LR Parsing
LALR Parser Generators

Outline

- Review of bottom-up parsing
- Computing the parsing DFA
- Using parser generators

Bottom-up Parsing (Review)

- A bottom-up parser rewrites the input string to the start symbol
- The state of the parser is described as

$$\alpha \mid \gamma$$

  - α is a stack of terminals and non-terminals
  - γ is the string of terminals not yet examined

- Initially: $\mid x_1 x_2 \ldots x_n$

The Shift and Reduce Actions (Review)

Recall the CFG: $E \to E + (E) \mid \text{int}$

A bottom-up parser uses two kinds of actions:

- Shift pushes a terminal from input on the stack

$$E + (\,|\, \text{int}) \Rightarrow E + (\text{int} \,|\,)$$

- Reduce pops 0 or more symbols off of the stack (production RHS) and pushes a non-terminal on the stack (production LHS)

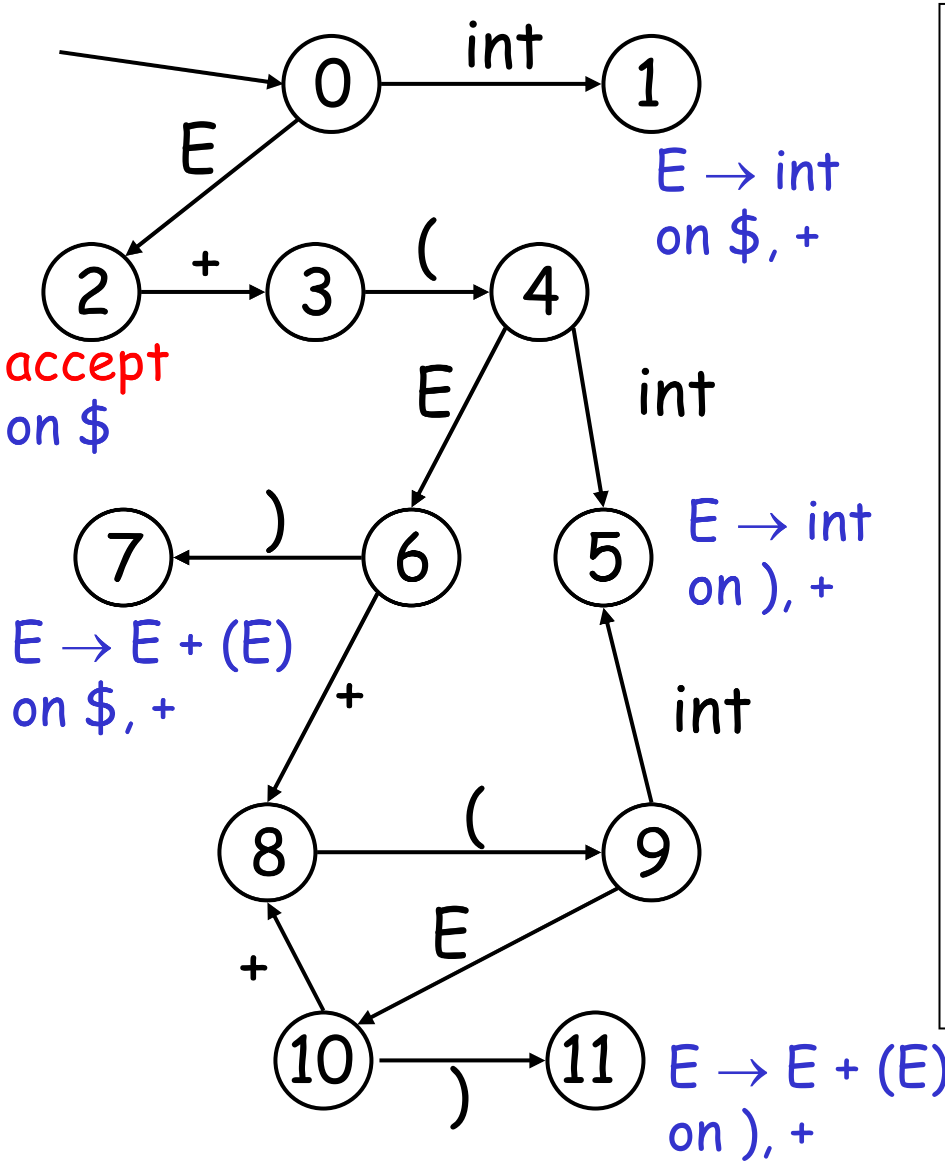
$$E + (\underline{E + (E)} \,|\,) \Rightarrow E + (\underline{E} \,|\,)$$

Key Issue: When to Shift or Reduce?

