UPPSALA UNIVERSITET

Project in Computational Science 2018

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Numerical simulation of solitons

Conclusion

The coupled Drinfeld-Sokolov-Wilson (DSW) equation has been analyzed and solved numerically using the SBP-SAT method.
We derive two sets of boundary conditions to the DSW equation and prove their linear well-posedness using the energy method. The result shows that the numerical solution converges to the analytical solution. It also strengthens the motivation for higher order SBP operators.

SBP-SAT method

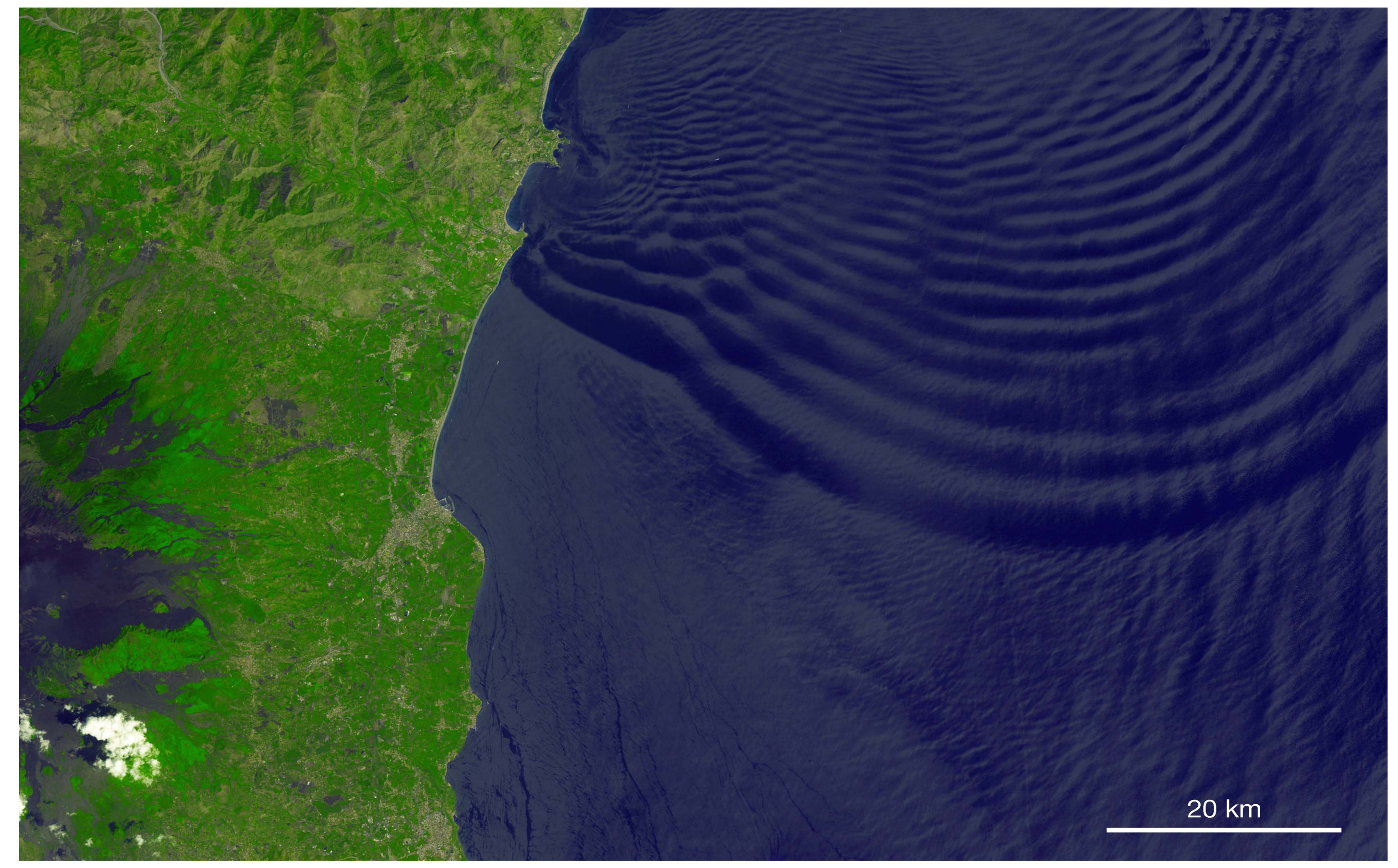
- A powerful method for boundary treatment;
- Combine
  - (+) *Summation-by-parts* (SBP) method: approximate governing equation,
  - (+) *Simultaneous-approximation-term* (SAT): impose boundary conditions;
- **Benefits**: well-controlled stability, high order of accuracy ⟶ computational efficiency, ability to capture more complex phenomena.

