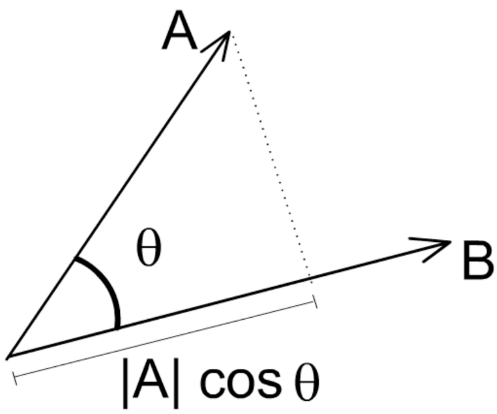
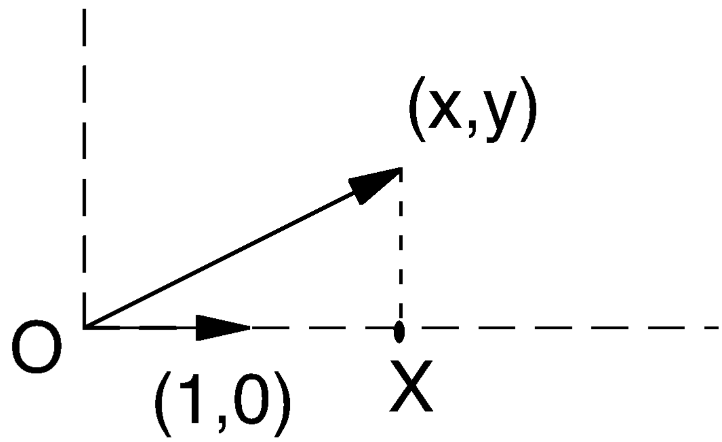
Explanation of the intersection algorithm for planes (section 5.2 in the project specification)

Before getting in the details, I will explain the geometric meaning of the dot product A•B when B is a unit vector, i.e. |B|=1. In that case, A•B = |A|cos θ, where θ is the angle between A and B. In other words A•B is the length of the projection of A onto the (extension) of B.

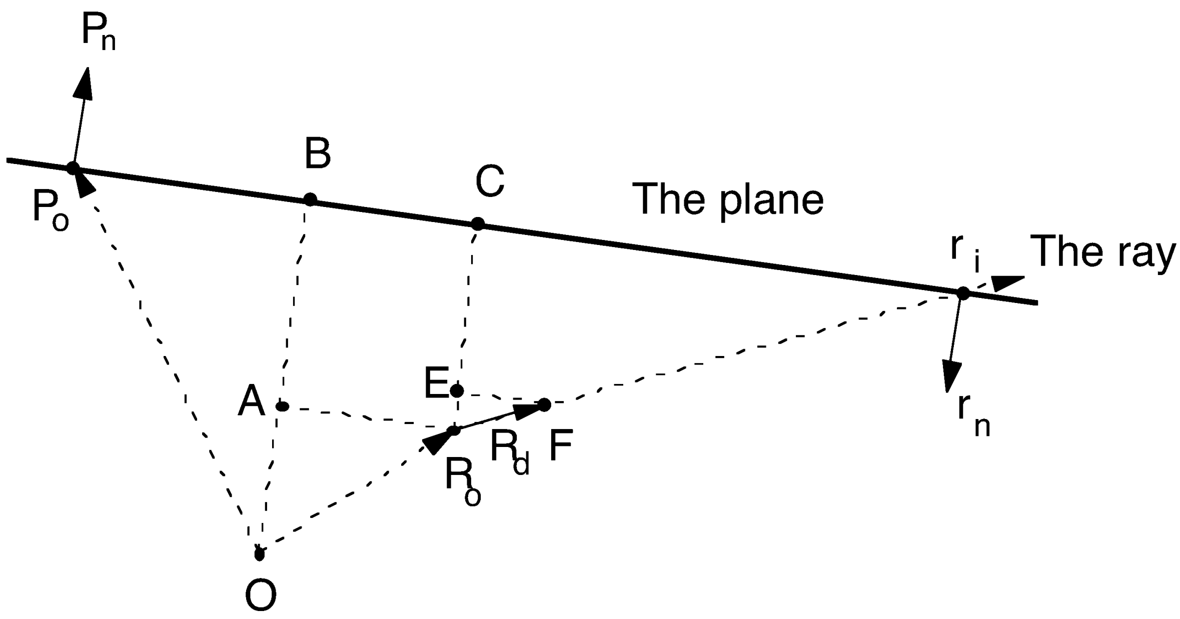


I will not prove this rigourously, but to see that this is the case, consider B to be the the unit vector along the x axis (1,0). Let A be the vector (x,y). The projection of A onto the x axis will be OX in the figure below, the length of OX is clearly x. Indeed A•B = 1x+0y = x, so the dot product has the intended meaning.



Now, we want to compute the intersection of a ray with a plane. As input, we'll have a point on the plane (P_o), a unit vector at right angles to the plane (P_n), the point where the ray starts (R_o) and a unit vector giving the direction of the ray (R_d). We want to compute the intersection point (r_i) and a vector normal to the plane at the intersection point (r_n).

I will consider equations 23-28 (from the project specification) in turn. The explanation refers to the following figure:



For simplicity, the plane is viewed from the side. O is the co-ordinate system origin (0,0,0). A, B, C, E and F are points I'll refer to in the course of the explanation.

(23) The shortest distance from O to the plane is a line intersecting the plane at right angles in point B. As P_n is also at right angles to the plane, The line OB will have the same direction as P_n. The co-ordinates of the point P_o can be regarded as a vector from O to P_o. The length of OB, is the length of the projection of P_o onto OB, which we can compute as $P_o \cdot P_n$, as P_n is a unit vector. This quantity is called D.

(24) The shortest distance from R_o to the plane is a line intersecting the plane at right angles in point C. As P_n is also at right angles to the plane, The line R_oC will have the same direction as P_n. We now project R_d onto the line R_oC. The projection will be R_oE. The distance from R_o to E is given by $P_n \cdot R_d$, as P_n is a unit vector. This quantity is called v_d. Note that if v_d is 0, R_d will be parallel to the plane and the ray will not intersect it.

(25) The co-ordinates of the point R_o can be regarded as a vector from O to R_o. We project this vector onto the line OB, giving OA. As P_n is a unit vector, the length of this projection (from O to A) is given by $P_n \cdot R_o$. As D is the distance from O to B, the distance from A to B is given as $D-P_n \cdot R_o$. This quantity is called v_o.

(26) Note that R_oABC is a rectangle, so the length of AB (v_o) is the same as the length of R_oC. The ratio of R_oC to R_oE is given by v_o/v_d. This quantity is called t.

(27) The triangles R_oEF and R_oCr_i have the same shape and share the corner R_o. Thus the ratio of R_oC to R_oE (t) is the same as the ratio of R_or_i to R_oF. In other words, the vector from R_o to r_i is the same as R_d*t. Adding R_o as a vector to R_d*t, we get r_i as a vector, i.e. $r_i = R_o+R_d*t$

(28) Finally, we should determine the vector r_n, normal to the intersection point. r_n should be a unit vector at right angles to the plane. We already have such a vector -- P_n. However, r_n should "point" towards the side of the plane from which the ray is coming, thus we may need to flip the direction of P_n. In the case in the figure, we should take r_n to be P_n. However, if P_n pointed in the other direction, we should take P_n as it is. We can distinguish these two cases by v_d being negative in the second case.