Pricing options in markets with jumps

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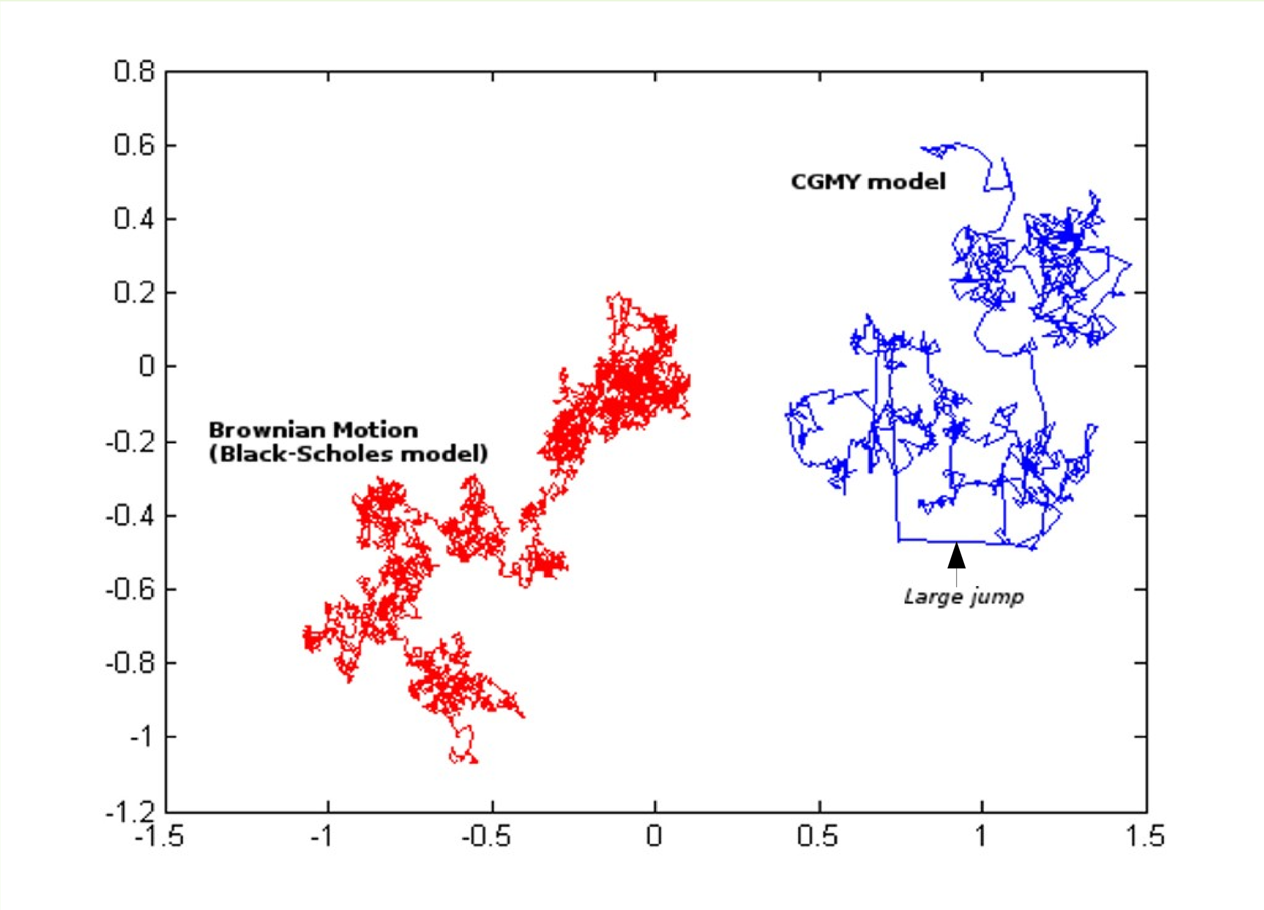
We investigate the **CGMY model** which, as opposed to the standard **Black-Scholes model**, accounts for jumps in the market.



