Course PM
1TD254: Finite element methods 2, 5.0 hp

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[Enter King Richard III]
King Richard III: –A horse! a horse! my kingdom for a horse!
Catesby: –Withdraw, my lord; I'll help you to a horse.
William Shakespeare: *Richard III*, Act 5, scene 4.

What is NOT in this document

- Exact dates and times
- Chapters to read, suitable exercises
- Pointers to the literature
- Actual laborations and projects

For these matters, please refer to the continuously updated course web-page at

`http://www.it.uu.se/edu/course/homepage/finmet2/vt14/`

Course contents

Key concepts: normed vector spaces, bilinear forms, L^p- and Sobolev-spaces, continuity & boundedness, completeness, dual space, weak derivatives, traces, representation theorems.

Finite element methods: elliptic boundary value problems, parabolic time-dependent initial value problems, convection-diffusion problem, stabilization, non-linear problems, linearization, system of PDEs, implementation and applications of finite element modeling.

Finite element theory: existence & uniqueness, well-posedness, interpolants, finite elements, higher order approximation, *a priori/posteriori* error bounds.

Learning objectives

This course will teach you how to solve problems formulated as partial differential equations (PDEs) with the finite element method and how to analyze the method itself. This method is important thanks to its *generality*: it applies in a wide variety of situations and has many important applications. For example, the FEM is used in computational methods in Biophysics, Chemical engineering, Electromagnetics, Fluid mechanics, General relativity, Geophysics, Structural mechanics. In this advanced level course we will focus on abstract analysis of elliptic problems, non-linear solution techniques, different kinds of finite elements, as well as finite elements for systems of PDEs.

To pass the course you should be able to

- P1 derive and discretize the variational formulation for a general elliptic operator using the finite element method,
- P2 use existing lemmas to prove well-posedness of the solution both in the continuous and in the discrete case,
- P3 derive basic *a priori* and *a posteriori* error estimates,
- P4 *(i)* define and use Lagrange finite element of higher order, *(ii)* given some set-up, define and use also bilinear quadrilateral-, Crouzeix-raviart, lowest order Raviart-Thomas, and lowest order Nédélec elements,
- P5 derive and use fix-point iteration/Picards method in an algorithm to solve non-linear elliptic problems,
- P6 implement the finite element method on a computer.

Note: P6 will be tested in the mandatory project and not in the written exam.

For higher grades you are also required to know how to

- H1 detail the equivalence between the finite element method and an abstract minimization problem,
- H2 derive and apply the Newton-Galerkin method in an algorithm to solve non-linear elliptic problems,
- H3 explain/motivate and perform *a priori* analysis of stabilization for convection-diffusion-type problems.

Teaching

The course consists of 7 lectures, 4 exercise classes and 5 computer labs. *Prepare each lecture and each class by reading through the material beforehand and by working through some of the exercises. This is much more efficient and saves a lot of time! You make the class less effective to yourself and to your fellow students by being unprepared! Instead, be prepared!*

Project There is one mandatory project.

	Details	Goals targeted
P1	Real applications	P6 + several others

Laborations The laborations are designed to be useful when later doing the projects.

	Details	Goals targeted
Lab 1	The finite element method	P1, P6, H3
Lab 2	Finite element method for non-linear problems	P5, P6, H2
Lab 3	System for partial differential equations	P1, P5, P6, H2

Final exam The final exam consists of 5 exam questions Q1–Q5. Each of these will test a few of the goals P1–P5 and/or H1–H3. Rather than stating a precise minimum number of points to pass, each question will be graded separately with respect to the goals it targets. Hence in order to *pass* the course you must demonstrate acceptable knowledge (that is, grade 3) on all basic goals P1–P5. For *higher* grades you should demonstrate *good/excellent* solutions for the basic goals, but *also* for H1–H3.

Since this method of grading might be a bit of a novelty, it will be discussed and motivated at the introductory lecture. We will also work through a couple of examples to see how it actually works. Should doubts still persist, never hesitate to bring it up in class or otherwise feel free to contact the teachers.

Goals	Question	Grade
P1	Q1(a),Q4(a)	5
P2	Q1(b),Q4(b)	5
P3	Q1(c+d)	5
P4	Q2(a),Q2(b)	3
P5	Q4(a),Q5(b)	4
H1	Q1(e),Q5(a)	4
H2	Q4(c),Q5(c)	5
H3	Q3(a),Q3(b)	F
	Total grade:	???

Table 1: The grading matrix for an imaginary exam with 5 exercises. *Example:* Question Q4 tests P1+2+5 and H2. -What grade should the student have?

7× pieces of advice from previous students

1. *"Read the book yourself before the lecture, it makes it easy to understand the difficult things during the lecture."*
2. *"The most challeging part are the different proofs in the course. My advice is to not just proof them but also reflect over what the consequence of the proof is for FEM. Also try to give the assignments alot of thought. Remember to discuss and analyse the results. The exam will become alot easier then."*
3. *"Don't be afraid to participate actively in the lectures and prepare yourself for each lecture by following the recommendations given at the end of the previous one. I regret not doing so."*
4. *"Get a good overview of all definitions."*
5. *"It's a hard course so follow the reading suggestions before the lectures. Take time to understand the assignments properly since they are very useful for the exam."*
6. *"Keep up with the pace during the whole course and do the recommended exercises and reading."*
7. *"The labs are really good so do your best to understand them and you will be well prepared for the exam."*