Introduction to Transportation Engineering

Car Following Models

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Approaches to Modeling Traffic

Microscopic
  Attempts to look into individual driving behaviors
  Vehicle-following models

Macroscopic
  Looks at the traffic as a fluid-flow or heat-transfer phenomena
  Vehicles are not identified individually but as a group of entities moving on the system

Technically, both microscopic and macroscopic models consider the human in their solutions
Car Following Models

• This problem has been studied for the past 45 years in traffic flow theory
• Started with the work of Gazis and Herman
• Models vehicles individually
• The vehicle and human interactions are modeled explicitly
• A driver follows another vehicle by judging:
  a) Distance
  b) Speed difference
  c) Reaction time
  d) Vehicle performance
Car Following Model

• Two cars follow each other on the same road

• The driver in the lead car has a speed known speed profile independent of the following car

• The driver in the following car adjusts to the behavior of the front car
Car Following Models

Nomenclature:

\( x_{fc} \) and \( x_{lc} \) are the positions of the following and lead cars, respectively

\( \dot{x}_{fc} \) and \( \dot{x}_{lc} \) are the speeds of the following and lead cars, respectively

\( \ddot{x}_{fc} \) is the acceleration of the following car. This is the control variable that the driver adjusts to keep up with the lead car (and avoid a collision)
Car Following Models

Assume the velocity profile of the lead car is known as a function of time.

**Proposed Driving Rule # 1**

The acceleration profile of the “following” car is just a function of the relative speeds of the two cars;

\[
\ddot{x}_{fc}(t + \tau) = k(\dot{x}(t)_{lc} - \dot{x}(t)_{fc})
\]

where: \( k \) is a gain constant of the response process

\( \ddot{x}_{fc}(t + \tau) \) is the acceleration of the following vehicle
$\dot{x}_{lc}(t)$ is the speed of the leading vehicle

$\dot{x}_{fc}(t)$ is the speed of the following vehicle
Car Following Models

Assume the velocity profile of the lead car is known as a function of time.

Proposed Driving Rule # 2

The acceleration profile of the “following” car is a function of the relative speeds and the relative distance between the two vehicles;

\[
\ddot{x}_{fc}(t + \tau) = \frac{k(x_{lc}(t) - \dot{x}_{fc}(t))}{(x_{lc}(t) - x_{fc}(t))}
\]  

(2)

where: \( k \) is a gain constant of the response process
\ddot{x}_{fc}(t + \tau) \text{ is the acceleration of the following vehicle adjusted for time lag } \tau

\dot{x}_{lc}(t) \text{ is the speed of the leading vehicle}

\dot{x}_{fc}(t) \text{ is the speed of the following vehicle}

x_{lc}(t) \text{ is the position of the leading vehicle}

x_{fc}(t) \text{ is the position of the following vehicle}
How Do We Use This Model?

• The models presented in equations (1) and (2) can be tested against critical maneuvers performed by the lead vehicle.

• The critical maneuvers tests conditions where a driver in the lead vehicle “brakes hard”, accelerates quickly, or follows an erratic velocity profile.

• The second order differential equation can be solved numerically if desired of with the use of the Matlab toolbox called Simulink.

• Simulink is a toolbox designed to solve differential equations of motion.
Set-Up of the Problem

- Assume the velocity profile of the leading vehicle is known (we “drive” this car to test the response of the following vehicle)
- Initially, assume no time lags in the acceleration response of the following vehicle ($\tau = 0$)
- Test an emergency braking maneuver executed by the first vehicle at $3 \text{ m/s}^2$
- Test a new scenario with a deterministic time lag response time of 0.75 seconds
- Verify that both cars do not collide
Simulink

- Simulink is a powerful toolbox to solve systems of differential equations
- Simulink has applications in Systems Theory, Control, Economics, Transportation, etc.
- The Simulink approach is to represent systems of ODE using block diagram nomenclature
- Simulink provides seamless integration with MATLAB. In fact, Simulink can call any MATLAB function
- Simulink interfaces with other MATLAB toolboxes such as Neural Network, Fuzy Logic, and Optimization routines
Simulink Building Blocks

• Simulink has a series of libraries to construct models.

