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Simulating a Bubble

Abstract

In this project a two-phase flow problem is investigated. The interface between two fluids that do not mix (immiscible) is obtained by the finite element method using a level-set approach. The accuracy of the interface curvature is analysed.

Figure: Water and air are immiscible fluids

Level-Set Method

The level set method allows to simulate flow of two immiscible fluids separated by a moving interface. The basic equation for the level set method is an advection equation:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0$$

It is referred to as "the level set equation", $\boldsymbol{u}$ is the advection velocity term.

The interface is represented by the 0.5 contour of ϕ. In the transition layer near the interface ϕ changes smoothly form 0 to 1.

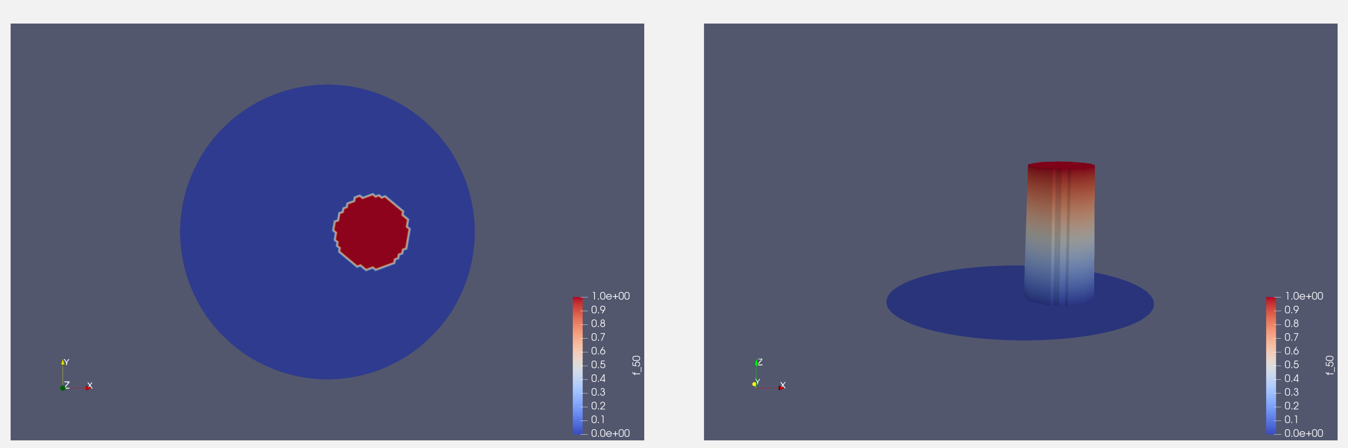


Figure: Solution, T = 0

Stabilization

- 2^{nd} order Residual-based artificial Viscosity (RV), μ,
- Anti-diffusive compression, $\Theta(\phi)$.

The advection equation takes the following form

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi - \nabla \cdot (\mu \Theta(\phi) \nabla \phi) = 0,$$

$$\Theta(\phi) = \left(1 - c_{comp} \frac{[(\phi - \phi_L)(\phi_R - \phi)]^+}{h|\nabla \phi|}\right)^+$$

