**Review:** Networks

The minimum cost network problem: an LP of the form

\[
\begin{align*}
\min_{x} & \quad c^T x \\
\text{subject to} & \quad Ax = b \\
& \quad 0 \leq x \leq u
\end{align*}
\]

- Each row of \( A \) corresponds to a **node** in the network
- Each column of \( A \) corresponds to an **arc** in the network
- Each component of \( x \) corresponds to the amount of flow along an arc, from one node to another
- Each column of matrix \( A \) is 1 precisely at one row (flow **out** at that node)
- Each column of matrix \( A \) is \(-1\) precisely at one row (flow **into** at that node)
- \( b_i > 0 \) **source**, \( b_i < 0 \) **sink**, \( b_i = 0 \) **transshipment**

**Special case I:** Transportation problem

- Special case of min-cost network flow
- Each node is either source or sink—no transshipment nodes.
- Each arc goes from a source to a sink

**Special case II:** Assignment problem

- Special case of transportation problem
- \( b_i = 1 \) for sources, \( b_i = -1 \) for sinks
- Classical example:
  - \( n \) workers assigned to \( n \) jobs
  - Benefit \( c_{ij} \) for assigning job \( j \) to worker \( i \)
  - Each worker exactly one job, each job exactly one worker
Note: the simplex method on a network problem yields an integer solution if data consists of integers.

Special case III: Shortest-path problem
- Determine fastest route between origin and destination
- Special case of min-cost network flow
- One +1 source, one −1 sink, all other transshipment
- Cost $c_{ij} \geq 0$ signifies length of arc

Special case IV: Max-flow problems
- Determines max amount of flow that can be moved through a network
- Special case of min-cost network flow
- Single source, single sink