Assignment 1: Linear Programming

Hand in a written report containing a short presentation of the problem, results, discussion, source code, and a print-out of the result from your Matlab sessions. Answer all question and comment your results. If the report is incomplete, I will return it for completion before starting to grade it. You may work alone or in groups of two (not three). Both persons in a group should contribute to the solution and the report. Discussions between the groups are encouraged. If you receive substantial help from another group, say so in the report. You are not allowed to copy solutions or computer codes from others. I prefer answering questions in person or by phone (instead of email). I do not wish to debug your code.

The absolute deadline for submitting this years assignments is 2006-11-13. You will obtain a 1/2-point bonus point on the final exam if a correct solution for this assignment is handed in at the latest 2006-09-22.

1. Solve the linear program

\[
\begin{align*}
\text{max } & \quad x_1 - x_2 - x_3 \\
\text{subject to } & \quad x_1 - 2x_2 + x_3 \leq 4, \\
& \quad x_1 - x_2 - 2x_3 \leq 2, \\
& \quad -x_1 + 2x_2 + 2x_3 \leq 2, \\
& \quad x_1, x_2, x_3 \geq 0
\end{align*}
\]

by the simplex method, accounting for every iteration. The problem should be solved "by hand", but you may use Matlab or some other tool to solve the linear system and to perform matrix-vector products, if you like. Verify your result by using the Matlab Optimization Toolbox routine linprog (note that the optimization toolbox is only available on the Sun-system). Type doc linprog (or help linprog) at the Matlab prompt to get information on how linprog is used. Also write down the dual problem to problem (1) and determine the solution to the dual problem from your previous computations.

2. The Skellefteå Subway System (SSS) has been awarded the contract to operate the subway in the Greater Skellefteå Metropolitan Area. The Traffic Planning Department has estimated that at least the following number of staff is needed each four-hour interval throughout a standard 24-hour period:

<table>
<thead>
<tr>
<th>Time period</th>
<th>Staff</th>
</tr>
</thead>
<tbody>
<tr>
<td>06–10</td>
<td>700</td>
</tr>
<tr>
<td>10–14</td>
<td>200</td>
</tr>
<tr>
<td>14–18</td>
<td>800</td>
</tr>
<tr>
<td>18–22</td>
<td>300</td>
</tr>
<tr>
<td>22–02</td>
<td>100</td>
</tr>
<tr>
<td>02–06</td>
<td>100</td>
</tr>
</tbody>
</table>

All staff works in 8-hour shifts. There are six possible shifts that start on the hour in the beginning of each 4-hour period in the table. All staff has the same hourly salary, except that the hourly salary is 50% higher between 20.00 and 24.00. Moreover, the hourly salary is 100% higher between 24.00 and 06.00.
(a) Find how many employees that are needed for each of the six shifts in order to minimize the salary costs for SSS, while at the same time satisfying the staffing requirements.

Note: When solving the resulting LP in Matlab, use the following command before calling \texttt{linprog}:

\begin{verbatim}
options = optimset('LargeScale','off');
\end{verbatim}

and call \texttt{linprog} with \texttt{options} as the last argument (see \texttt{doc linprog} to more info).

(b) The staffing company \texttt{BILLIARDIA} offers external staff at a more flexible schedule, with 4-hour shift starting on the hour in the beginning of each 4-hour period in the table. The hourly cost for using \texttt{BILLIARDIA}'s staff is a flat (that is, the same at all hours) 80 % higher than the daytime cost for SSS's own staff. Management wants to know if there is any benefit of using \texttt{BILLIARDIA}.

Find out the optimal mix of SSS and \texttt{BILLIARDIA} personnel in order to minimize the salary costs for SSS, while at the same time satisfying the staffing requirements. Is there any financial benefit compared to (a)?

(c) Try running problem (a) with the default large-scale algorithm (a so-called interior-point method) instead of the algorithm of simplex type which is used by setting \texttt{options} as in (a). To use the large-scale algorithm, set

\begin{verbatim}
options = optimset('LargeScale','on');
\end{verbatim}

and call \texttt{linprog} with \texttt{options} as the last argument. Compare with the result you obtained in (a) and discuss the reason for a possible difference!