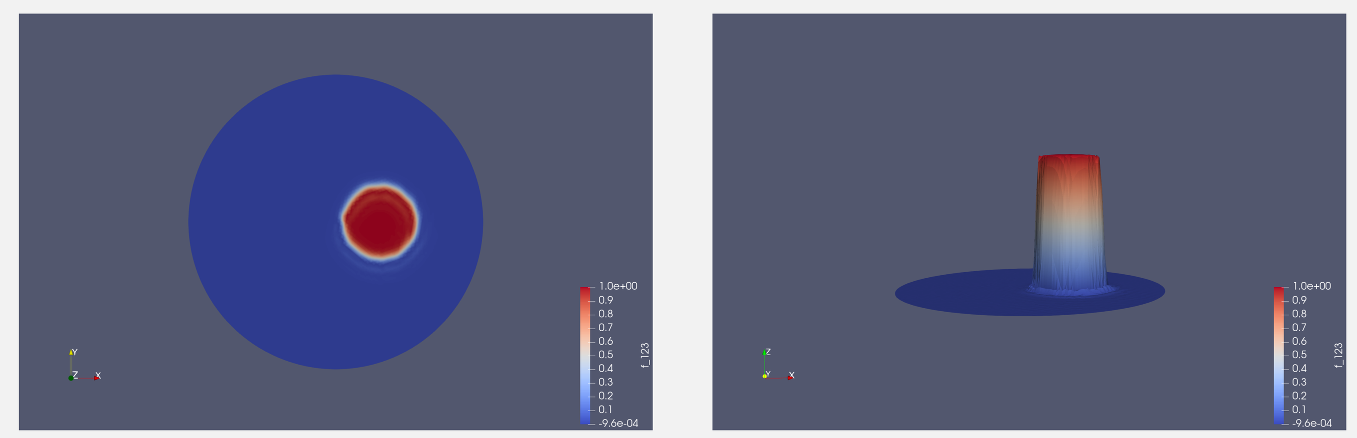
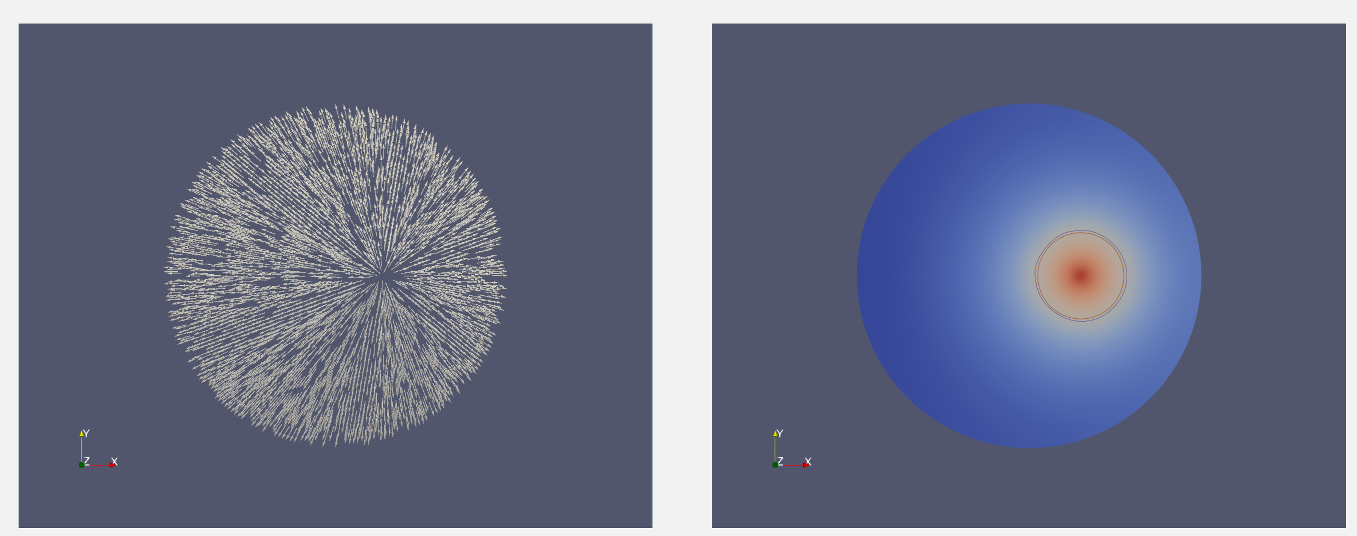


Figure: Solution, T = 1

Normal - Curvature

$$\bar{n} = -\frac{\nabla \phi}{||\nabla \phi||_{\ell^2}}, \qquad \kappa = \nabla \cdot \bar{n}$$



(a) Normal $\bar{n}$, T = 1 (b) Curv. κ, T = 1

Two-Phase Flow Model

The stabilized Level-Set is advected by the velocity from the incompressible Navier-Stokes (INS) equation. Resulting in the system describing two-phase flow.

$$\begin{cases} \rho(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot (\nu(\nabla \mathbf{u} + \nabla^T \mathbf{u})) \\ \nabla \cdot \mathbf{u} = 0 \\ \frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi - \nabla \cdot (\mu \Theta(\phi) \nabla \phi) = 0. \end{cases}$$

