Review: the KKT Conditions

A nonlinear program:

\[
\min f(x) \text{ subject to } \quad g_i(x) \geq 0, \ i = 1, \ldots, m
\]

(NLP)

The **Lagrangian** for (NLP) is the function

\[
\mathcal{L}(x, \lambda) = f(x) - \sum_{i=1}^{m} \lambda_i g_i(x),
\]

where \( \lambda^T = (\lambda_1, \ldots, \lambda_m) \) is the **Lagrange multiplier vector**.
Theorem (1st-order necessary conditions)

Assume $x^*$ is a regular, local minimizer to (NLP). Then there is a Lagrange multiplier vector $\lambda^*$ such that

$$\nabla f(x^*) = \sum_{i=1}^{m} \lambda_i^* \nabla g_i(x^*)$$

(dual feasible)

$$\lambda_i^* \geq 0 \quad i = 1, \ldots, m$$

(dual feasible)

$$g_i(x^*) \geq 0 \quad i = 1, \ldots, m$$

(primal feasible)

$$\lambda_i^* g_i(x^*) = 0 \quad i = 1, \ldots, m$$

(complementary slackness)

Note:

$$\nabla f(x^*) = \sum_{i=1}^{m} \lambda_i^* \nabla g_i(x^*) \iff \nabla_x \mathcal{L}(x^*, \lambda^*) = 0$$

October 5, 2007
Geometrical interpretation of the KKT conditions:

At a local minimum, the gradient is in the cone of the normals to the active constraints

*Example:* $\bar{x}$ local minimum. Note that $\nabla f(\bar{x})$ is “between” $\nabla g_1(\bar{x})$ and $\nabla g_2(\bar{x})$, the normals associated with the active constraints $g_1(\bar{x}) = 0$, $g_2(\bar{x}) = 0$