UPPSALA UNIVERSITET

PDE-Constrained Optimization Using Physics-Informed Neural Networks

Summary

A physics-informed neural network (PINN) model is created by extending a standard neural network to incorporate physical constraints into the training process. The parameters of the PINN are optimized and the effectiveness of PINNs as a tool for solving partial differential equations are investigated.

Problem formulation

The aim is to create a PINN model to solve the inverse problem

$$\underset{c(x)}{\text{minimize}} \sum_{i=1}^{N} |u(t_i, \hat{x}) - \hat{u}_i|^2$$

constrained by the second order wave equation

$$\begin{aligned} u_{tt} &= (c(x)u_x)_x, && 0 \le x \le 1, \quad t > 0 \\ u(t,x) &= \sin(\pi x) + \cos(\pi x), && 0 \le x \le 1, \quad t = 0 \\ u_t(t,x) &= 0, && 0 \le x \le 1, \quad t = 0, \\ u(t,x) &= 0, && x = 0, 1, \quad t > 0, \end{aligned}$$

where $\hat{u}_i = u(t_i, \hat{x})$ is the exact solution at $\hat{X}$ and c(x) is the wave speed function.

The PINN model

The neural network consists of an input layer, one or multiple hidden layers and an output layer. All layers consists of a set of neurons. The network parameters are initialized using an **initializer** and at each neuron an **activation function** is applied to define the output given a set of inputs from the parameters of the previous layer.

The **training data** consists of simulated data points spread out over the boundaries and inside domain by a point generator engine.

A **loss function** is used to evaluate the accuracy of the predicted solution together with the predicted wave speed function.

Partial derivatives of the predicted solution is computed with **automatic differentiation**. The derivatives of the neural network together with the wave speed function are extracted as functions of their input data.

An **optimizer** is used to update both the parameters of the network and wave speed function to reduce the loss.

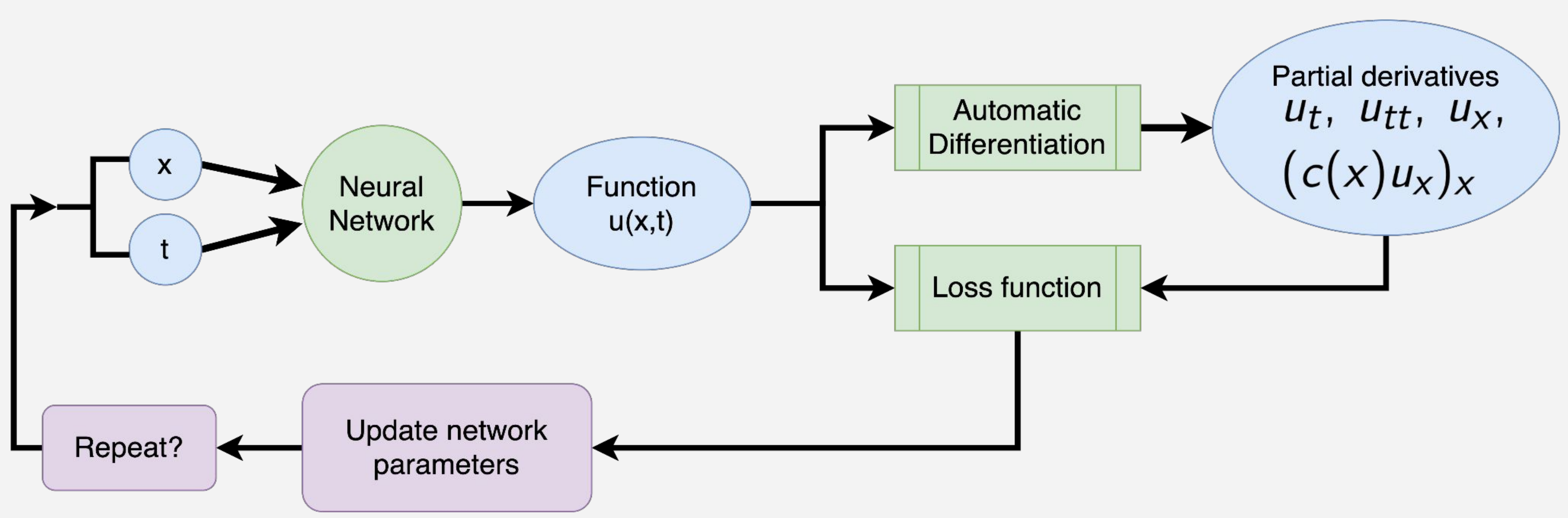
Model Parameters

The network parameters are optimized for the inverse problem. The coefficients for c(x) is stored in the network as trainable parameters. For the optimization the reference c(x) is set to c(x)=1.

Number of hidden layers	10
Neurons per hidden layer	40
Data generation engine	Halton
Optimizer	L-BFGS
Initializer	GlorotUniform
Activation function	Hyperbolic tangent
Number of collocation points	15000
Number of data points for reference data	250
Number of BC data points	800
Number of IC data points	200

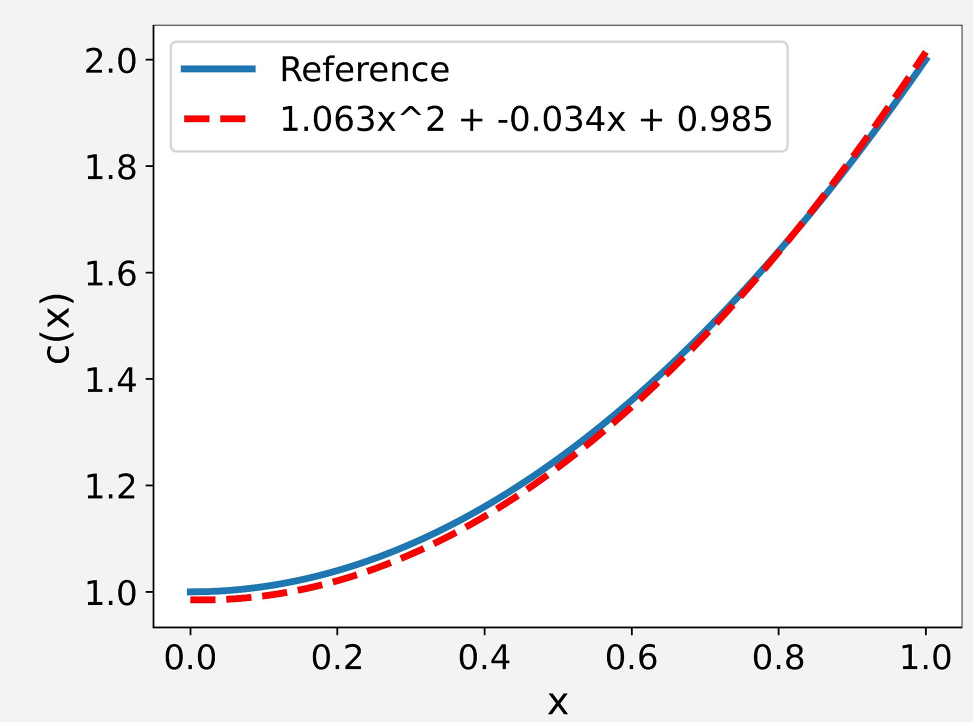
Parameter choices for the PINN model.

Physics-Informed Neural Network



Flowchart overview of the PINN model.

Results



Correct wave speed function c(x)= x^2 + 1 along with the identified one obtained by learning three unknown coefficients.

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