- Idea: use a deterministic finite automaton (DFA) to decide when to shift or reduce
  - The input is the stack
  - The language consists of terminals and non-terminals

- We run the DFA on the stack and we examine the resulting state X and the token tok after |
  - If X has a transition labeled tok then <u>shift</u>
  - If X is labeled with "$A \to \beta$ on tok" then <u>reduce</u>

LR(1) Parsing: An Example



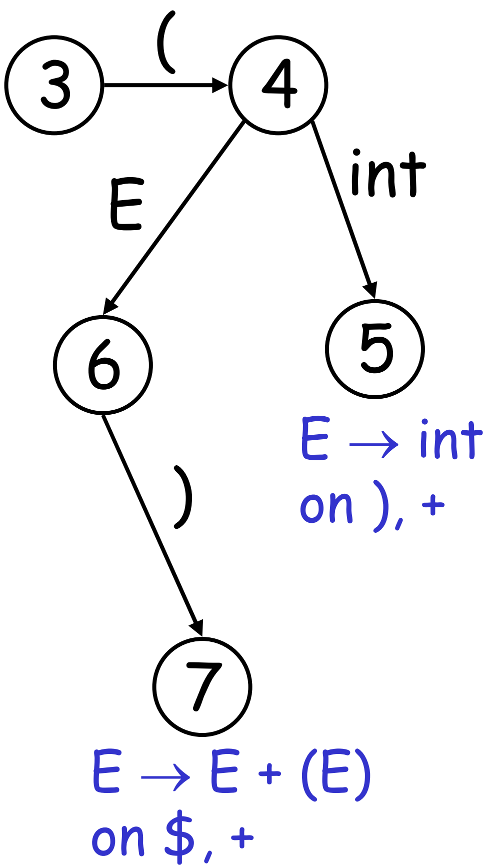
<table>
<tr><td>| int + (int) + (int)$</td><td>shift</td></tr>
<tr><td>int | + (int) + (int)$</td><td>E → int</td></tr>
<tr><td>E | + (int) + (int)$</td><td>shift (x3)</td></tr>
<tr><td>E + (int |) + (int)$</td><td>E → int</td></tr>
<tr><td>E + (E |) + (int)$</td><td>shift</td></tr>
<tr><td>E + (E) | + (int)$</td><td>E → E+(E)</td></tr>
<tr><td>E | + (int)$</td><td>shift (x3)</td></tr>
<tr><td>E + (int |)$</td><td>E → int</td></tr>
<tr><td>E + (E |)$</td><td>shift</td></tr>
<tr><td>E + (E) | $</td><td>E → E+(E)</td></tr>
<tr><td>E | $</td><td>accept</td></tr>
</table>

Representing the DFA

- Parsers represent the DFA as a 2D table
  (Recall table-driven lexical analysis)
- Lines correspond to DFA states
- Columns correspond to terminals and non-terminals
- Typically columns are split into:
  - Those for terminals: the action table
  - Those for non-terminals: the goto table

Representing the DFA: Example

The table for a fragment of our DFA:



	int	+	(	)	$	E
...						
3			s4			
4	s5					g6
5		$r_{E \rightarrow int}$		$r_{E \rightarrow int}$		
6	s8		s7			
7		$r_{E \rightarrow E+(E)}$			$r_{E \rightarrow E+(E)}$	
...						

sk is shift and goto state k
$r_{X \rightarrow \alpha}$ is reduce
gk is goto state k

The LR Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
  - This is wasteful, since most of the work is repeated

- To avoid this, we remember for each stack element on which state it brings the DFA

- LR parser maintains a stack

$$\langle \text{sym}_1, \text{state}_1 \rangle \ . \ . \ . \ \langle \text{sym}_n, \text{state}_n \rangle$$

state_k is the final state of the DFA on $\text{sym}_1 \ldots \text{sym}_k$

The LR Parsing Algorithm

```
let I = w$ be initial input
let j = 0
let DFA state 0 be the start state
let stack = ⟨ dummy, 0 ⟩
  repeat
    case action[top_state(stack), I[j]] of
      shift k: push ⟨ I[j++], k ⟩
      reduce X → A:
        pop |A| pairs,
        push ⟨ X, goto[top_state(stack), X] ⟩
      accept: halt normally
      error: halt and report error
```

Key Issue: How is the DFA Constructed?

