

Global Optimization



Lecture Outline

- Global flow analysis
- Global constant propagation
- Liveness analysis

Local Optimization

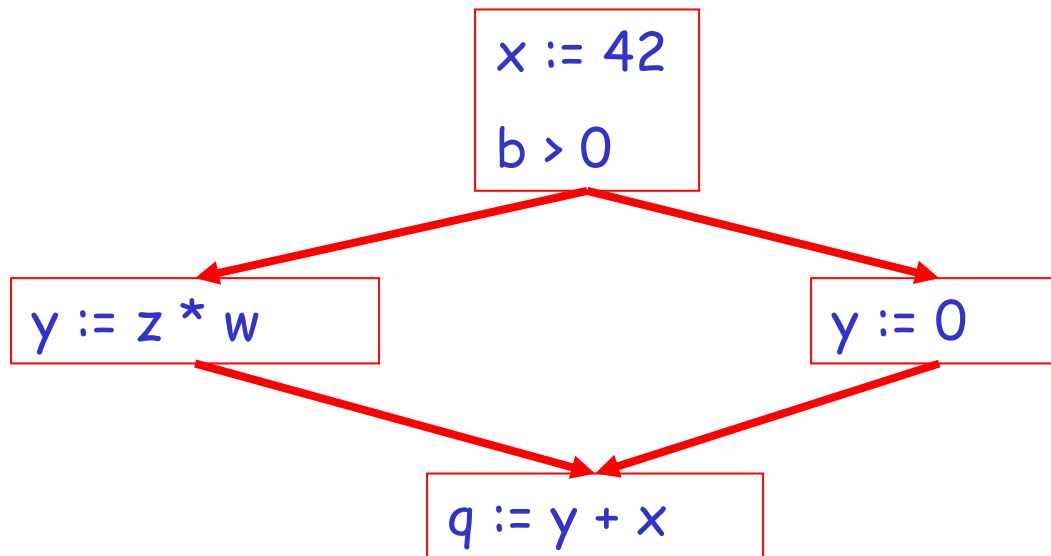
Recall the simple basic-block optimizations

- Constant propagation
- Dead code elimination

$x := 42$		$x := 42$		$x := 42$
$y := z * w$		$y := z * w$		$y := z * w$
$q := y + x$		$q := y + 42$		$q := y + 42$

