

# Evolving solitons with SBP-SAT

- constructing stable high order methods for solving non-linear PDEs

## Background

Solitons are a type of wave that travels without losing its shape or energy. They arise for example in quantum mechanical systems or shallow water systems, and can be modelled by stiff non-linear PDEs.

A tsunami is an example of a soliton which it may be of interest to model



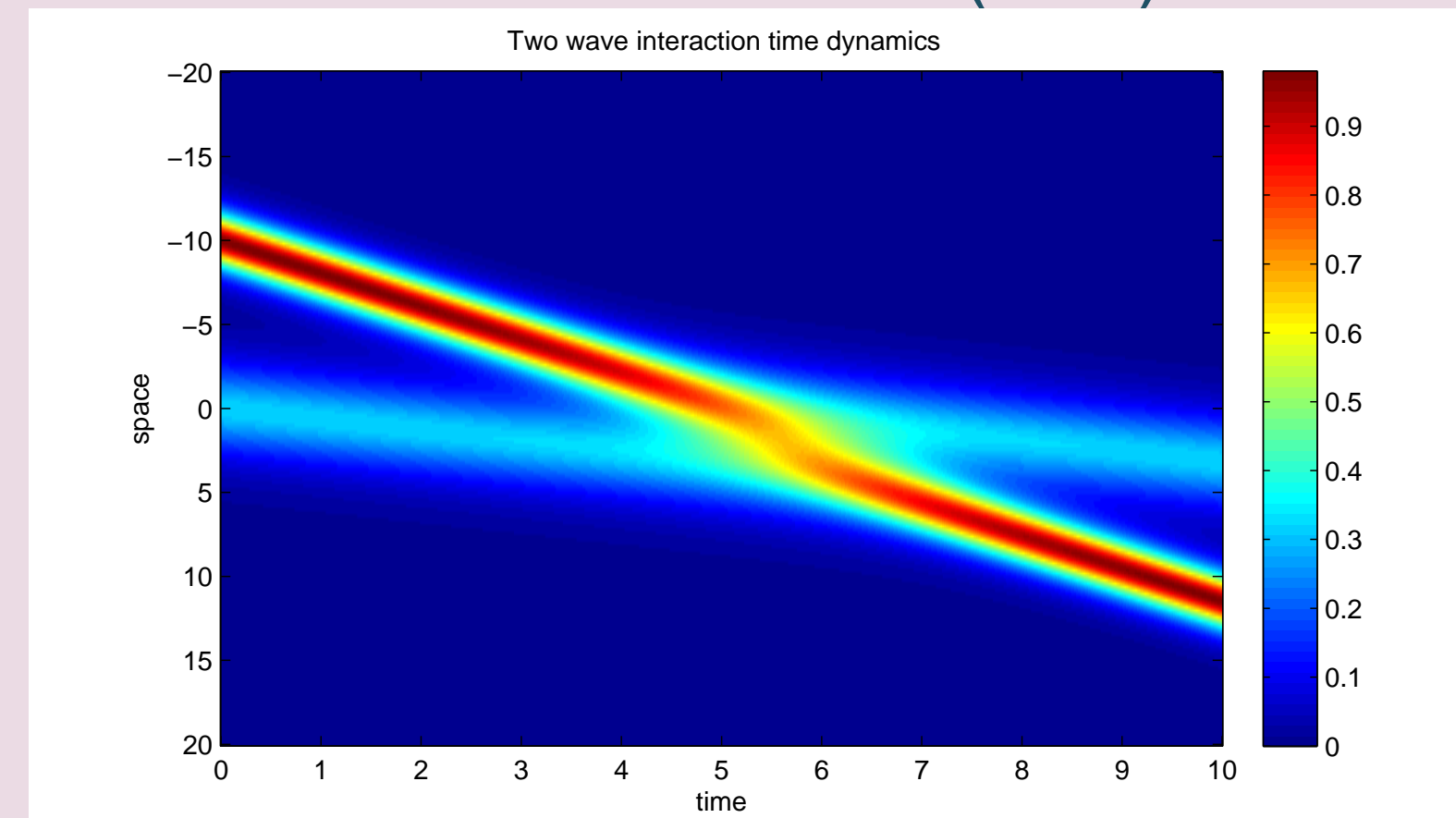
Modelling these systems numerically proves difficult; many grid points are required in both time and space. Also, it can be difficult to handle boundary conditions in a stable manner, which is required to prove that the method actually solves the right problem.