The phenomenon of solitons

Solitons (or *solitary waves*) are special kinds of waves with a distinct set of features arising from a balance between nonlinear effects and dispersive effects. The characteristics of a soliton are:

- when propagating at a constant speed, the shape of the soliton remains unchanged;
- when a collison occurs between two solitons, both solitons emerge unchanged.

The study on soliton plays a vital role in various areas (e.g., nonlinear optics, biophysics) and also for the construction of more complex models - plasma waves, shock waves, tornados, hydrodynamics, etc.



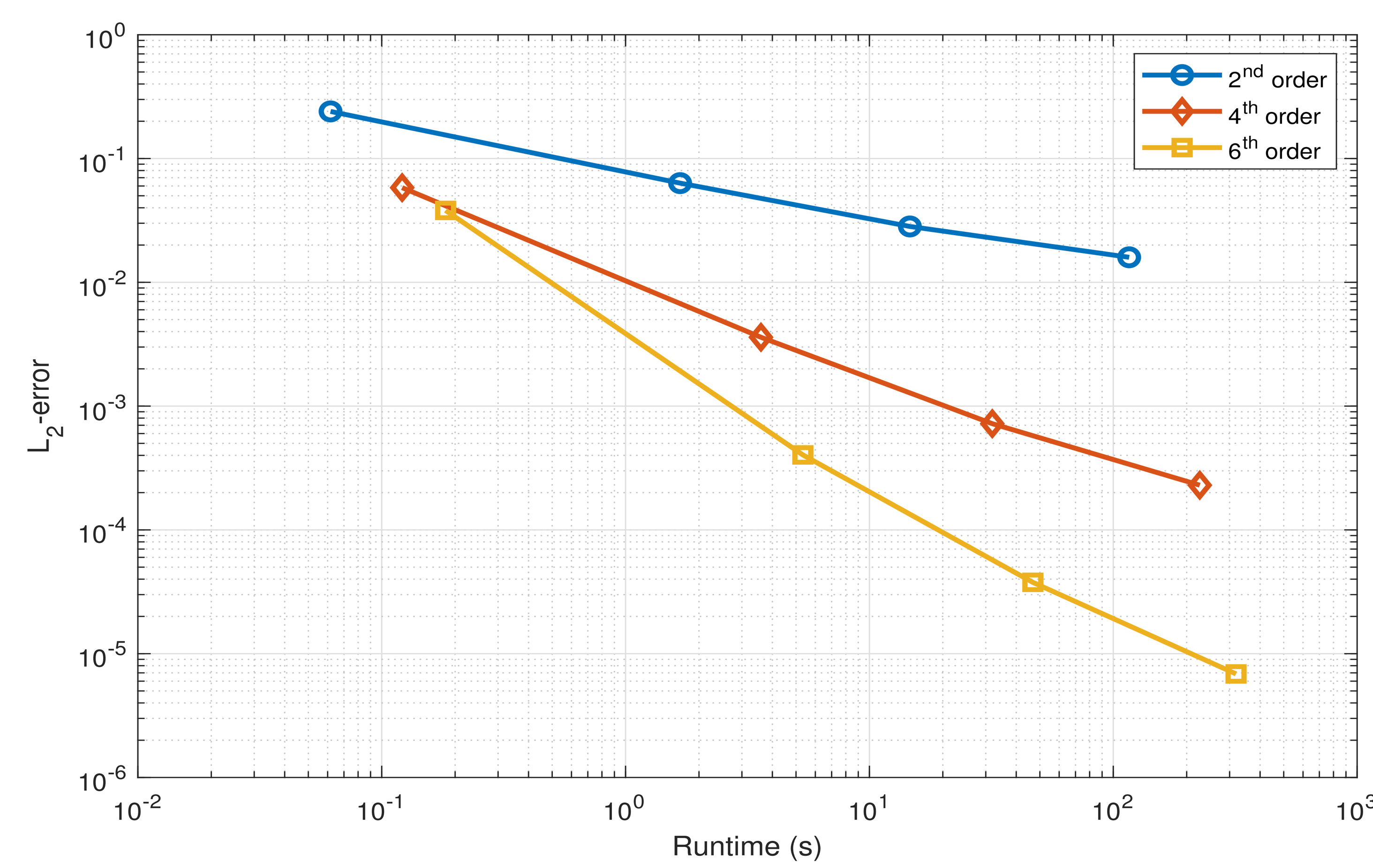
Solitary waves observed in Messina Strait, Italy (Credit: NASA's Earth Observatory.)