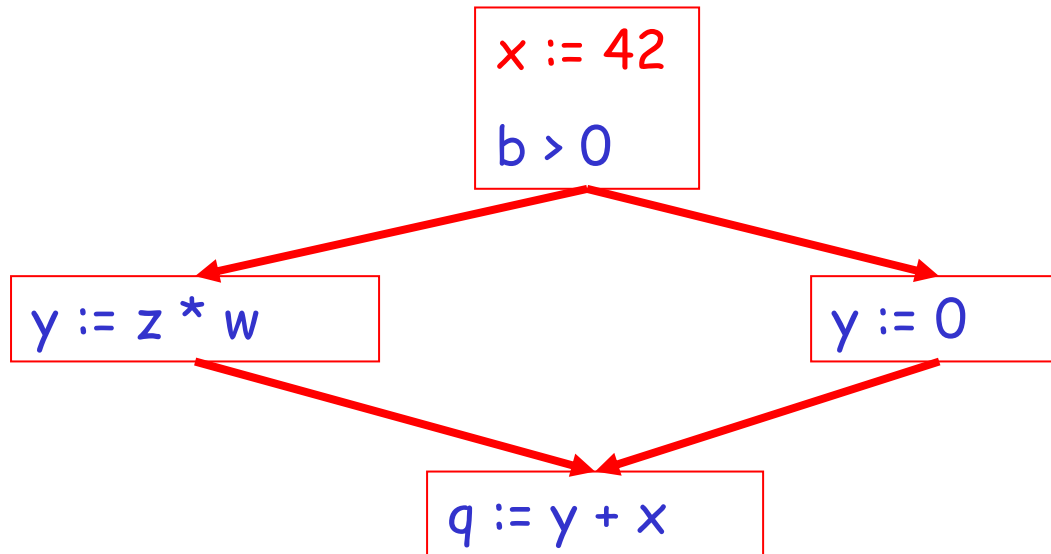
Global Optimization

These optimizations can be extended to an entire control-flow graph



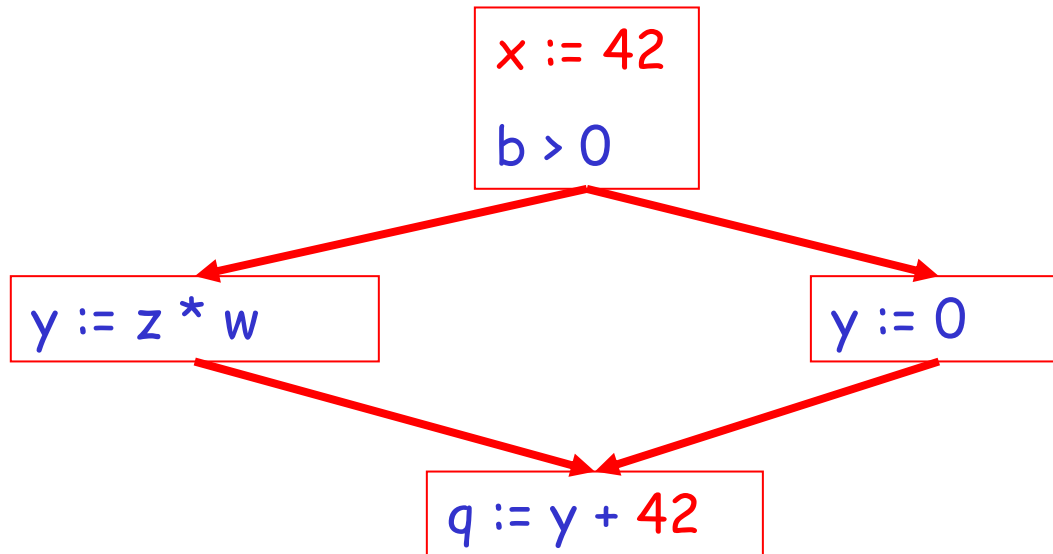
Global Optimization

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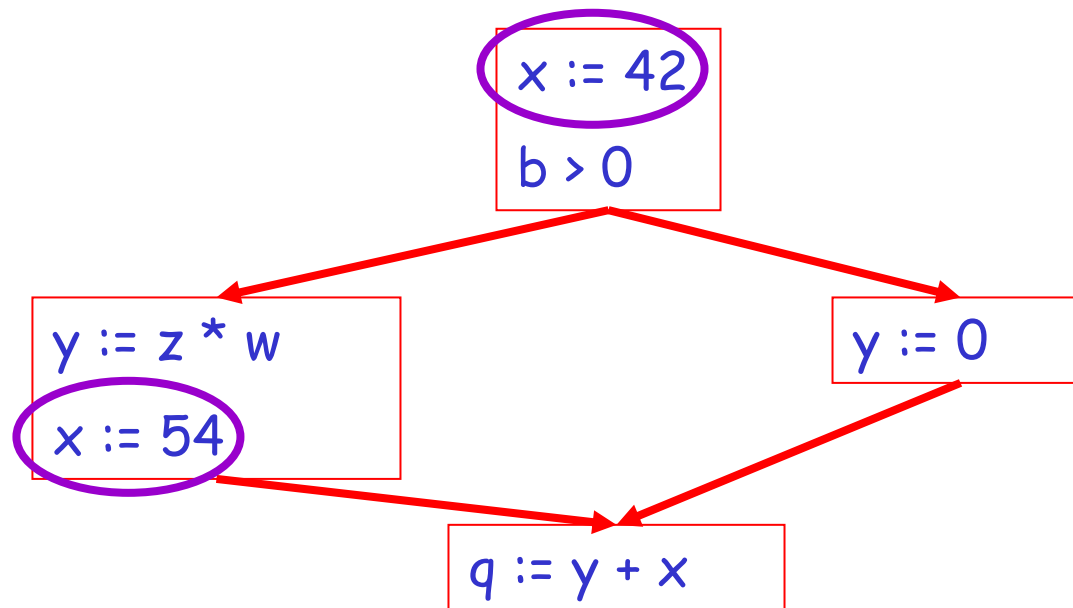
Global Optimization

These optimizations can be extended to an entire control-flow graph



Correctness

- How do we know whether it is OK to globally propagate constants?
- There are situations where it is incorrect:

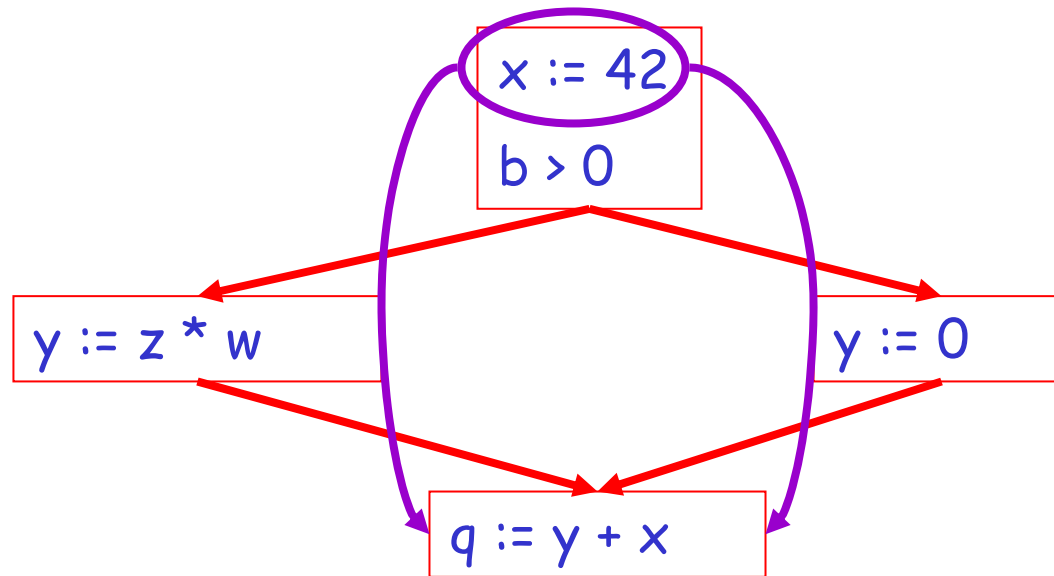


Correctness (Cont.)

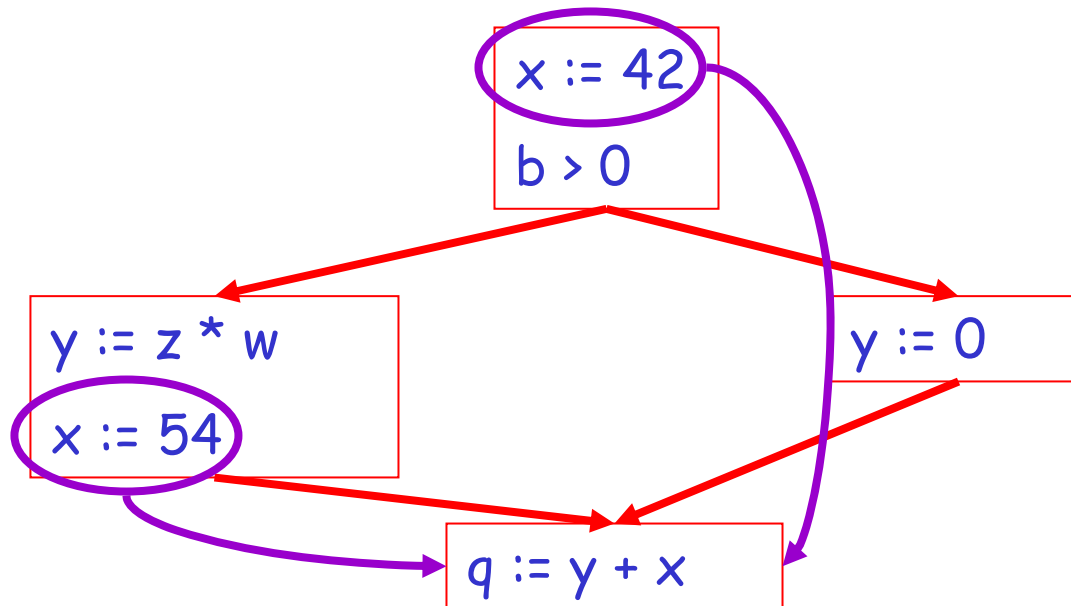
To replace a use of x by a constant k we must know that the following property $**$ holds:

*On every path to the use of x ,
the last assignment to x is $x := k$ $**$*

Example 1 Revisited



Example 2 Revisited



Discussion

- The correctness condition is not trivial to check
- “All paths” includes paths around loops and through branches of conditionals
- Checking the condition requires *global analysis*
 - An analysis that determines how data flows over the entire control-flow graph of a function/method

Global Analysis

Global optimization tasks share several traits:

- The optimization depends on knowing a property P at a particular point in program execution
- Proving P at any point requires knowledge of the entire function body
- Property P is typically undecidable !
- It is OK to be conservative: If the optimization requires P to be true, then want to know either
 - that P is definitely true, or
 - that we don't know whether P is true
- It is always safe to say "don't know"
 - We try to say do not know as rarely as possible

Global Analysis (Cont.)