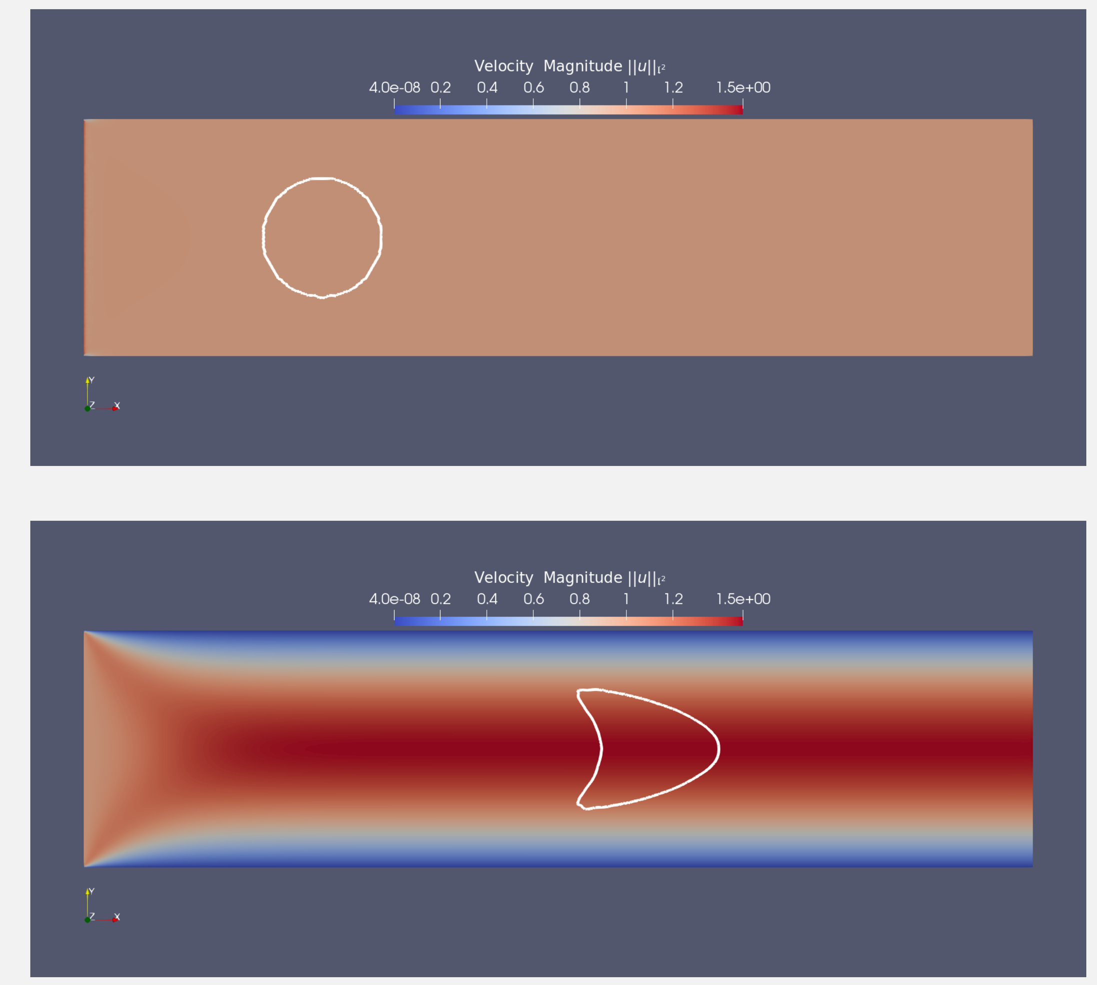


Figure: Two-Phase flow solution at T = 0, 1.

White curve represents the interface.

Conclusions

- Antidiffusion produces a smooth and sharp solution.
- The ϕ convergence rate obtained by the Antidiffusion, measured in the $\ell^2 - \textit{norm}$, is $q = 0.58$.
- The solution obtained from the coupled INS problem is deformed, due to the higher velocity values in the middle of the channel.