- The stack describes the context of the parse
  - What non-terminal we are looking for
  - What production RHS we are looking for
  - What we have seen so far from the RHS

- Each DFA state describes several such contexts
  - E.g., when we are looking for non-terminal E, we might be looking either for an int or an E + (E) RHS

LR(0) Items

- An <u>LR(0) item</u> is a production with a "$\shortmid$" somewhere on the RHS

- The LR(0) items for $T \rightarrow (E)$ are

  $T \rightarrow \shortmid\ (E)$

  $T \rightarrow (\ \shortmid E)$

  $T \rightarrow (E \shortmid\ )$

  $T \rightarrow (E) \shortmid$

- The only LR(0) item for $X \rightarrow \varepsilon$ is $X \rightarrow \shortmid$

LR(0) Items: Intuition

- An item $[X \to \alpha \mid \beta]$ says that the parser
  - is looking for an X
  - has an α on top of the stack
  - expects to find a string derived from β next in the input

- Notes:
  - $[X \to \alpha \mid a\beta]$ means that a should follow
    - Then we can shift it and still have a viable prefix
  - $[X \to \alpha \mid]$ means that we could reduce X
    - But this is not always a good idea !

LR(1) Items

- An <u>LR(1) item</u> is a pair:

$$X \to \alpha \mid \beta,\ a$$

  - $X \to \alpha\beta$ is a production
  - a is a terminal (the lookahead terminal)
  - LR(1) means 1 lookahead terminal
- $[X \to \alpha \mid \beta, a]$ describes a context of the parser
  - We are trying to find an X followed by an a, and
  - We have (at least) α already on top of the stack
  - Thus we need to see next a prefix derived from βa

Note

- The symbol I was used before to separate the stack from the rest of input
  - α I γ, where α is the stack and γ is the remaining string of terminals
- In items, I is used to mark a prefix of a production RHS:

$$X \to \alpha \,\text{I}\, \beta, \quad a$$

  - Here β might contain non-terminals as well
- In either case the stack is on the left of I

Convention

- We add to our grammar a fresh new start symbol S and a production $S \to E$
  - Where E is the old start symbol
- The initial parsing context contains:

$$S \to \mid E \;,\; \$$$

  - Trying to find an S as a string derived from E$
  - The stack is empty

LR(1) Items (Cont.)

- In context containing

$$E \rightarrow E + \mid (E) \quad , +$$

  - If (follows then we can perform a shift to context containing

$$E \rightarrow E + (\mid E) \quad , +$$

- In context containing

$$E \rightarrow E + (E) \mid \quad , +$$

  - We can perform a reduction with $E \rightarrow E + (E)$
  - But only if a + follows

LR(1) Items (Cont.)

- Consider the item

$$E \rightarrow E + (\mid E) \;,\; +$$

- We expect a string derived from $E \;) \; +$
- Our example has two productions for E

$$E \rightarrow \text{int} \quad \text{and} \quad E \rightarrow E + (E)$$

- We describe this by extending the context with two more items:

$$E \rightarrow \mid \text{int} \qquad , \;)$$

$$E \rightarrow \mid E + (E) \;,\;)$$

The Closure Operation

- The operation of extending the context with items is called the closure operation

```
Closure(Items) =
  repeat
    for each [X → α • Yβ, a] in Items
      for each production Y → γ
        for each b in First(βa)
          add [Y → • γ, b] to Items
  until Items is unchanged
```

Constructing the Parsing DFA (1)