- *Global dataflow analysis* is a standard technique for solving problems with these characteristics
- Global constant propagation is one example of an optimization that requires global dataflow analysis

Global Constant Propagation

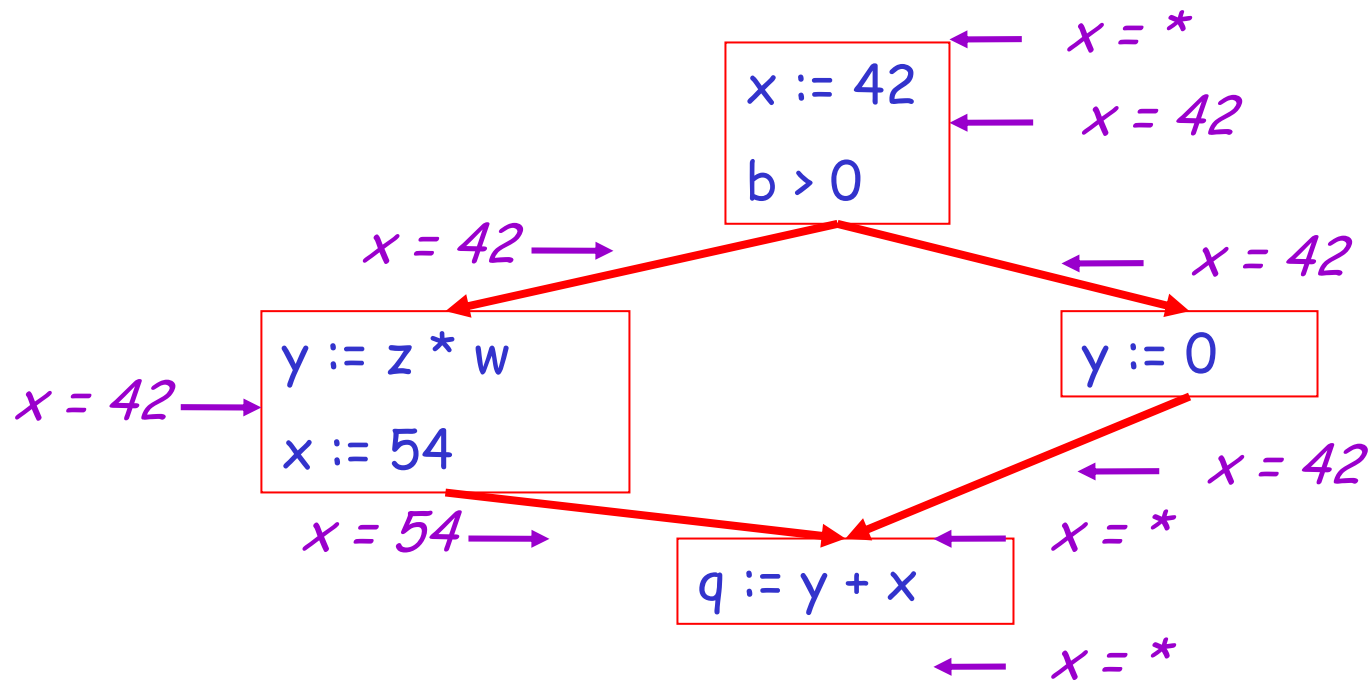
- *On every path to the use of x , the last assignment to x is $x := k$ ***
- Global constant propagation can be performed at any point where property ** holds
- Consider the case of computing ** for a single variable x at all program points

Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with x at every program point

<i>value</i>	<i>interpretation</i>
#	This statement never executes
c	$x = \text{constant } c$
*	Don't know whether x is a constant

Example



Using the Information

- Given global constant information, it is easy to perform the optimization
 - Simply inspect the $x = ?$ associated with a statement using x
 - If x is constant at that point replace that use of x by the constant
- But how do we compute the properties $x = ?$

The Analysis Idea

The analysis of a (complicated) program can be expressed as a combination of simple rules relating the change in information between adjacent statements

Explanation

- The idea is to “push” or “transfer” information from one statement to the next
- For each statement s , we compute information about the value of x immediately before and after s

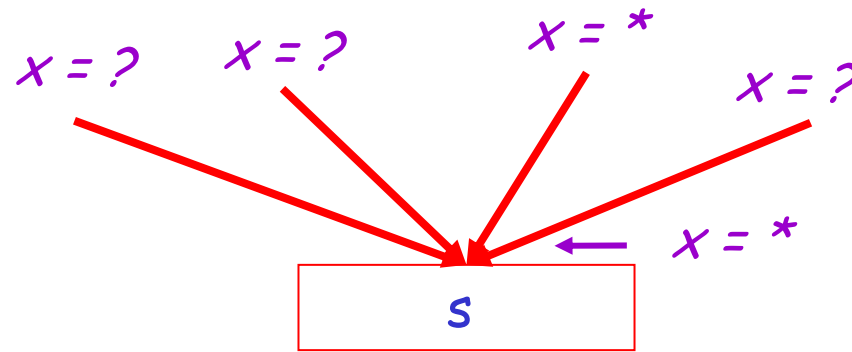
$C_{in}(x,s)$ = value of x before s

$C_{out}(x,s)$ = value of x after s

Transfer Functions

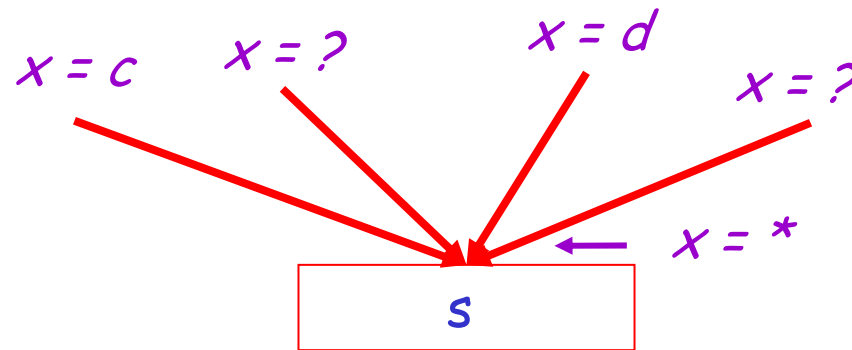
- Define a transfer function that transfers information from one statement to another
- In the following rules, let statement s have as immediate predecessors statements p_1, \dots, p_n

