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# Verification of optimality for portfolio consumption problems

The classic investors problem is to find an investment strategy such that the investor gets maximum expected utility from the investment. Often it is not hard to find candidates for such optimal strategies, but verifying mathematically that these indeed are optimal strategies can be very difficult.

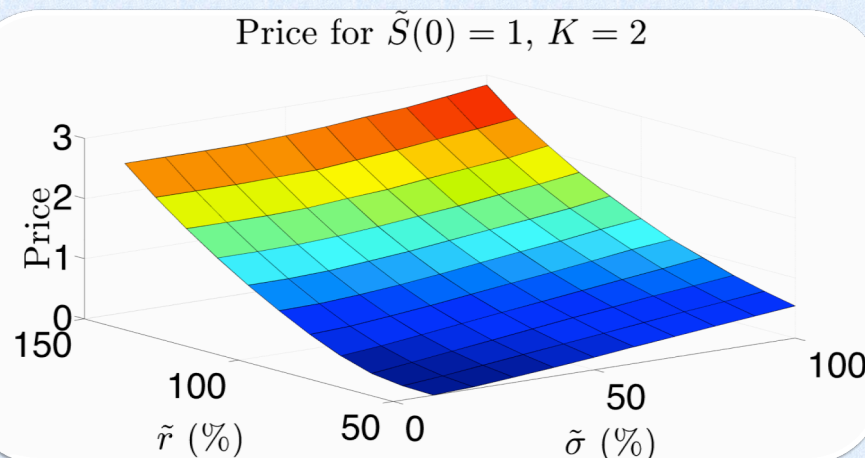
There are examples where this can be done, but if the methods use demand the problem be solved explicitly there can be problems when trying to generalize the approach. Such problems occur when the market is assumed to be incomplete.

To solve the investors problem we can use dynamic programming to obtain a Hamilton-Jacobi-Bellman equation. This partial differential equation looks difficult to solve but for some models it can be reduced to a linear partial differential equation and solved explicitly.

$$\sup_{\pi \in \mathbb{R}^K, c \geq 0} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \mathcal{D}^{\pi, c} G(t, w, x) \right\} = 0$$

$$G(T, w, x) = \frac{w^{1-\gamma}}{1-\gamma}$$

$$\begin{aligned} \mathcal{D}^{\pi, c} G(t, w, x) &= G_t + w [\pi^T \theta(x) + r(x)] G_w \\ &\quad - c G_w + \mu^X(t)^T G_x + 1/2 w^2 \|\pi\|^2 G_{ww} \\ &\quad + w(\sigma^X \pi)^T G_{wx} + 1/2 \text{tr}[\Sigma^X G_{xx}^T]. \end{aligned}$$



Incompleteness makes it difficult to model markets. One way to see this is in option pricing. Even if the assets for real options are highly correlated to traded assets the price can take almost any value. Using Monte-Carlo methods and martingale pricing we see in the plot that even when restricting to "reasonable" parameter values the price of the option can still vary significantly.

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