**Review:** The simplex algorithm. Starts with a basic feasible solution $x^T = (x_B^T \ x_N^T)$, where $x_B = B^{-1}b = \hat{b} > 0$, $x_N = 0$. Matrices $B$ and $N$ contain the columns of $A$, and the columns of $B$ is a basis in $\mathbb{R}^m$. Also, let $c^T = (c_B^T \ c_N^T)$.

1. (optimality) Compute the *simplex multipliers* $y = B^{-T}c_B$ (solve $B^Ty = c_B$) and the *reduced costs* $\hat{c}_N^T = c_N^T - y^TN$. If $\hat{c}_N^T \geq 0$ stop (optimality found), else select some component $p$ of $x_N$ where $\hat{c}_{p,N}^T < 0$. (Corresponding column $a_p$ of $N$ will be brought into $B$.)

2. (step) Compute $\hat{a}_p = B^{-1}a_p$ (solve $B\hat{a}_p = a_p$). If $\hat{a}_p \leq 0$ stop (unbounded). Else, find the $i^*$ that attains $\min \left\{ \frac{\hat{b}_i}{\hat{a}_{i,p}} : \hat{a}_{i,p} > 0 \right\}$.

3. (update) Interchange column $i^*$ in $B$ and column $p$ in $N$. Reorder $x_B, x_N, c_B$ and $c_N$. 