Rule 1



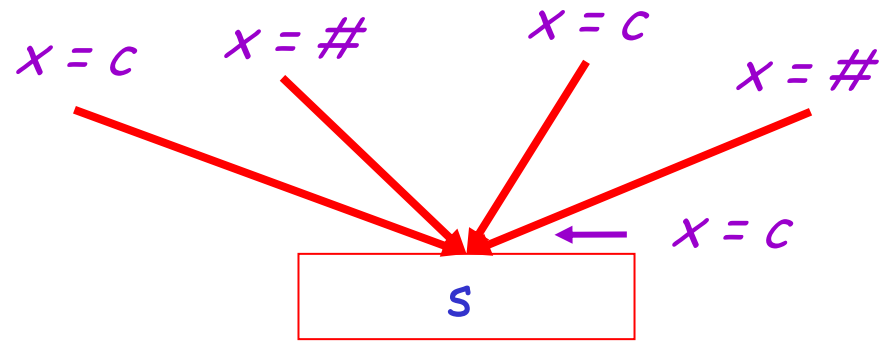
if $C_{\text{out}}(x, p_i) = *$ for any i , then $C_{\text{in}}(x, s) = *$

Rule 2



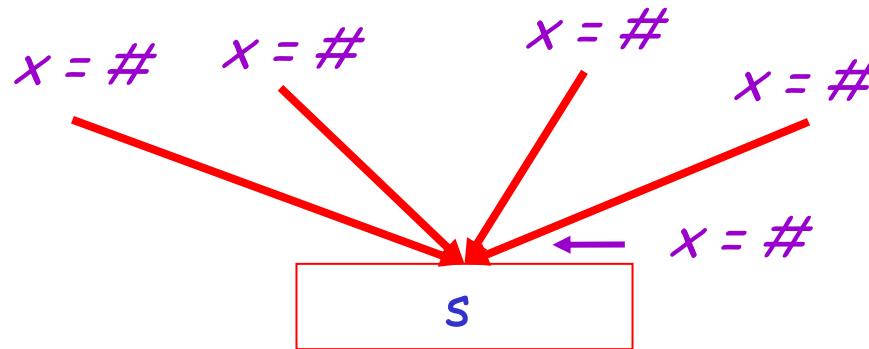
If $C_{out}(x, p_i) = c$ and $C_{out}(x, p_j) = d$ and $d \neq c$
then $C_{in}(x, s) = *$

Rule 3



if $C_{\text{out}}(x, p_i) = c$ or $\#$ for all i ,
then $C_{\text{in}}(x, s) = c$

Rule 4

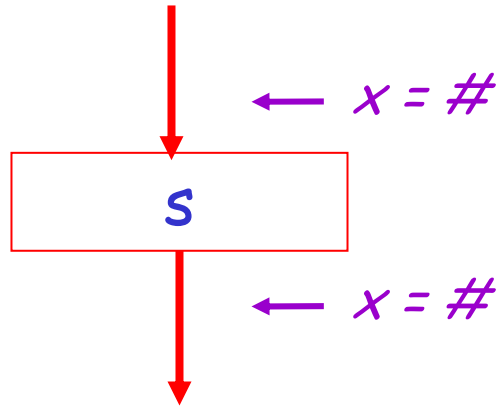


if $C_{\text{out}}(x, p_i) = \#$ for all i ,
then $C_{\text{in}}(x, s) = \#$

The Other Half

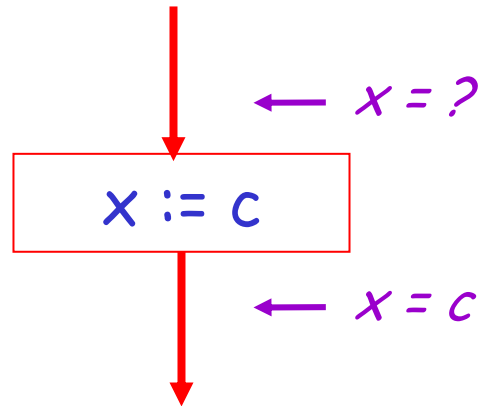
- Rules 1-4 relate the *out* of one statement to the *in* of the successor statement
 - they propagate information forward across CFG edges
- We also need rules relating the *in* of a statement to the *out* of the same statement
 - to propagate information across statements

Rule 5



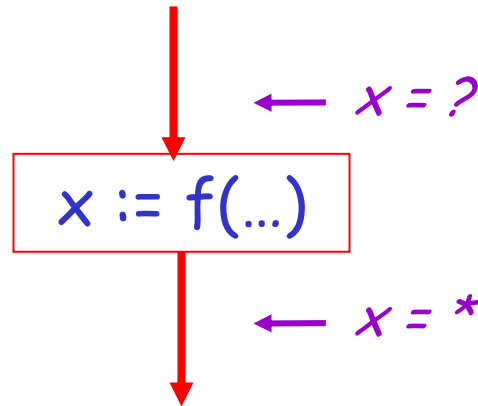
$$C_{\text{out}}(x, s) = \# \text{ if } C_{\text{in}}(x, s) = \#$$

Rule 6



$C_{\text{out}}(x, x := c) = c$ if c is a constant

Rule 7

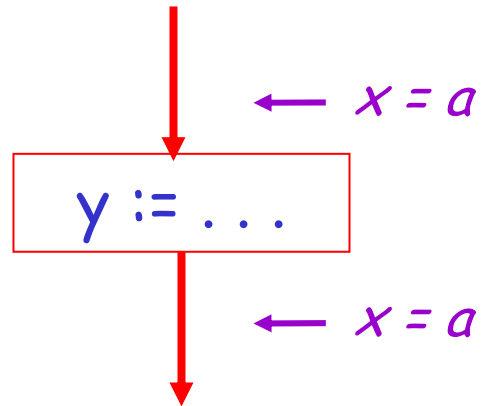


where f is a function other than the one being analyzed

$$C_{\text{out}}(x, x := f(\dots)) = *$$

This rule says that we do not perform inter-procedural analysis (i.e. we do not look at what other functions do)