$E \rightarrow E + (E) \mid int$

- Construct the start context:

$$\text{Closure}(\{S \rightarrow \bullet E, \$\})$$

$$\begin{array}{ll} S \rightarrow \bullet E & , \$ \\ E \rightarrow \bullet E+(E) & , \$ \\ E \rightarrow \bullet int & , \$ \\ E \rightarrow \bullet E+(E) & , + \\ E \rightarrow \bullet int & , + \end{array}$$

- We abbreviate as:

$$\begin{array}{ll} S \rightarrow \bullet E & , \$ \\ E \rightarrow \bullet E+(E) & , \$/+ \\ E \rightarrow \bullet int & , \$/+ \end{array}$$

Constructing the Parsing DFA (2)

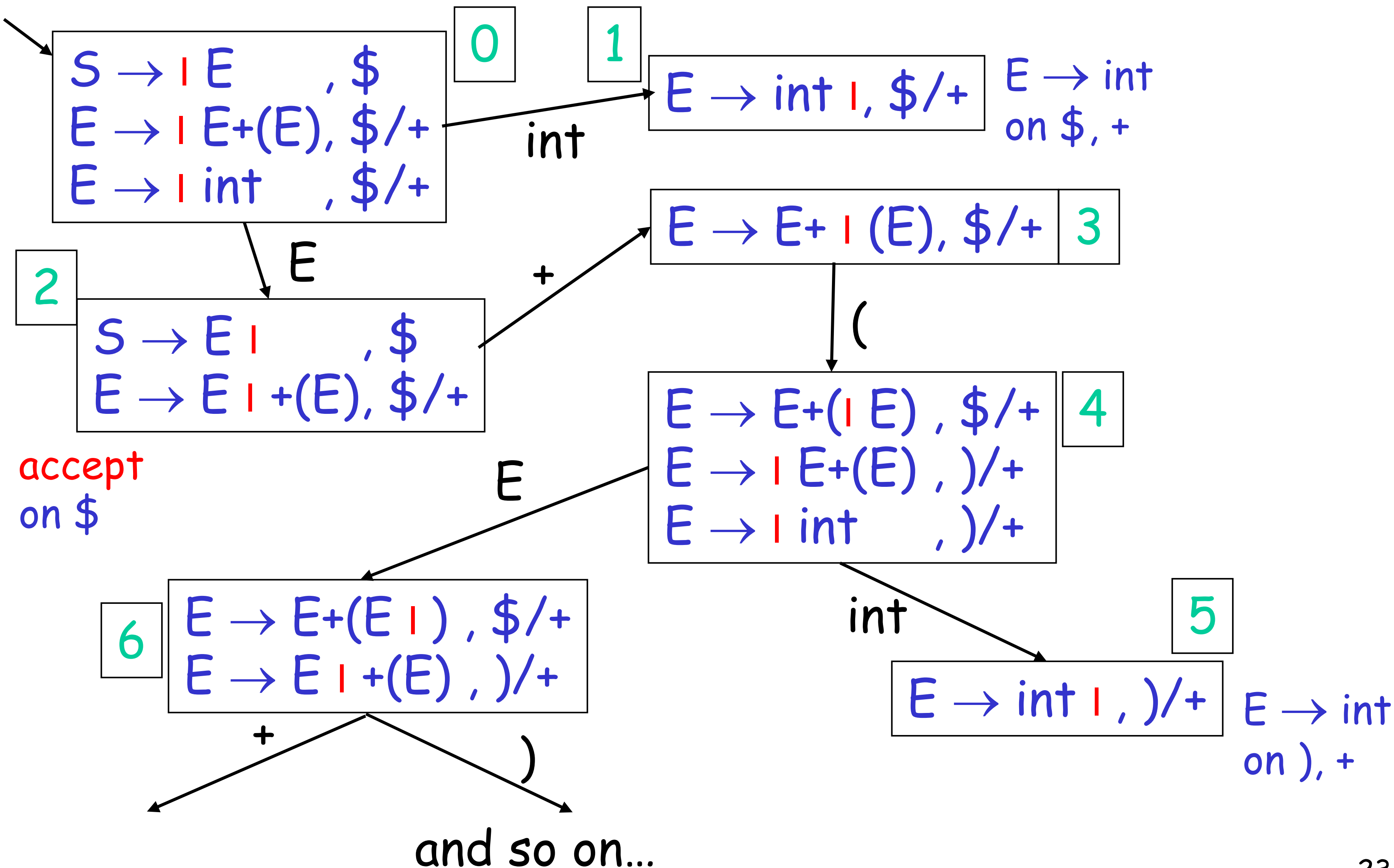
- A DFA state is a closed set of LR(1) items
- The start state contains $[S \to \mid E \;, \$]$
- A state that contains $[X \to \alpha \mid, b]$ is labeled with "reduce with $X \to \alpha$ on b"
- And now the transitions …

