1. Derive the dual problem and solve it geometrically. This is possible since there are only two constants. The dual is

\[
\begin{align*}
\max & -z_1 + z_2 \\
3z_1 + 2z_2 & \leq 6 \\
2z_1 + 6z_2 & \leq 15 \\
2z_1 + 3z_2 & \leq 6 \\
z_1 - z_2 & \leq 1 \\
z_1, z_2 & \geq 0.
\end{align*}
\] (1)

After plotting the constraints in the plane it is evident that the maximum is in the corner between \(z_1 = 0\) and \(2z_1 + 3z_2 = 6\), i.e. \(z_1 = 0, z_2 = 2\). The optimum is \(b^Tz = c^Tx = 2\), and since the third equation is satisfied as an equality the corresponding primal variable \(z_3\) is basic. Since \(z_2 \neq 0\) the second primal constraint equation is satisfied as an equality and \(x_3 = 1/3\). The other variables \(x_j = 0, j = 1, 2, 4\) by complementarity.

2. (a) Rewrite \(Ax \leq b\) in standard form with slack variables

\[
\begin{align*}
\min & c^T x, \ Ax \leq b \rightarrow \\
\min & c^T x^+ - c^T x^- \\
Ax^+ - Ax^- + Is & = b \\
x^+ & \geq 0, x^- \geq 0, s \geq 0.
\end{align*}
\]

The dual constraints are

\[
\begin{align*}
z^T A & \leq c^T \\
-z^T A & \leq -c^T \\
Iz & \leq 0
\end{align*}
\]

and the dual is \(\max b^Tz\) such that \(z^T A = c^T, z \leq 0\).

(b) By complementarity we have

\[
x_i(z^T A_i - c_i) = 0 \rightarrow (z^T A - c^T)x_i = 0 \rightarrow z^T A x - c^T x = z^T b - c^T x = 0.
\]

3. The network can be depicted as:
Notice the extra node 8, that balance the network.

The minimum cost linear program is:

\[
\begin{align*}
\text{min} & \quad 36x_{1,3} + 27x_{1,4} + 33x_{2,3} + 29x_{2,4} + 2x_{3,4} + 21x_{3,5} + 15x_{3,6} + 13x_{3,7} + 2x_{4,3} + 16x_{4,5} + 16x_{4,6} + 17x_{4,7} \\
\text{subject to} & \\
-x_{1,3} - x_{1,4} - x_{1,8} &= -350 \quad \text{(node 1)} \\
-x_{2,3} - x_{2,4} - x_{2,8} &= -250 \quad \text{(node 2)} \\
x_{1,3} + x_{2,3} + x_{4,3} - x_{3,4} - x_{3,5} - x_{3,6} - x_{3,7} &= 0 \quad \text{(node 3)} \\
x_{1,4} + x_{2,4} + x_{3,4} - x_{4,3} - x_{4,5} - x_{4,6} - x_{4,7} &= 0 \quad \text{(node 4)} \\
x_{3,5} - x_{4,5} &= 150 \quad \text{(node 5)} \\
x_{3,6} + x_{4,6} &= 200 \quad \text{(node 6)} \\
x_{3,7} + x_{4,7} &= 220 \quad \text{(node 7)} \\
x_{1,8} + x_{2,8} &= 30 \quad \text{(node 8)} \\
x_{3,4} &\leq 35, \quad x_{4,3} \leq 35, \quad (\text{capacities})
\end{align*}
\]

4. The function \( \Phi(\alpha) = f(x_0 - \alpha \nabla f(x_0))/\nabla^2 f(x_0) \). First compute the correction \( \Delta x \) by Newton’s method at \( x_0 = 2 \). Then \( \Delta x = -\nabla f(x_0)/\nabla^2 f(x_0) = -3/8 \). Construct the line \( l(\alpha) = \Phi(0) + \alpha \Phi'(0) = 2 - \alpha \cdot 0.27/8 \). Let \( \alpha_1 = 1 \). Then check if \( \Phi(\alpha) \leq l(\alpha) \) for \( x_j = x_0 + \Delta x \) and \( \alpha = \alpha_j \). If not then \( \Delta x := \beta \Delta x, \quad \alpha_j+1 = \beta \alpha_j \), and compute \( x_{j+1} = x_0 + \Delta x \). Otherwise, accept the value \( x_j \) as the next iterate. Since \( x_1 = 1.625, \quad \alpha_1 = 1, \quad l(\alpha_1) = 1.325 \) and \( \Phi(\alpha_1) = f(x_1) = 0.0158 \), the accepted value by line search by back tracking is \( x = 1.625 \) after one try.

5. Let

\[ r_j(a, b) = \sin(at + b). \]

The objective function is

\[ f(a, b) = (\sum_{j=1}^{n} r_j^2)^{1/2}. \]

The search direction \( p \) in Gauss-Newton’s algorithm solves

\[ \nabla r \nabla r^T p = -\nabla r \]
where
\[
\nabla r^T(a, b) = \begin{pmatrix}
0 & \cos(b) \\
\cos(a + b) & \cos(a + b) \\
2\cos(2a + b) & \cos(2a + b)
\end{pmatrix}.
\]

Then the algorithm for \( y_k = (a, b)^T \) is a repetition of the following statements
\[
\nabla r(y_k) \nabla r(y_k)^T p_k = -\nabla r(y_k) r(y_k)
y_{k+1} = y_k + p_k
k \leftarrow k + 1
\]

In our case we have
\[
(a_0, b_0) = (1, 1), \quad r_0^T = (0.0515, 0.0493, 0.2911),
\nabla r_0 \nabla r_0^T = \begin{pmatrix}
4.0935 & 2.1333 \\
2.1333 & 1.4452
\end{pmatrix},
(\nabla r_0 r_0)^T = (-0.5969, -0.2809),
\]

and \((a_1, b_1) = (1.1930, 0.9095)\). In the next step we have
\[
\nabla r_1^T = (-0.0008, 0.0019, -0.0033),
\nabla r_1 \nabla r_1^T = \begin{pmatrix}
4.1630 & 2.2100 \\
2.2100 & 1.6107
\end{pmatrix},
(\nabla r_1 r_1)^T = (0.0055, 0.0018),
\]

and \((a_2, b_2) = (1.1903, 0.9121)\) after two steps of Gauss-Newton and the method seems to converge.

6. (a) **Decrease**: fewer drastic changes in the parameters, may not reach extreme points of parameter space, may not move away from local minima. **Increase**: too frequent destruction of promising genes close to global minimum.

(b) **Decrease**: faster computation of new generation, smaller part of parameter space is covered by one generation. **Increase**: better coverage of parameter space, good genes have more difficulties to dominate, more computational work.

(c) **Decrease**: less probability of accepting increase in energy, harder to escape local minimum. **Increase**: higher probability to increase energy in a step, easier to move from local minimum.