Numerical modelling of quantum solitons

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Background

Non-linear PDE may allow solitons, solitary waves, to form and propagate. These solitons can carry information in quantum computers or propagate as tsunamis on the ocean. Simulation of such solitons requires high-order numerical methods.

Method

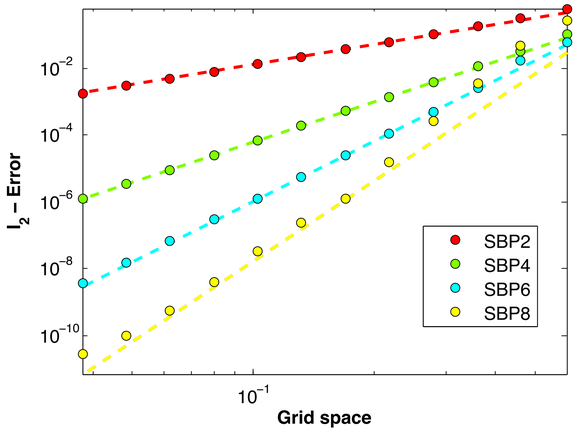
High-order SBP-SAT finite difference schemes were developed for the Gross-Pitaevskii and the non-linear Klein-Gordon equations, including accurate boundary treatment. The schemes were shown to be accurate, stable and efficient for simulation of solitons.

The Gross-Pitaevskii equation

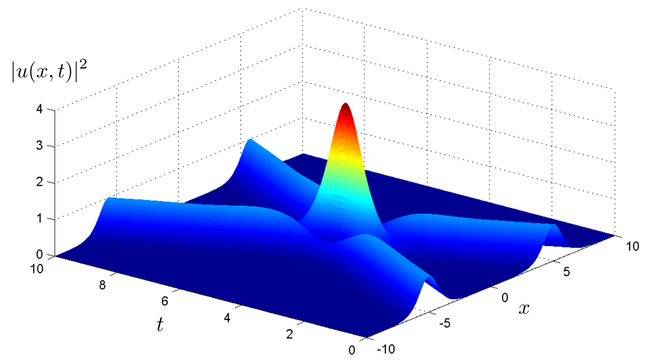
$$iu_t = -\tfrac{1}{2}u_{xx} + V(x,t)u + \tilde{g}|u|^2 u$$

The Klein-Gordon equation

$$u_{tt} = u_{xx} - m^2 u + |u|^{p-2}u$$

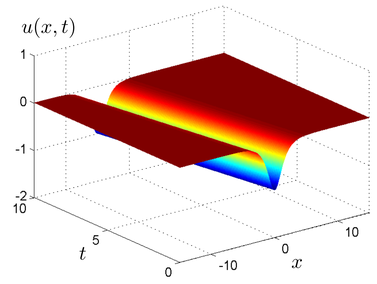


Convergence to an analytical solution of the Klein-Gordon solver for different orders of accuracy. Dashed lines show expected convergence.



Simulation of the Gross-Pitaevskii equation for two colliding solitons in time and space.

A soliton of the Klein-Gordon equation propagating in time and space.



Conclusion

The SBP-SAT method can be used to accurately simulate solitons for the two equations.

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