Assignment 3

Programming Theory

The program below computes the number of nodes in a tree of height $h$ where each node has $n$ children. For example; if each node has 3 children, a tree of hight 0 has 1 node and a tree of height 4 has a total of 121 nodes.

Use the iterative command theorem to translate the correctness of the program into the validity of a set of logical formulas and prove that the program is correct.

\[
\begin{align*}
\{Q: \ n > 0 \land h \geq 0\} &\\
\ r, x, y := 1, 0, 1; &\\
\{\text{inv } P: \ r = \Sigma i \ : \ 0 \leq i < x + 1 : \ n^i \land 0 \leq x \leq h \land y = n^x\} &\\
\{\text{bound } t: \ h - x\} &\\
\textbf{do} &\ x < h \quad \rightarrow \quad y := y \cdot n; &\\
&\ r, x := r + y, x + 1 &\\
\textbf{od} &\\
\{R: \ r = \Sigma i \ : \ 0 \leq i < h + 1 : \ n^i\} &
\end{align*}
\]