• Libraries have object blocks that encapsulate code and behaviors.

• Connectors between blocks establish causality and flow of information in the model.
Simulink Interface

- The main application of Simulink is to model continuous systems
- Perhaps systems that can be described using ordinary differential equations
Typical Simulink Libraries

Shown are some typical Simulink libraries.
Sample Simulink Library (Windows and UNIX)

The new Simulink interface in Windows uses standard graphical interfaces (Java-based interface)
Example 1. First-Order Kinematic Model

We would like to solve the first-order differential equation shown below in Simulink

\[ \frac{dV}{dt} = k_1 - k_2 V \]  \hspace{1cm} (3)

where: \( V \) is the speed of the vehicle, \( k_1 \) and \( k_2 \) are model constants.

The values of the model parameters are: \( k_1 = 4.0 \) and \( k_2 = 0.1 \) with units for \( V \) in m/s and for \( \frac{dV}{dt} \) in m/s\(^2\).
The following plot shows the Simulink model solution for the first order differential equation

\[ \frac{dV}{dt} = k_1 - k_2 V \]
Procedure to Create a Simulink Model

The Simulink blocks needed for this model are found in four distinct Simulink libraries:

- **Constant block** in the Simulink Sources library

- **Product** and **Subtraction** blocks in the Simulink Math Operations library

- **Integrator block** in Simulink Continuous library
Procedure to Create a Simulink Model (cont.)

• **Scope block** in Simulink Sinks library

Once the blocks are available in the new model window, they can be “wired”

Wiring a model implies connecting the output connection of each block with the input connection of another block

- Input connection of integrator block
- Output connection of integrator block
- After wiring
Creating the Simulink Model

• Recognize the number of terms inside the differential equation to be solved. For the example below we have two terms in the right-hand side

• Each term requires a series of operations to evaluate it. Your Simulink model will require one mathematical block for each operation. For example, the product operation between $k_2$ and $V$ requires a Simulink block

\[
d\frac{V}{dt} = k_1 - k_2 V
\]
Completing the Simulink Model

- In the model of interest we need to integrate the output of the term $k_1 - k_2 V$ because this is the rate of change of vehicle speed with respect to time. We accomplish this using an integrator block (see figure)

$$\frac{dV}{dt} = k_1 - k_2 V$$
Completing the Simulink Model

- The output of the integrator block is $V$ (the vehicle speed) which is the signal needed by the product block multiplying $k_2$ and $V$. This forms a first-order negative feedback loop.

$$\frac{dV}{dt} = k_1 - k_2 V$$
Completing the Simulink Model

• The output of the integrator block ($V$) can be displayed in a scope block. This scope block is labeled “Plot Velocity” in the model.

\[
\frac{dV}{dt} = k_1 - k_2 V
\]
Simulation Model Settings

• Before a differential equation is solved numerically we need to define the simulation configuration parameters.

• These parameters are needed to tell the solver the initial and final times of the simulation and the numerical integration procedure to be used.
Simulation Model Settings (cont.)

• For this simple differential equation we use a start time of zero (t=0) and 40 seconds as the stop time (t=40). This means the equation will be solved between these limits.

• The numerical differential equation solver used is a variable step ODE45 solve (default)
Initial Conditions

• Solving a differential equation requires the specification of initial conditions (Initial Value Problems). Initial conditions are required for all state variables of the system (e.g., variables to be integrated).

