UPPSALA UNIVERSITET
AVDELNINGEN FÖR SYSTEMTEKNIK
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Some Answers to Computer exercise 1 for the course Empirical Modelling

1

2

2.1

2.2 Preparation exercises

Calculate the static gain for the system (you may also want to test that the two ways above to calculate the static gain give the same result)

Answer:

$$y_s = \frac{B(1)}{A(1)} = \frac{b_1}{1+a_1} = \frac{1}{1-0.8} = 5$$

How could we find the static gain in a bode plot?
Answer:
Since $y_s = \frac{B(1)}{A(1)} = H(1) = H(e^{i\omega=0})$, the static gain is seen in the Bode magnitude plot for $\omega = 0$ (that is, the same as for continuous time systems). In practice, the static gain can be found from the magnitude ('gain') for low frequencies in the Bode diagram (low frequency asymptot).

Calculate $\{g(k)\}_{k=0}^{2}$ for the system (that is for the case $b_1 = 1$, and $a_1 = -0.8$)
Answer:

$$g(0) = 0,\ g(1) = b_1 = 1,\ g(2) = -b_1 a_1 = 0.8$$

2.3 Transient analysis

Compare the steady state value of the noise free system with the calculated static gain done in the preparation exercise.
Answer:
Good agreement (the output from the system reaches, as expected, the value five).

How would you estimate the static gain if you only had noisy data from a step response (as in the figure from the macro above)?
Answer:
Estimate the mean value (after the output has settled from the step response)

2.4 Correlation analysis

Compare the estimated impulse response $\{gest(k)\}_{k=0}^{2}$ with the true values calculated in the Preparation exercise.

Answer:
Typical result (note that the result will differ from different realizations) $gest(0) = 0.05$, $gest(1) = 0.93$, and $gest(2) = 0.7$. Hence we get a fairly close agreement with the true values (see previous exercise).

Which one of the following system properties may be directly visible in the impulse response: Static gain, Number of poles or Time delay?
Answer:
The time delay (which corresponds to the number, nn of impulse response coefficients fulfilling $\{gest(k) \approx 0\}_{k=0}^{nn}$.) Note that an estimate of the uncertainty of the parameters are also presented in the plot.
The uncertainty of the estimate depends on, for example, the number of data, and the noise level. Few data and a high noise level will give poor estimate where the time delay may be hard to see.

2.5 Spectral analysis

Discuss how M affects the estimate. What value on M do you find reasonable?
Answer:
Low M (for example $M = 10$), smooth estimate but significant errors. High M (for example $M = 100$), noisy estimate but a low systematic error. A fairly good estimate is achieved with M in the region 20-30. Note that if no argument is given to *spa* the used default value is

```
\min(length(DATA)/10,30)
```

See `help(spa)`.

3 Parametric methods

3.1

3.1.1

3.1.2 Exercises

Try a few different number of data points (for example 10, 100 and 1000). Does the estimated model seem to converge to the true system as the number of data points increases (check both parameter values and Bode plots)?
Answer:
Yes the estimated model seems to converge to the true system as the number of data points increases (which is expected from theory, see equation (12.49) in the book).

3.2 The least squares method and non white noise

Use the macro above and enter S2 as answer to the last question. Is it possible to accurately estimate the system $\mathcal{S}_2$ with the least squares estimate of the ARX model? In this exercise, use many data points, for example $N = 1000$ (or higher!).
Answer:

No the estimated model seems not to converge to the true system even when the number of data points is high. From the theory we know that the ARX model estimated with the LS method only give correct estimate if the noise is white and enters as for the system $\mathcal{S}_1$) and hence in this case we should expect a bias (systematic error) which will *not* vanish even as the number of data points goes to infinity..

Is the difference between the true and estimated system (the bias") at low frequencies smaller or larger when the input is low pass filtered compared to when the input signal is white noise? Does this make sense?

Answer:
The bias error for low frequencies are smaller when the input signal has a low frequency character compared to when the input signal is white noise (a white noise sequence has a constant spectral density).
Heuristic explanation: In general a model is calibrated so that the squared sum of the prediction errors are minimized. If the signal is of low frequency character, the model will have good accuracy for low frequency in order to minimize the criteria.

In equation (12.46) it is seen how the estimate is affected by the input spectral density $\Phi_u(\omega)$.

4 Examples of linear regression

In this section we give some exercises of linear regression models. Do them during the lab occasion if you have time left or later at your own.

4.1 A simple example

Calculate the least squares estimate

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T Y$$

What estimate do you get?

Answer:
$\hat{\theta} = (1\ 2\ 3)^T$

Also try the following simple Matlab command (where Φ is called `Phi`) for the least squares estimate

```
thetaHat= Phi\Y
```

Do you get the same estimate?
Answer:
Yes!
More detail on the backslash operator is given by `help mldvivide`.

4.2 Prediction of the word population by a polynomial model

What model order do you think gives the most realistic prediction?
Answer:
A low order polynomial, second or third order, seems to give the most realistic prediction. For high polynomial orders ('overfitting'), very bad predictions (even negative population) may occur!

4.3 Bad data

Do the elements become large if e_i is small? If so, the variance of the estimated parameters will be large and the standard least squares method is not useful!

Answer:

Yes. Note that the variance of the estimate is given by $\lambda(\Phi^T\Phi)^{-1}$