The DFA Transitions

- A state "State" that contains $[X \to \alpha \,|\, y\beta, b]$ has a transition labeled y to a state that contains the items "**Transition**(State, y)"
  - y can be a terminal or a non-terminal

Transition(State, y)
    Items = $\varnothing$
    for each $[X \to \alpha \,|\, y\beta, b]$ in State
        add $[X \to \alpha y \,|\, \beta, b]$ to Items
    return Closure(Items)

Constructing the Parsing DFA: Example



LR Parsing Tables: Notes

- Parsing tables (i.e., the DFA) can be constructed automatically for a CFG

- But we still need to understand the construction to work with parser generators
  - E.g., they report errors in terms of sets of items

- What kind of errors can we expect?

Shift/Reduce Conflicts

- If a DFA state contains both

  $[X \to \alpha \mid a\beta, b]$ and $[Y \to \gamma \mid, a]$

- Then on input "a" we could either
  - Shift into state $[X \to \alpha a \mid \beta, b]$, or
  - Reduce with $Y \to \gamma$

- This is called a *shift-reduce conflict*

Shift/Reduce Conflicts

- Typically due to ambiguities in the grammar
- Classic example: the dangling else

  $S \rightarrow$ if E then S | if E then S else S | OTHER
- Will have DFA state containing

  [S $\rightarrow$ if E then S I, else]

  [S $\rightarrow$ if E then S I else S, x]
- If else follows then we can shift or reduce
- Default (yacc, ML-yacc, bison, etc.) is to shift
  - Default behavior is as needed in this case

More Shift/Reduce Conflicts

- Consider the ambiguous grammar

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

- We will have the states containing

$$[E \rightarrow E * \,|\, E,\ +] \qquad\qquad [E \rightarrow E * E \,|,\ +]$$
$$[E \rightarrow \,|\, E + E,\ +] \quad \Rightarrow^{E} \quad [E \rightarrow E \,|\, + E,\ +]$$
$$\ldots \qquad\qquad\qquad \ldots$$

- Again we have a shift/reduce on input +
  - We need to reduce (* binds more tightly than +)
  - Recall solution: declare the precedence of * and +

More Shift/Reduce Conflicts

- In yacc declare precedence and associativity:

```
%left +
%left *
```

- Precedence of a rule = that of its last terminal

  See yacc manual for ways to override this default

- Resolve shift/reduce conflict with a <u>shift</u> if:
  - no precedence declared for either rule or terminal
  - input terminal has higher precedence than the rule
  - the precedences are the same and right associative

Using Precedence to Solve S/R Conflicts

- Back to our example:

$$[E \to E * \mid E,\ +] \qquad [E \to E * E \mid,\ +]$$

$$[E \to \mid E + E,\ +] \stackrel{E}{\Rightarrow} [E \to E \mid + E,\ +]$$

$$\ldots \qquad \ldots$$

- Will choose reduce because precedence of rule $E \to E * E$ is higher than of terminal $+$

Using Precedence to Solve S/R Conflicts

- Same grammar as before

$$E \rightarrow E + E \;\mid\; E * E \;\mid\; \text{int}$$

- We will also have the states

$$[E \rightarrow E + \mid E,\ \ +] \qquad\qquad [E \rightarrow E + E \mid,\ \ +]$$

$$[E \rightarrow \mid E + E,\ \ +] \quad \Rightarrow^{E} \quad [E \rightarrow E \mid + E,\ \ +]$$

$$\ldots \qquad\qquad\qquad\qquad \ldots$$

- Now we also have a shift/reduce on input +
  - We choose reduce because $E \rightarrow E + E$ and + have the same precedence and + is left-associative

Using Precedence to Solve S/R Conflicts

- Back to our dangling else example

  [S → if E then S I, else]

  [S → if E then S I else S, x]

- Can eliminate conflict by declaring else having higher precedence than then
- But this starts to look like "hacking the tables"
- Best to avoid overuse of precedence declarations or we will end with unexpected parse trees

Precedence Declarations Revisited

The term "precedence declaration" is misleading!