The properties of the CGMY model are given by its **characteristic function**:

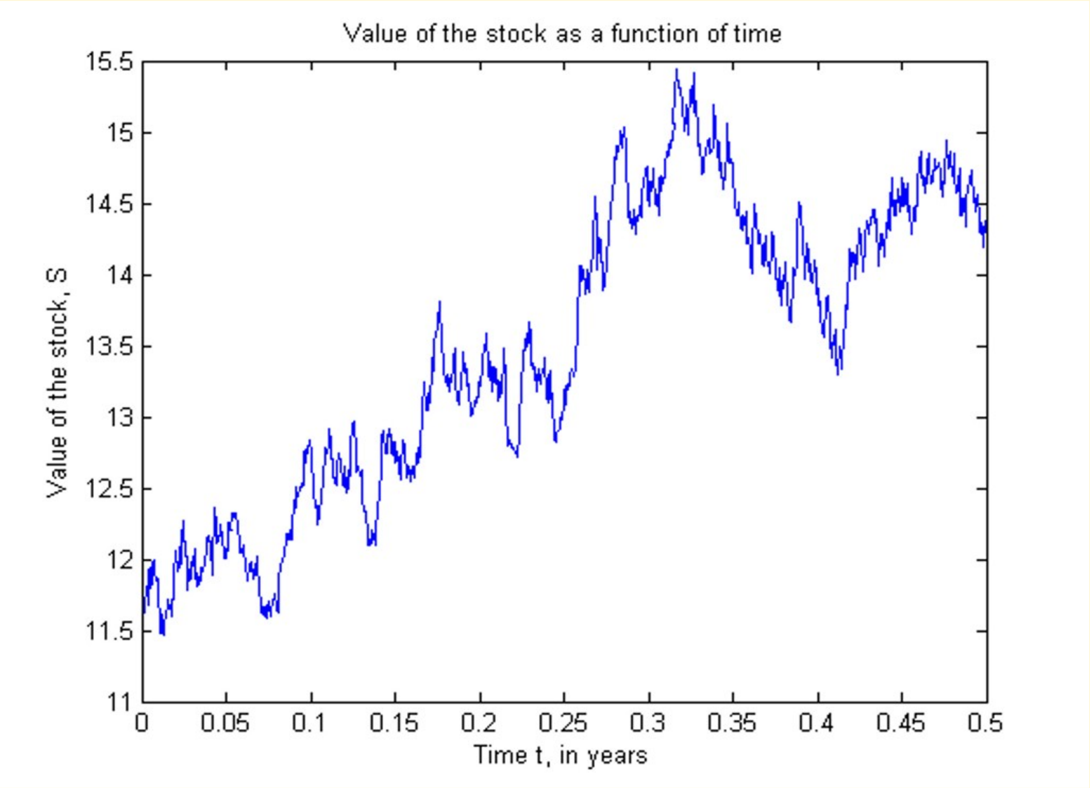
$$\Psi_{CGMY}(\xi)=C\,\Gamma(Y)\left[(M-i\xi)^{Y}-M^{Y}+(G+i\xi)-G^{Y}\right]$$

Approach A: Stochastic Differential Equation (SDE)

The SDE,

$$d(\ln S_t)=(r-v)\,dt+dL_t^{P}$$

is solved numerically using a **Monte-Carlo** method which produces the following simulation of a risky asset's price:



Approach B: Fractional Partial Differential Equation (FPDE)

We solve the following FPDE numerically

$$\frac{\partial V(x,t)}{\partial t}+(r-v)\frac{\partial V(x,t)}{\partial x}+C\Gamma(-Y)e^{Mx}D^{Y}_{[x,\infty]}\left(e^{-Mx}V(x,t)\right)$$
$$+C\Gamma(-Y)e^{-Gx}D^{Y}_{[-\infty,x]}\left(e^{Gx}V(x,t)\right)=\left(r+C\Gamma(-Y)\left(M^{Y}+G^{Y}\right)\right)V(x,t)$$

using **Grünwald-Letnikov.** For example the left fractional derivative is approximated:

$$D^{Y}_{[-\infty,x]}\left(V(x,t)\right)=\frac{1}{h^{Y}}\left[\sum_{k=0}^{i+1}\omega_k V^{n}_{i-k+1}\right]$$

Conclusion: The CGMY model provides a more general approximation of the market than the Black-Scholes model. Due to the FPDE resulting in full matrices and the SDE having infinite activity as well as different jump sizes, these models are more complicated to solve numerically.