The SBP-SAT method was developed to address these problems. It allows handling the boundary conditions in a manner that guarantees stability. At the same time it has a high order of accuracy, whereby the number of grid points can be reduced.

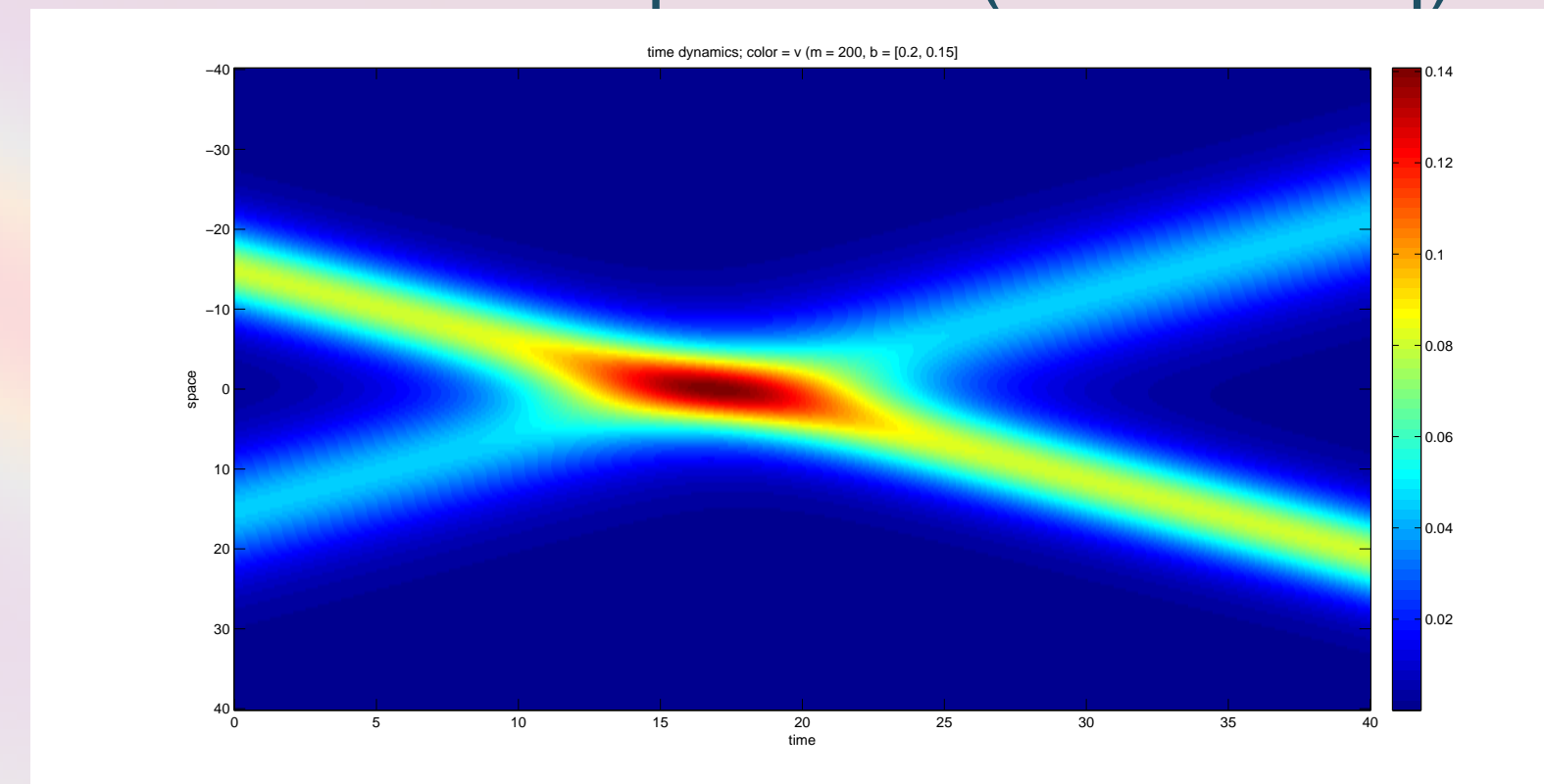
For the project to become applicable to real physics problems, interesting extensions would include implementing the method for higher dimensional equations and working out how to add a non-homogenous seabed.