Rule 8



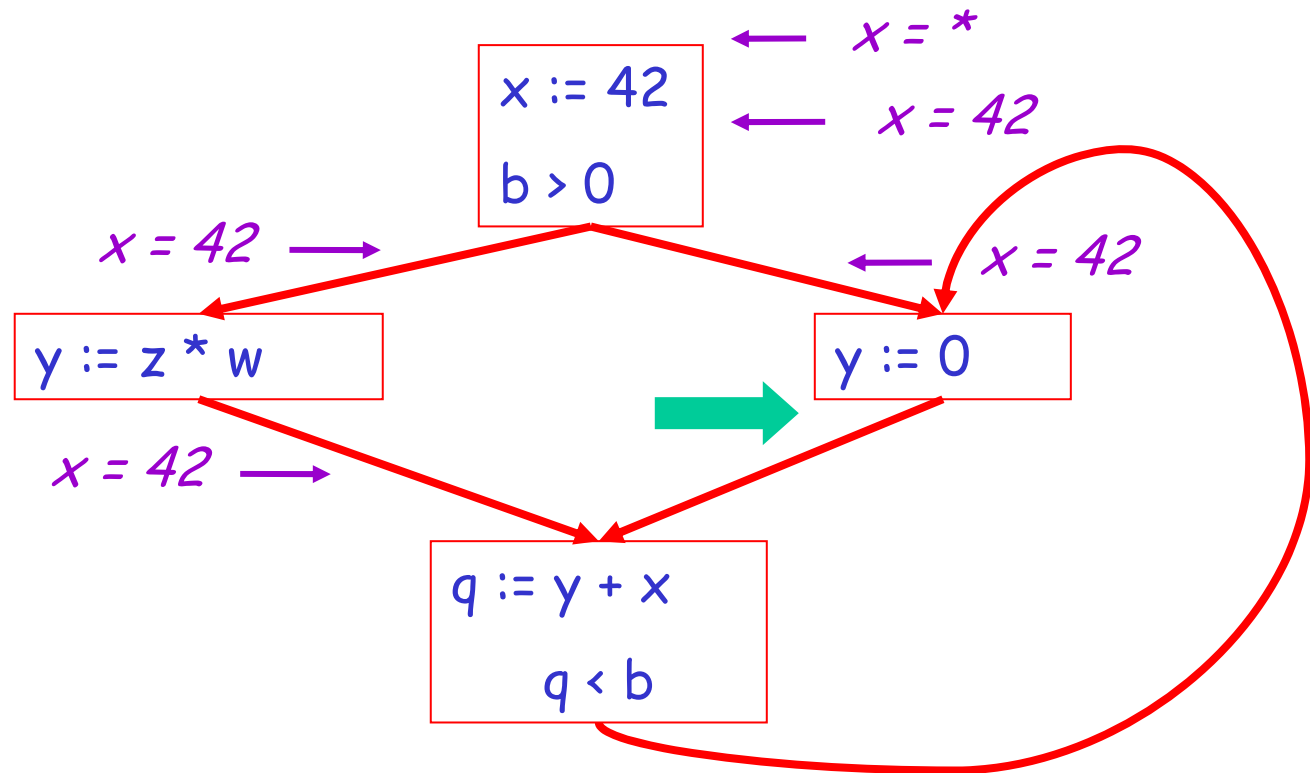
$$C_{\text{out}}(x, y := \dots) = C_{\text{in}}(x, y := \dots) \text{ if } x \neq y$$

An Algorithm

1. For every entry s to the function, set $C_{in}(x, s) = *$
2. Set $C_{in}(x, s) = C_{out}(x, s) = \#$ everywhere else
3. Repeat until all points satisfy 1-8:
Pick s not satisfying 1-8 and update using the appropriate rule

The Value

To understand why we need #, look at a loop



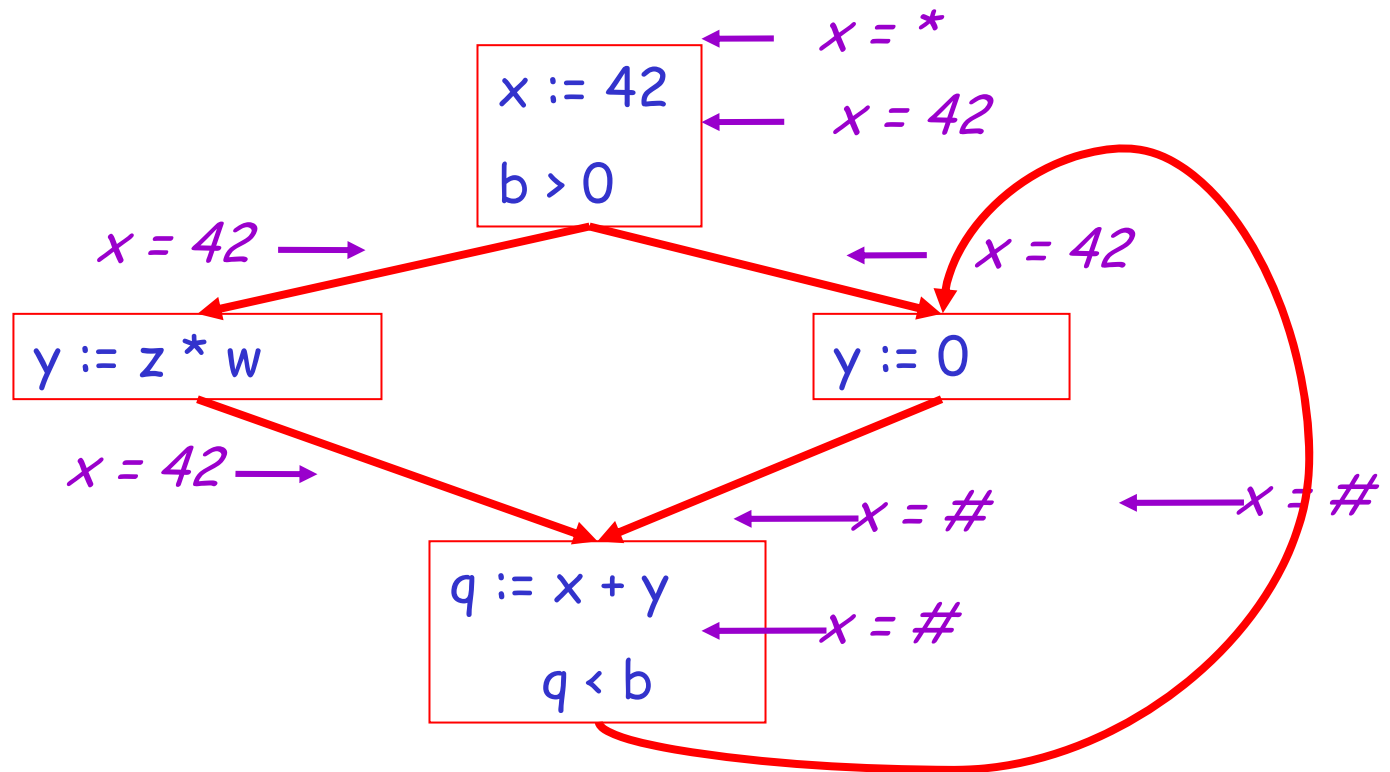
Discussion

- Consider the statement $y := 0$
- To compute whether x is constant at this point, we need to know whether x is constant at the two predecessors
 - $x := 42$
 - $q := y + x$
- But information for $q := y + x$ depends on its predecessors, including $y := 0$!