• There is only one state variable in this system ($V$). Specify initial conditions by opening the integrator block.

\[ \frac{dV}{dt} = K_1 - k_2 V \]
Simulating and Getting Results

- To solve the differential equation just “start” the model from the “Simulation” pull down menu (in Windows there is “play” button in the Simulink window)

- See the model results opening each scope block

Note: All plots in a scope are functions of time
Example 2 - Car Following Problem

Solve the second-order differential equation of motion representing the acceleration profile of the “following” vehicle (acceleration is function of the relative speeds of the two cars)

\[
\ddot{x}_{fc}(t + \tau) = k(\dot{x}(t)_{lc} - \dot{x}(t)_{fc})
\]  

(4)

where: \( k \) is a gain constant of the response process

\[
\ddot{x}_{fc}(t + \tau)
\]

is the acceleration of the following vehicle

\[
\dot{x}_{lc}(t)
\]

is the speed of the leading vehicle
\[ \dot{x}_{fc}(t) \] is the speed of the following vehicle
Simulink Representation of the Car-Following Problem

Car Following Problem
(No time lag and saturation limits for acceleration)

Integrate Acceleration of following car

Integrate speed of lead car

Combine Distances

Distance Profiles
Car-Following Model Output

The time space diagram below illustrates an emergency braking maneuver for the leading vehicle.

![Time space diagram with vehicle trajectories](image)
Car-Following Model Output

Velocity profiles for lead and following vehicles

Speed (m/s)
Car-Following Experiment

**Goal:** to determine the practical capacity of an interstate highway using a car-following model

**Method:**

a) Simulate a critical maneuver (such as hard braking)

b) Determine if vehicles collide given known initial positions and speeds (initial conditions)

c) Repeat the experiment in part (b) for various initial conditions
Car-Following Experiment

d) Assuming drivers use common sense, the minimum spacing is the distance between cars that avoids the collision by some desired margin

NOTE: In real practice, several critical maneuvers could be tried to obtain a safe spacing between successive cars. The issue is whether people driving are so predictable.
Car Following Experiment

Assumptions:

a) Cars travel at 30 m/s initially

b) Lead car initial position is at datum point (0 meters)

c) Lead car brakes at 3 m/s\(^2\) to avoid an obstacle

d) Following car brakes behind lead car

e) Car size is 5.8 meters in length

f) Minimum desired distance between cars at end of critical maneuver is one car length (5.8 meters)
Car Following Example

Minimum safe distance between two stopped vehicles is (11.6 meters - two car lengths)
## Car Following Experiment

Table containing outcomes of car-following experiment

<table>
<thead>
<tr>
<th>Following Car Initial Position (m) from Datum Point</th>
<th>Distance Between Front Bumpers at Closest Point of Approach (m)</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>70</td>
<td>Safe</td>
</tr>
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</tr>
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<td>No</td>
</tr>
<tr>
<td>-30</td>
<td>0</td>
<td>No</td>
</tr>
</tbody>
</table>
Interpretation of Results

The experiment suggest that the critical spacing to avoid an accident is between 40 meters and 50 meters.

Further refinement of the experiment suggests 42 meters is the critical spacing to avoid an accident.

Starting with an initial condition of 30 m/s for the lead and following cars, this implies a headway of 1.4 seconds per successive vehicle.

The approximate capacity of the highway for this critical maneuver would be 2,514 vehicles per hour per lane.

**NOTE:** This value is optimistic since no time lags have been factored in the man-machine system modeled.
Car-Following Model with Delay

A pure transport delay block is added to the original model simulating the transport lag dynamics of a man-machine system.
Output of the Car-Following Model with Delay

- If the following car is 50 meters behind the lead car, clearly a collision (spacing < 11.6 m.) occurs if $\tau = 1.5\text{s}$. 

Space (m)
Car Following Experiment (Again)

Repeat (for homework the experiment) including the time lag (to be conservative use a 2.5 second reaction time)

Other Assumptions:

a) Cars travel at 30 m/s initially

b) Lead car initial position is at datum point (0 meters)

c) Lead car brakes at 3 m/s² to avoid an obstacle

d) Following car brakes behind lead car
Car Following Experiment (Again)

e) Car size is 5.8 meters in length

f) Minimum desired distance between cars at end of critical maneuver is one car length (5.8 meters)