Review: The simplex algorithm

Starts with a basic feasible solution $x^\mathsf{T} = (x_B^\mathsf{T}\ x_N^\mathsf{T})$, where $x_B = B^{-1}b = \hat{b} > 0$, $x_N = 0$. Matrices B and N contain the columns of A, and the columns of B is a basis in $\mathbb{R}^m$. Also, let $c^\mathsf{T} = (c_B^\mathsf{T}\ c_N^\mathsf{T})$.

1. (optimality) Compute the *simplex multipliers* $y = B^{-T}c_B$ (solve $B^\mathsf{T}y = c_B$) and the *reduced costs* $\hat{c}_N^\mathsf{T} = c_N^\mathsf{T} - y^\mathsf{T}N$. If $\hat{c}_N^\mathsf{T} \geq 0$ **stop** (optimality found), else select some component p of x_N where $\hat{c}_{p,N}^\mathsf{T} < 0$. (Corresponding column a_p of N will be brought into B.)
2. (step) Compute $\hat{a}_p = B^{-1}a_p$ (solve $B\hat{a}_p = a_p$). If $\hat{a}_p \leq 0$ **stop** (unbounded). Else, find the i^* that attains $\min\left\{ \dfrac{\hat{b}_i}{\hat{a}_{i,p}} :\ \hat{a}_{i,p} > 0 \right\}$.
3. (update) Interchange column i^* in B and column p in N. Reorder x_B, x_N, c_B and c_N.