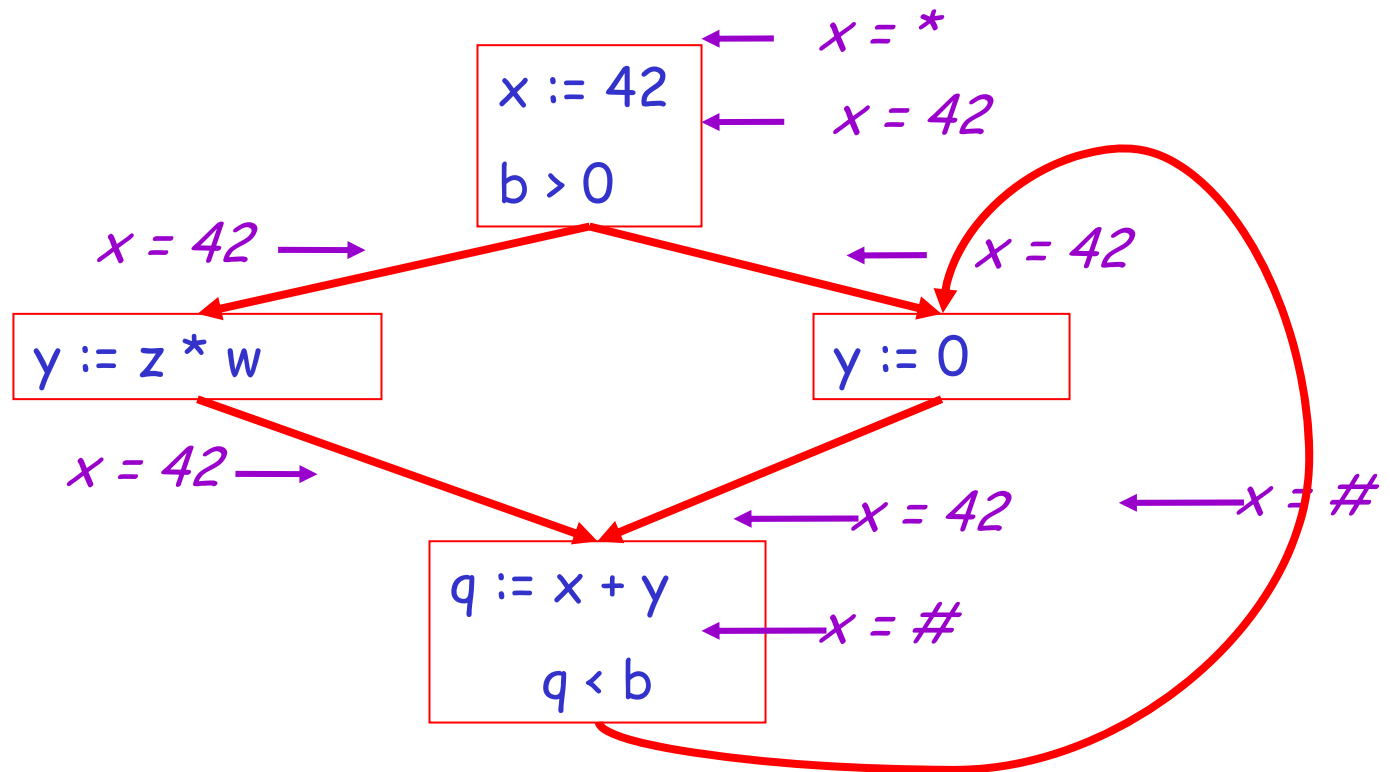
The Value # (Cont.)

- Because of cycles, all points must have values at all times
- Intuitively, assigning some initial value allows the analysis to break cycles
- The initial value # means "So far as we know, control never reaches this point"

