Assignment 1: Linear Programming

Hand in a written report containing a short presentation of the problem, results, discussion, and source code. Remember to answer all question and comment your results. You may work alone or in groups of two (not three). The absolute deadline for submitting this years assignments is 2004-11-04. You will obtain a 1/2-point bonus point on the nal exam if a correct solution for this assignment is handed in at the latest 2004-09-22.

1. Solve the linear program

\[
\begin{align*}
\min & \quad -x_2 + x_3 \\
\text{subject to} & \quad x_2 - 2x_3 \leq 2, \\
& \quad x_1 - x_2 + 3x_3 \leq 2, \\
& \quad -x_1 + x_2 \leq 4, \\
& \quad x_1, x_2, x_3 \geq 0
\end{align*}
\]

by the simplex method, accounting for every iteration. The problem should be solved “by hand”, but you may use Matlab or some other tool to solve the linear system and to perform matrix–vector products, if you like. Verify your result by using the Matlab Optimization Toolbox routine \texttt{linprog} (note that the optimization toolbox is only available on the Sun-system). Type \texttt{help linprog} at the Matlab prompt to get information on how \texttt{linprog} is used. Also write down the dual problem and determine its solution from your previous computations.

2. The Skellefteå Subway System (SSS) has obtained the contract to operate the subway in the Greater Skellefteå Metropolitan Area. The Traffic Planning Department has estimated that at least the following number of staff is needed each four-hour interval throughout a standard 24-hour period:

<table>
<thead>
<tr>
<th>Time period</th>
<th>Staff</th>
</tr>
</thead>
<tbody>
<tr>
<td>06–10</td>
<td>700</td>
</tr>
<tr>
<td>10–14</td>
<td>200</td>
</tr>
<tr>
<td>14–18</td>
<td>800</td>
</tr>
<tr>
<td>18–22</td>
<td>300</td>
</tr>
<tr>
<td>22–02</td>
<td>100</td>
</tr>
<tr>
<td>02–06</td>
<td>100</td>
</tr>
</tbody>
</table>

All staff works in 8-hour shifts. There are six possible shifts that start on the hour in the beginning of each 4-hour period in the table. All staff has the same hourly salary, except that the hourly salary is 60 % higher between 22.00 and 6.00.

(a) Find how many employees that are needed for each of the six shifts in order to minimize the salary costs for SSS, while at the same time satisfying the staffing requirements.

Note: When solving the resulting LP in Matlab, use the following command before calling \texttt{linprog}:
options = optimset('LargeScale','off');

and call linprog with options as the last argument (see help linprog to more info).

(b) The staffing company Billiardia offers external staff at a more flexible schedule, with 4-hour shift starting on the hour in the beginning of each 4-hour period in the table. The hourly cost for using Billiardia’s staff is a flat (that is, the same at all hours) 80% higher than the daytime cost for SSS’s own staff. Management wants to know if there is any benefit of using Billiardia. Find out the optimal mix of SSS and Billiardia personnel in order to minimize the salary costs for SSS, while at the same time satisfying the staffing requirements. Is there any financial benefit compared to (a)?

(c) Try running problem (a) with the default large-scale algorithm (a so-called interior-point method) instead of the algorithm of simplex type which is used by setting options as in (a). To use the large-scale algorithm, set

options = optimset('LargeScale','on');

and call linprog with options as the last argument. Compare with the result you obtained in (a) and discuss the reason for a possible difference!