These declarations do not define precedence: they define conflict resolutions

- I.e., they instruct shift-reduce parsers to resolve conflicts in certain ways
- These two are not quite the same!

Reduce/Reduce Conflicts

- If a DFA state contains both

  $[X \to \alpha \,\iota, a]$ and $[Y \to \beta \,\iota, a]$

  - Then on input "a" we don't know which production to reduce

- This is called a *reduce/reduce conflict*

Reduce/Reduce Conflicts

- Usually due to gross ambiguity in the grammar
- Example: a sequence of identifiers

$$S \rightarrow \varepsilon \mid id \mid id\ S$$

- There are two parse trees for the string id

$$S \rightarrow id$$

$$S \rightarrow id\ S \rightarrow id$$

- How does this confuse the parser?

More on Reduce/Reduce Conflicts

- Consider the states

$$
\begin{array}{lcl}
 & & [S \to id\ \bullet, \quad \$] \\
[S' \to \bullet\ S, \quad \$] & & [S \to id\ \bullet\ S, \quad \$] \\
[S \to \bullet, \quad \$] & \Rightarrow^{id} & [S \to \bullet, \quad \$] \\
[S \to \bullet\ id, \quad \$] & & [S \to \bullet\ id, \quad \$] \\
[S \to \bullet\ id\ S, \quad \$] & & [S \to \bullet\ id\ S, \quad \$]
\end{array}
$$

- Reduce/reduce conflict on input $\$$

$$S' \to S \to id$$

$$S' \to S \to id\ S \to id$$

- Better to rewrite the grammar as: $S \to \varepsilon \mid id\ S$

Using Parser Generators

- Parser generators automatically construct the parsing DFA given a CFG
  - Use precedence declarations and default conventions to resolve conflicts
  - The parser algorithm is the same for all grammars (and is provided as a library function)
- But most parser generators do not construct the DFA as described before
  - Because the LR(1) parsing DFA has 1000s of states even for a simple language

LR(1) Parsing Tables are Big

- But many states are similar, e.g.

$\boxed{1}$ $\boxed{E \to \text{int}\ \mathbf{.},\ \$/+}$ $E \to \text{int}$ on $\$$, +

and

$\boxed{5}$ $\boxed{E \to \text{int}\ \mathbf{.},\ )/+}$ $E \to \text{int}$ on), +

- Idea: merge the DFA states whose items differ only in the lookahead tokens
  - We say that such states have the same core
- We obtain

$\boxed{1'}$ $\boxed{E \to \text{int}\ \mathbf{.},\ \$/+/)}$ $E \to \text{int}$ on $\$$, +,)

The Core of a Set of LR Items

Definition: The core of a set of LR items is the set of first components

- Without the lookahead terminals

- Example: the core of

$$\{[X \to \alpha \mid \beta, b], [Y \to \gamma \mid \delta, d]\}$$

is

$$\{X \to \alpha \mid \beta,\ Y \to \gamma \mid \delta\}$$

LALR States

- Consider for example the LR(1) states

$$\{[X \to \alpha \,\mathsf{I},\ a],\ [Y \to \beta \,\mathsf{I},\ c]\}$$

$$\{[X \to \alpha \,\mathsf{I},\ b],\ [Y \to \beta \,\mathsf{I},\ d]\}$$

- They have the same core and can be merged
- The merged state contains:

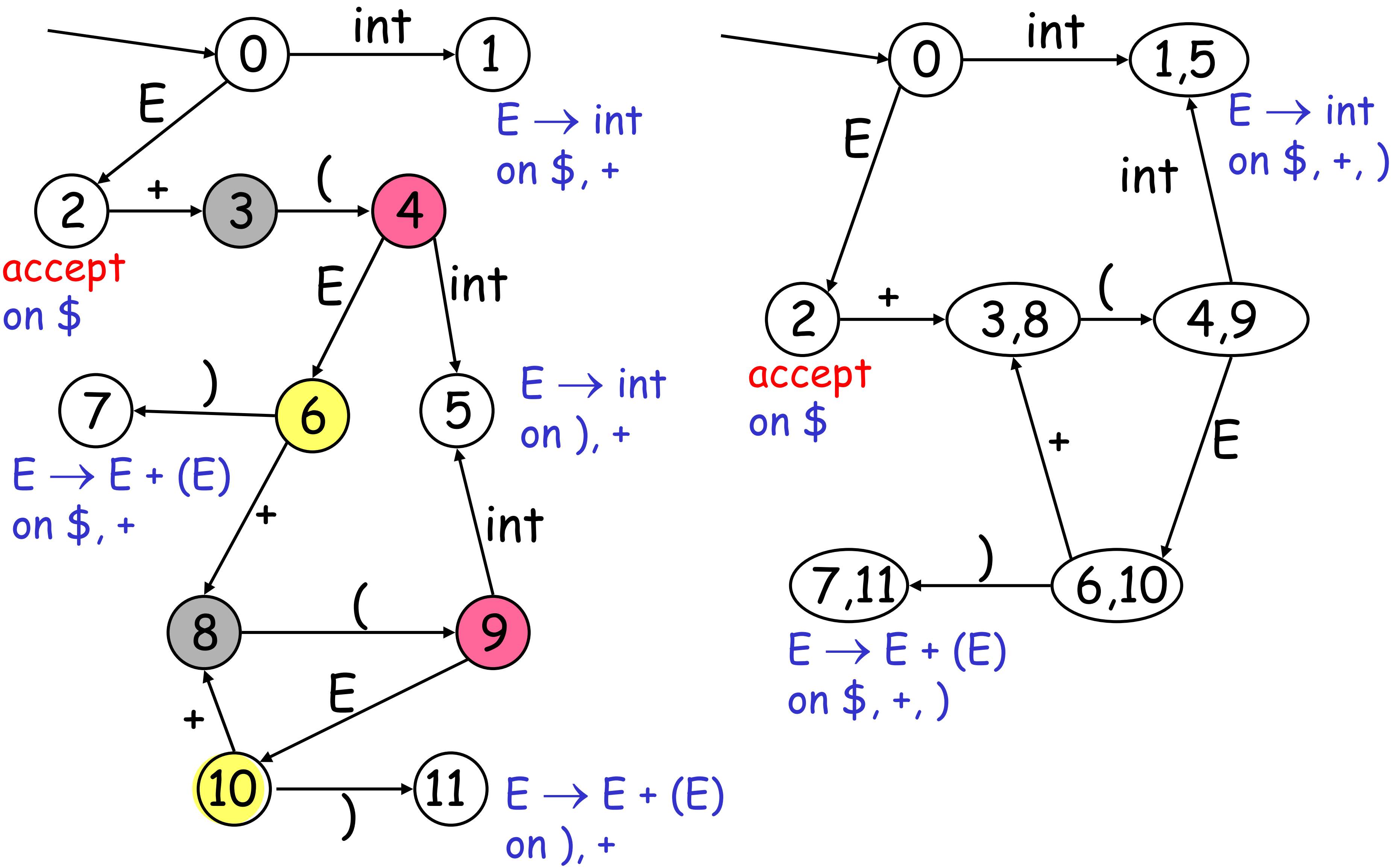
$$\{[X \to \alpha \,\mathsf{I},\ a/b],\ [Y \to \beta \,\mathsf{I},\ c/d]\}$$

- These are called LALR(1) states
  - Stands for LookAhead LR
  - Typically 10 times fewer LALR(1) states than LR(1)

A LALR(1) DFA

- Repeat until all states have distinct core
  - Choose two distinct states with same core
  - Merge the states by creating a new one with the union of all the items
  - Point edges from predecessors to new state
  - New state points to all the previous successors

Conversion LR(1) to LALR(1): Example.