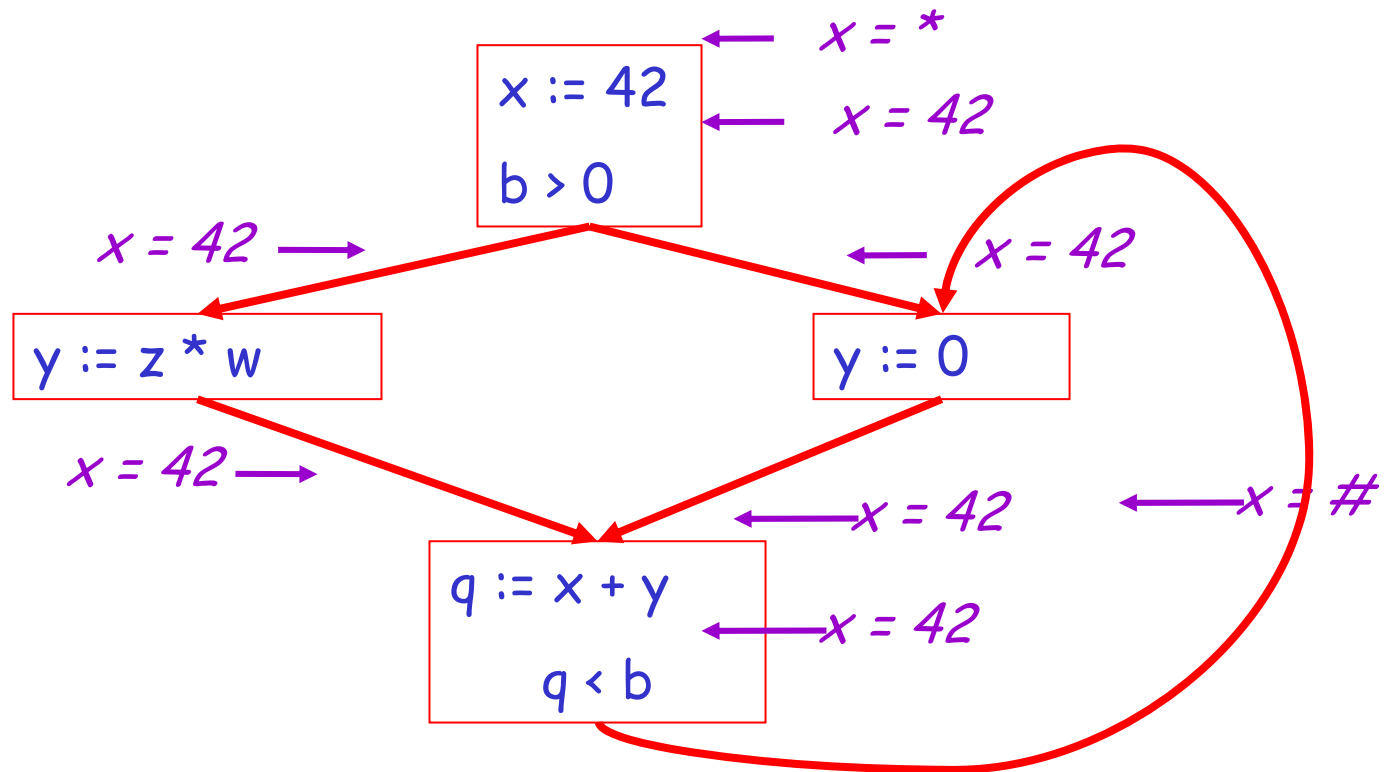
Example



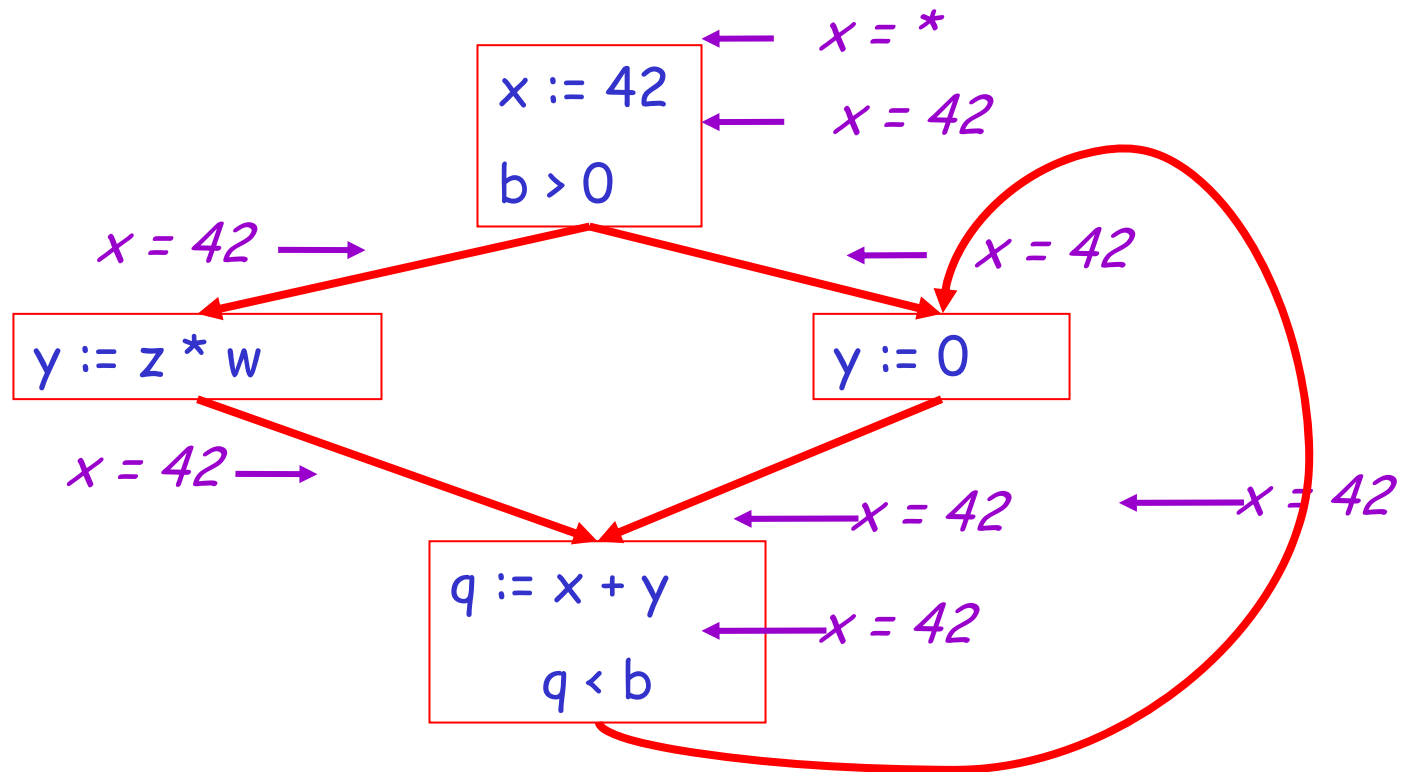
Example



Example



Example

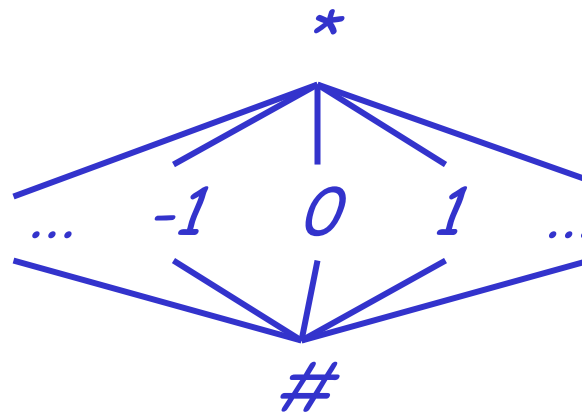


Orderings

- We can simplify the presentation of the analysis by ordering the values

$$\# < c < *$$

- Drawing a picture with "lower" values drawn lower, we get



Orderings (Cont.)

- * is the greatest value, # is the least
 - All constants are in between and incomparable
- Let *lub* be the least-upper bound in this ordering
- Rules 1-4 can be written using lub:
$$C_{in}(x, s) = \text{lub} \{ C_{out}(x, p) \mid p \text{ is a predecessor of } s \}$$

Termination

- Simply saying "repeat until nothing changes" doesn't guarantee that eventually we reach a point where nothing changes
- The use of lub explains why the algorithm terminates
 - Values start as # and only *increase*
 - # can change to a constant, and a constant to *
 - Thus, $C_-(x, s)$ can change at most twice

Termination (Cont.)

