertical. Also, the positions of the two carts cannot both be controllable output signals, because one of the carts must always balance the seesaw in order to keep the system at rest, and is thus dependent on the other cart. Hence, the only candidates for controllable output signals are the angle of the seesaw and the position of one of the carts. Here the position of the pendulum cart is considered to be the most interesting signal. Thus, reference signals are introduced for the position of the pendulum cart and the angle of the seesaw.

\textbf{Task 5.3} Use the controller obtained in the previous task on the real system again. When the transients have decayed, do a set point change for both reference signals by moving the slide bars that appear in one of the MATLAB windows. The controller obtained by \texttt{lqgseesaw} is designed to give a static gain equal to one from the reference signal to the output.
Question 5.1: Do the control errors vanish? If not, explain why.

Answer:

5.4 Integral action

The standard approach to eliminate control errors is to introduce integral action in the control loop. This can be done in several ways. Here the approaches in Section 2.2 will be used.

Task 5.4 Apply the approach with constant disturbances on the input signal. Use kqintin to obtain the controller. Set \( R_{\text{eq}} = r_{\nu} I \), where \( r_{\nu} \) is a positive scalar. Use the same \( Q_1, Q_2, R_1 \) and \( R_2 \) as obtained above. Tune the controller by changing \( r_{\nu} \).

Question 5.2: Use tfseesaw to study the singular values of the sensitivity function. Do the integrators show in the diagram?

Answer:

Try the controller on the real system and do the set point changes again.
**Question 5.3:** Do the control errors vanish?

**Answer:**

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**Task 5.5** Use the approach where the control errors\(^3\) are integrated. The function \texttt{lqgintout} gives the corresponding controller. Set \(Q_e = \rho_e I\), where \(\rho_e\) is a positive scalar. Tune the controller by changing \(\rho_e\). Use the same \(Q_1, Q_2, R_1\) and \(R_2\) as obtained above (\(Q_2\) may also be changed). Try the controller on the real system and do the set point changes again.

**Question 5.4:** Do the control errors vanish?

**Answer:**

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**Task 5.6 - Optional**

Compare the two controllers obtained in the two previous tasks. Use \texttt{sensfunc} to obtain LTI objects representing \(S(s)\) and \(G_{vy}(s)\) for the two controllers.

Notice that the output \(y = [\xi_w \quad \xi_p \quad \theta \quad \alpha]^T\) (ignoring the measurement noise) and the disturbance \(v_1 = [v_{1w} \quad v_{1p} \quad v_{1\theta} \quad v_{1\alpha}]^T\) can be interpreted as vectors\(^4\). Therefore the notions of output and disturbance directions are justified, meaning the directions of the output and disturbance vectors respectively.

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\(^3\)It is the control errors for the pendulum cart position and the seesaw angle that are integrated.

\(^4\)This is true for the input as well.
**Question 5.5:** Study $G_{vy}(s)$ for the two controllers. In which disturbance directions do constant disturbances have no effect on the output statically? Hint: The functions `degain` and `svd` are useful here.

**Answer:**

**Question 5.6:** Study $S(s)$ for the two controllers. In which output directions will the static control error be zero? Hint: The functions `degain` and `svd` are useful here.

**Answer:**

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