The governing equation

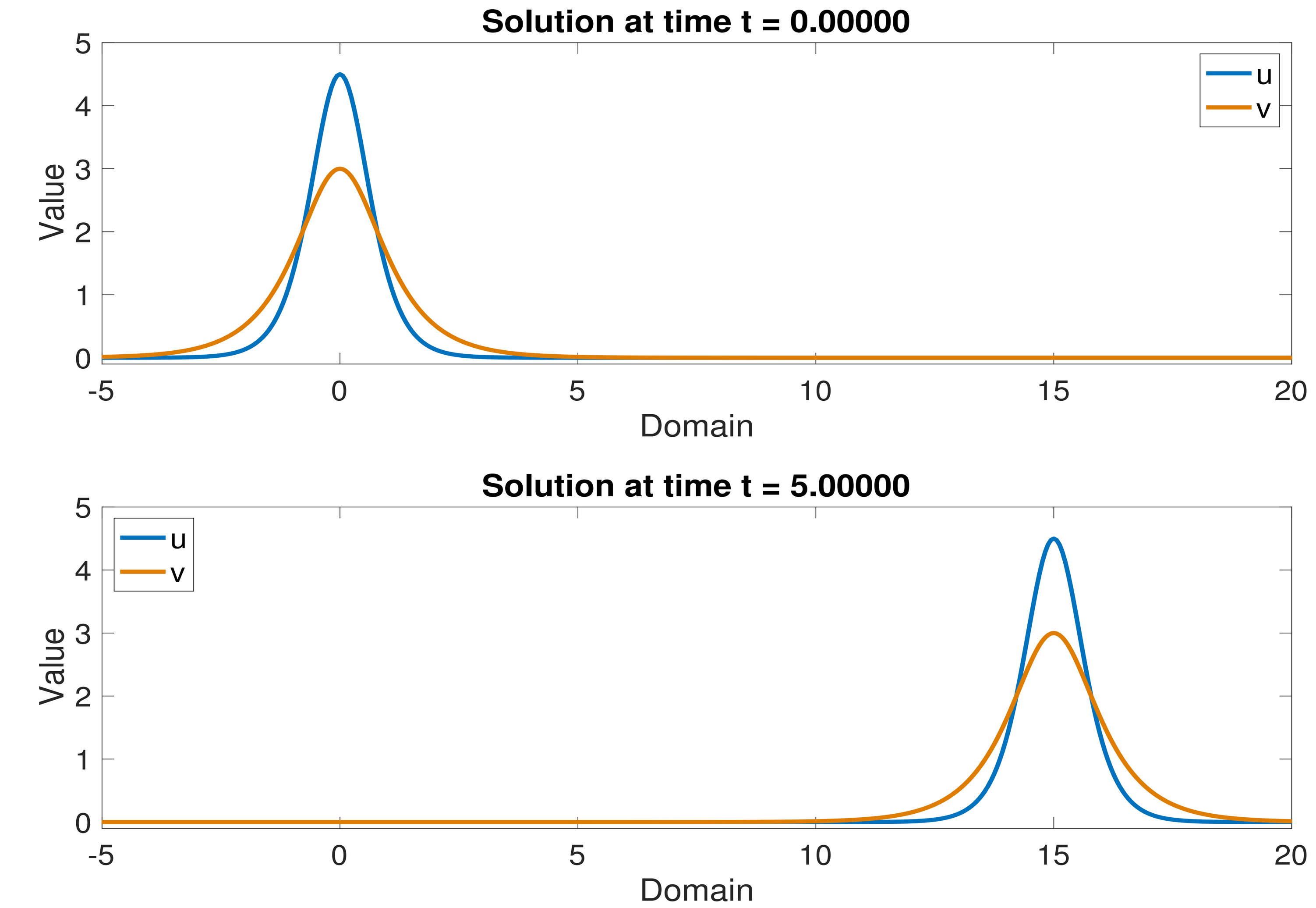
We study the soliton solution of the coupled Drinfeld-Sokolov-Wilson equation,

$$\mathbf{u_t + 3vv_x = 0}$$
$$\mathbf{v_t + vu_x + 2uv_x + 2v_{xxx} = 0.}$$

The system consists of three nonlinear terms and a single dispersive, third order term. It is the balance between the nonlinear behaviour and the dispersion that gives rise to soliton solutions.



Efficiency plot comparing the second-, fourth-, and sixth-order SBP approximations



Soliton solution to the DSW equation at time t = 0 and t = 5.

Results

- A set of Dirichlet boundary conditions and a set of charateristic boundary conditions are derived. The approximations to the corresponding initial boundary value problems are obtained with SBP-SAT method. Linear stability is proven.
- The numerical solution converges to the exact solution with the expected convergence rate.
- The simulation is performed with different orders of accuracy SBP-SAT operators. Higher order accurate approximations are shown to be more efficient in terms of runtime-error.
- We do a numerical study when the solution collides with non-periodic boundaries.

Future works

- Extend the currently shown linear stability for the DSW equation to possibly strict nonlinear estimate.
- Optimize the discrete time-stepping procedure.

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