Thus the algorithm is linear in program size

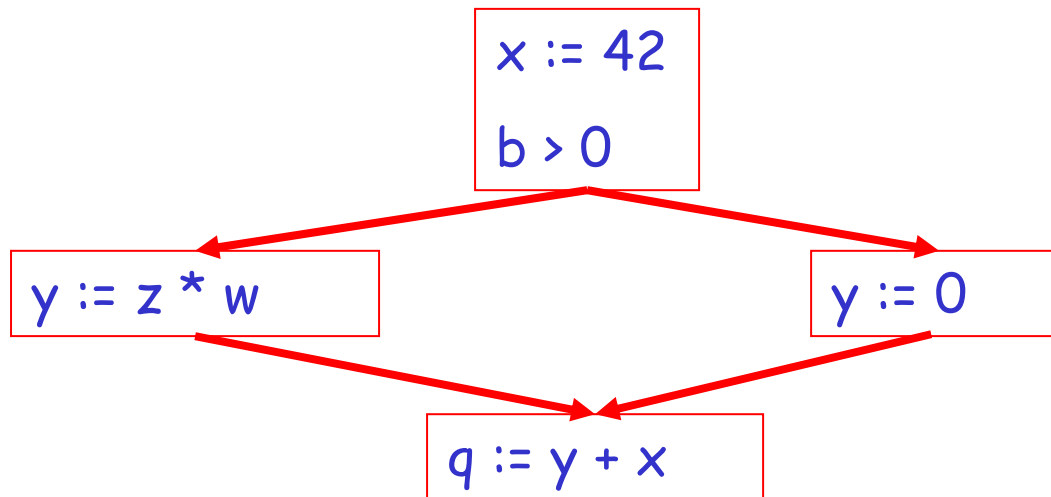
Number of steps = // worst case

Number of $C_+(\dots)$ values computed * 2 =

Number of program statements * 4

Liveness Analysis

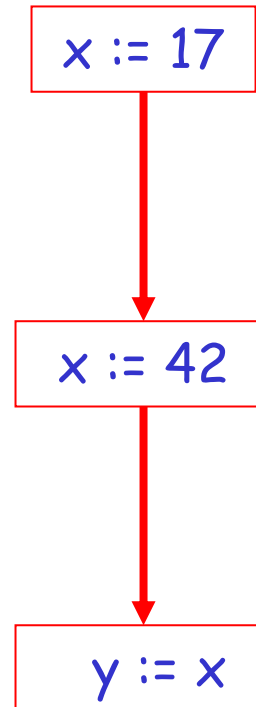
Once constants have been globally propagated, we would like to eliminate dead code



After constant propagation, `x := 42` is dead (assuming `x` is not used elsewhere)

Live and Dead Variables

- The first value of x is *dead* (never used)
- The second value of x is *live* (may be used)
- Liveness is an important concept for the compiler



Liveness

A variable x is live at statement s if

- There exists a statement s' that uses x
- There is a path from s to s'
- That path has no intervening assignment to x

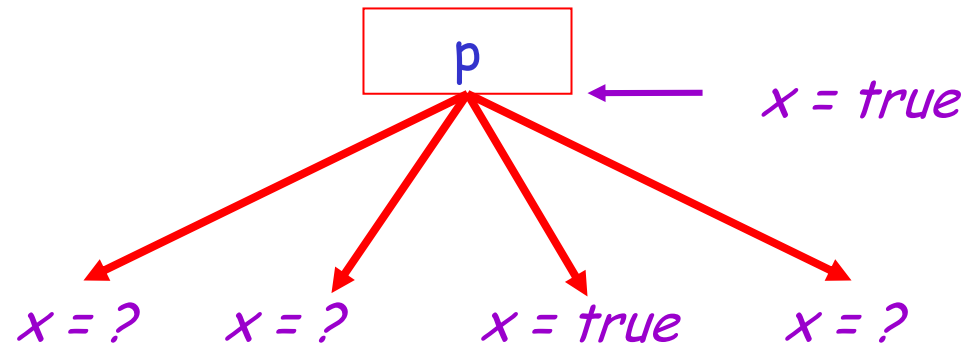
Global Dead Code Elimination

- A statement $x := \dots$ is dead code if x is dead after the assignment
- Dead statements can be deleted from the program
- But we need liveness information first . . .

Computing Liveness

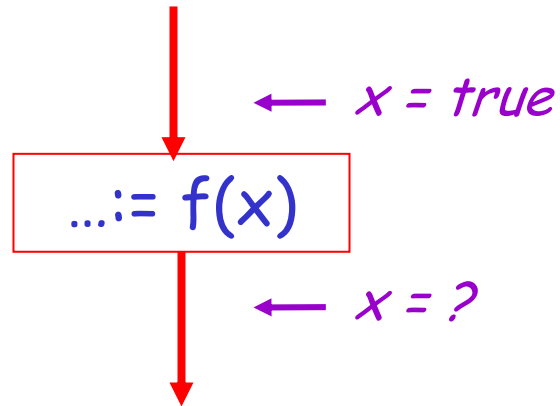
- We can express liveness in terms of information transferred between adjacent statements, just as in copy propagation
- Liveness is simpler than constant propagation, since it is a boolean property (true or false)

Liveness Rule 1



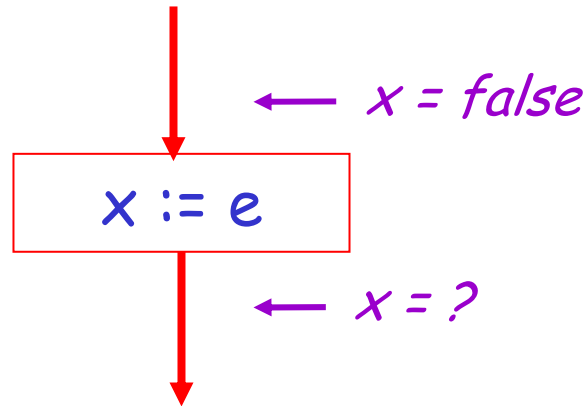
$$L_{out}(x, p) = \bigvee \{ L_{in}(x, s) \mid s \text{ a successor of } p \}$$

Liveness Rule 2



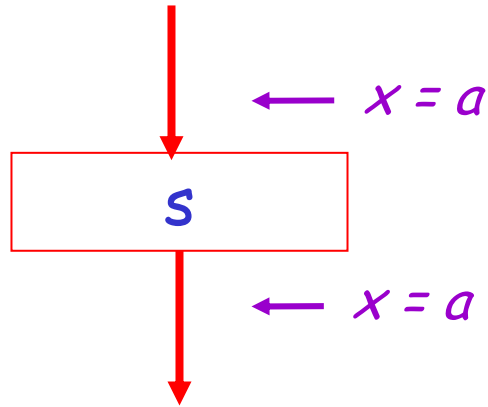
$L_{in}(x, s) = \text{true}$ if s refers to x on the RHS

Liveness Rule 3



$L_{in}(x, x := e) = \text{false}$ if e does not refer to x

Liveness Rule 4



$L_{in}(x, s) = L_{out}(x, s)$ if s does not refer to x

Algorithm

1. Let all $L_*(\dots) = \text{false}$ initially
2. Repeat until all statements s satisfy rules 1-4
Pick s where one of 1-4 does not hold and
update using the appropriate rule

Termination

- A value can change from **false** to **true**, but not the other way around
- Each value can change only once, so termination is guaranteed
- Once the analysis information is computed, it is simple to eliminate dead code

Forward vs. Backward Analysis

We have seen two kinds of analysis:

- An analysis that enables constant propagation:
 - this is a *forwards* analysis: information is pushed from inputs to outputs
- An analysis that calculates variable liveness:
 - this is a *backwards* analysis: information is pushed from outputs back towards inputs

Global Flow Analyses

- There are many other global flow analyses
- Most can be classified as either forward or backward
- Most also follow the methodology of local rules relating information between adjacent program points