The LALR Parser Can Have Conflicts

- Consider for example the LR(1) states

$$\{[X \rightarrow \alpha\, \mathsf{I},\ a], [Y \rightarrow \beta\, \mathsf{I},\ b]\}$$

$$\{[X \rightarrow \alpha\, \mathsf{I},\ b], [Y \rightarrow \beta\, \mathsf{I},\ a]\}$$

- And the merged LALR(1) state

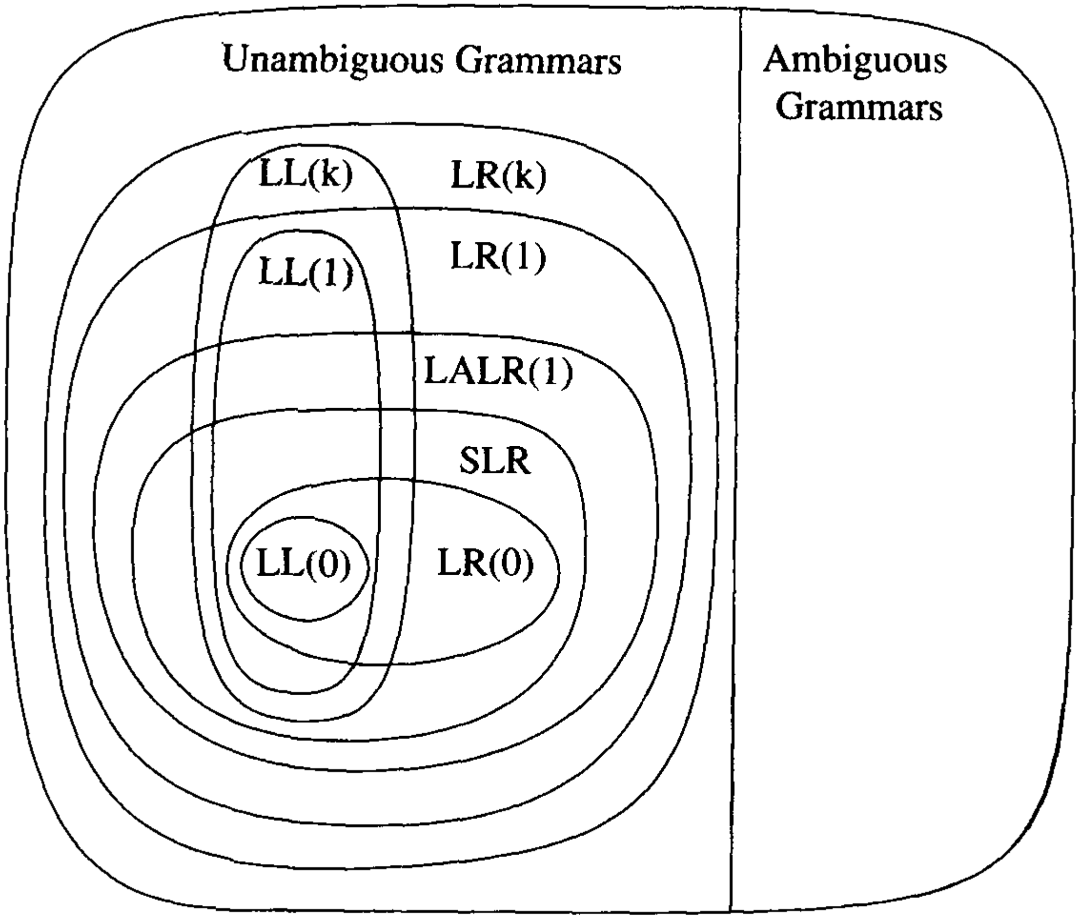
$$\{[X \rightarrow \alpha\, \mathsf{I},\ a/b], [Y \rightarrow \beta\, \mathsf{I},\ a/b]\}$$

- Has a <u>new</u> reduce/reduce conflict
- In practice such cases are rare

LALR vs. LR Parsing: Things to keep in mind

- LALR languages are not natural
  - They are an efficiency hack on LR languages
- Any reasonable programming language has a LALR(1) grammar
- LALR(1) parsing has become a standard for programming languages and parser generators

A Hierarchy of Grammar Classes



From Andrew Appel, "Modern Compiler Implementation in ML"