## Simulations

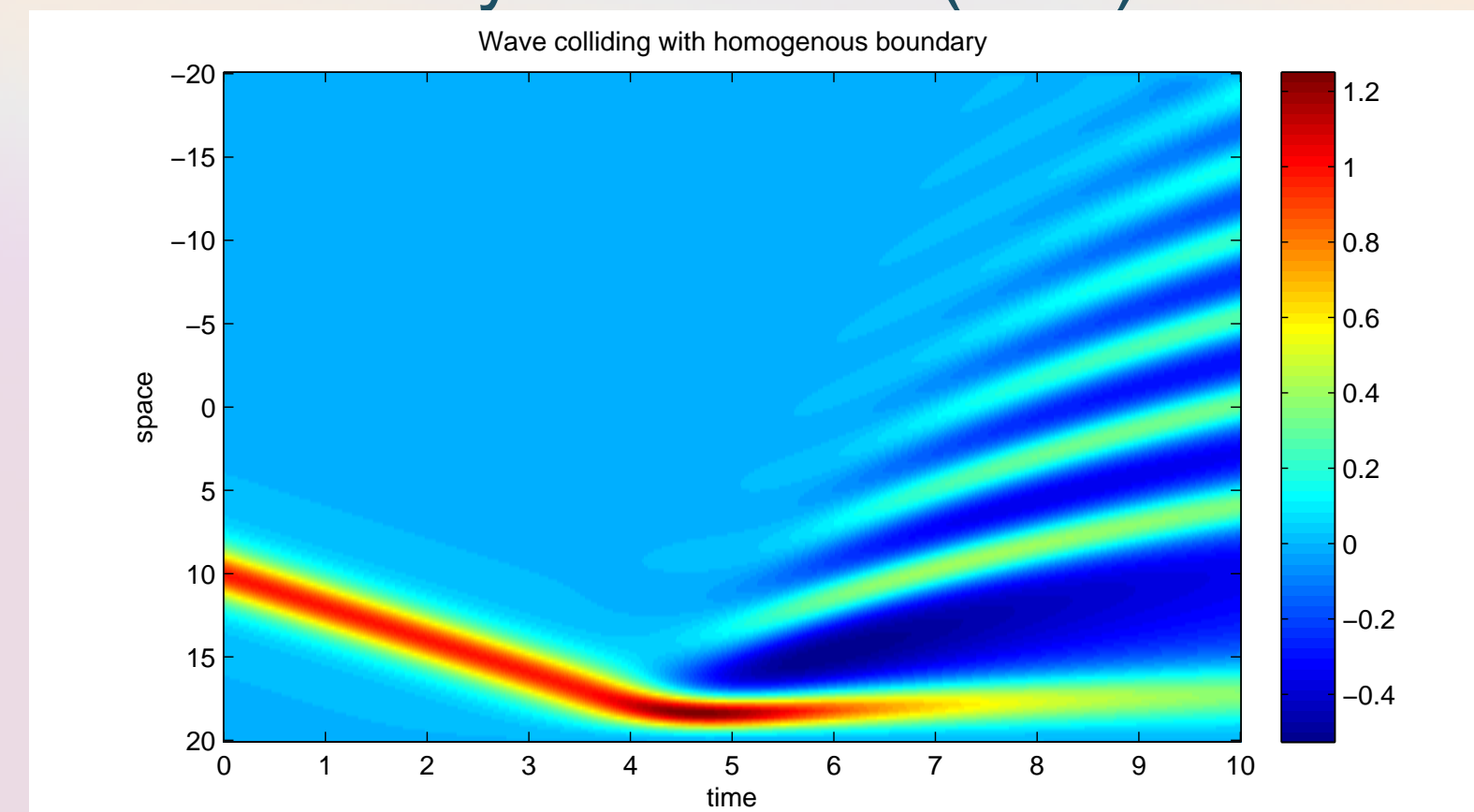
SBP solution to two wave interaction (KdV)



Central finite differences solution to two wave problem (Boussinesq)



SBP solution to boundary interaction (KdV)



In the dynamics graphs above, the horizontal axis represents time and the vertical axis represents space; the color is the value of the solution.

## Conclusions

Two finite difference methods; the SBP-SAT method and a central finite difference method are investigated with respect to accuracy and stability. As test problem, two equations modelling shallow wave systems are used; Korteweg-de Vries' equation:

$$u_t = -6uu_x - u_{xxx}$$

and Boussinesq's equation:

$$u_{tt} = u_{xx} - 3(u^2)_{xx} - \alpha u_{xxxx}$$

- The central difference schemes are proven to be stable and of predicted accuracy for the periodic problem.
- The SBP-SAT scheme is proven to be stable for a well posed initial and boundary value problem for de-Vries' equation.
- Accuracy is found to be mostly high, see figure below.

Results of convergence study for SBP-SAT on initial boundary value test problem:

