A Monte-Carlo Generator for Hyperons:

Facilitating the Search for an Explanation of the Asymmetric Universe

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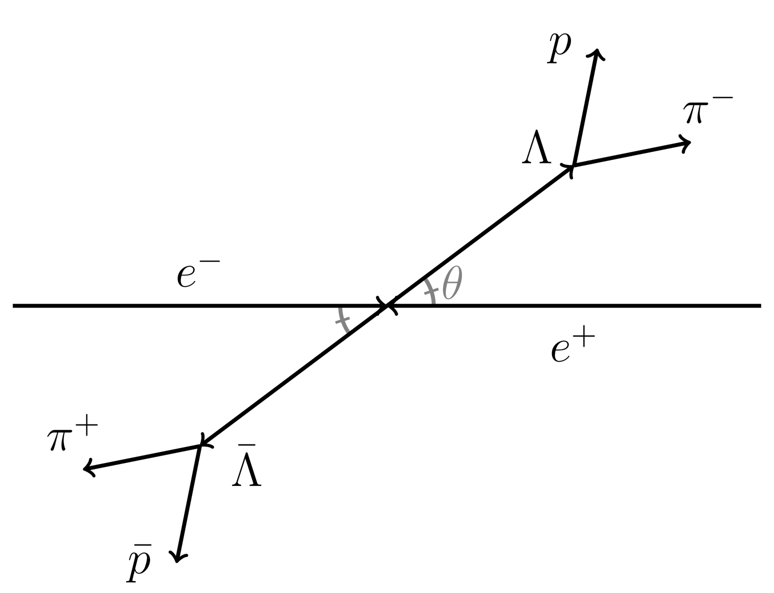
Introduction to Problem

A fundamental question in physics is why there is so much more matter than antimatter in our universe given that, according to the Big Bang theory, equal amounts of each were created. One theory is that of *baryogenesis* - a hypothesised process that would have generated this asymmetry dynamically from a symmetric initial state. In order for baryogenesis to have taken place there must be a fundamental asymmetry between the laws governing matter and antimatter, known as *charge-parity (CP) violation*. The nuclear physics group at Uppsala University is searching for CP violation by studying the decay of hyperon-antihyperon $(\Lambda/\bar{\Lambda})$ pairs produced in the annihilation of electron-positron pairs.



- $e^{\mp}$: electron/positron
- $\Lambda/\bar{\Lambda}$: hyperon/antihyperon
- $p/\bar{p}$: proton/antiproton
- $\pi^{\mp}$: negatively/positively charged pion

Figure 1. Graphical representation of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction. The collision of an e^- with a e^+ produces a $\Lambda/\bar{\Lambda}$ pair, which decay into (p, π^-) and $(\bar{p}, \pi^+)$ pairs respectively.

The angles $\xi = (\theta, \theta_1, \phi_1, \theta_2, \phi_2)$ describe the paths of the particles produced in the collision and subsequent decays depicted in Figure 1, where θ gives the direction of motion of the $\Lambda/\bar{\Lambda}$ hyperons, and (θ_i, ϕ_i) give the polar angles of the direction of the $p/\bar{p}$.

The distribution of ξ given physical parameters $\beta = (\eta, \Delta\Phi, \alpha_1, \alpha_2)$ is given by:

$$\begin{aligned}\mathcal{W}(\xi) =& \mathcal{F}_0(\xi) + \eta\mathcal{F}_5(\xi) \\ &+ \alpha_1\alpha_2\Big(\mathcal{F}_1(\xi) + \sqrt{1-\eta^2}\cos(\Delta\Phi)\mathcal{F}_2(\xi) + \eta\mathcal{F}_6(\xi)\Big) \\ &+ \sqrt{1-\eta^2}\sin(\Delta\Phi)\Big(\alpha_1\mathcal{F}_3(\xi) + \alpha_2\mathcal{F}_4(\xi)\Big),\end{aligned}$$

where the $\mathcal{F}_i$ are trigonometric functions on ξ [4].

For this project we have created a Monte-Carlo generator in Python to generate the angles ξ, given the parameters β, and based on the distribution $\mathcal{W}$. If $\alpha_1 \neq \alpha_2$, this would provide evidence of CP violation.

Monte-Carlo Methods

We use Monte-Carlo methods to generate samples from the distribution $\mathcal{W}$. We have explored two Monte-Carlo methods:

Hit-and-Miss [1]

- Draws samples from uniform distribution, but discards them if they fall outside the desired sampling distribution
- May be inefficient if many samples are discarded
- Samples are independent

Metropolis [5]

- Markov chain where every step is an attempt to move about the sample space
- Efficient for multidimensional distributions
- No need to integrate density for normalisation
- Samples are correlated as some attempts are rejected

Proof of Concept

Before implementing a generator for the full 5-dimensional distribution, we first constructed generators for simpler 1- and 3-dimensional cases that consider angles (θ) and $(\theta, \theta_1, \phi_1)$ respectively. These have much simpler distributions, and so are a good way to test the functionality of a Monte-Carlo generator.

Results

Figure 2 shows the value of $R = \sqrt{\tau\frac{1-\eta}{1+\eta}}$, where τ is a physical parameter, as estimated by the method of moments [3] on data generated using hit-and-miss Monte-Carlo on the 5-dimensional distribution $\mathcal{W}$. The red line shows the true value of R based on the value of η used to generate the data.

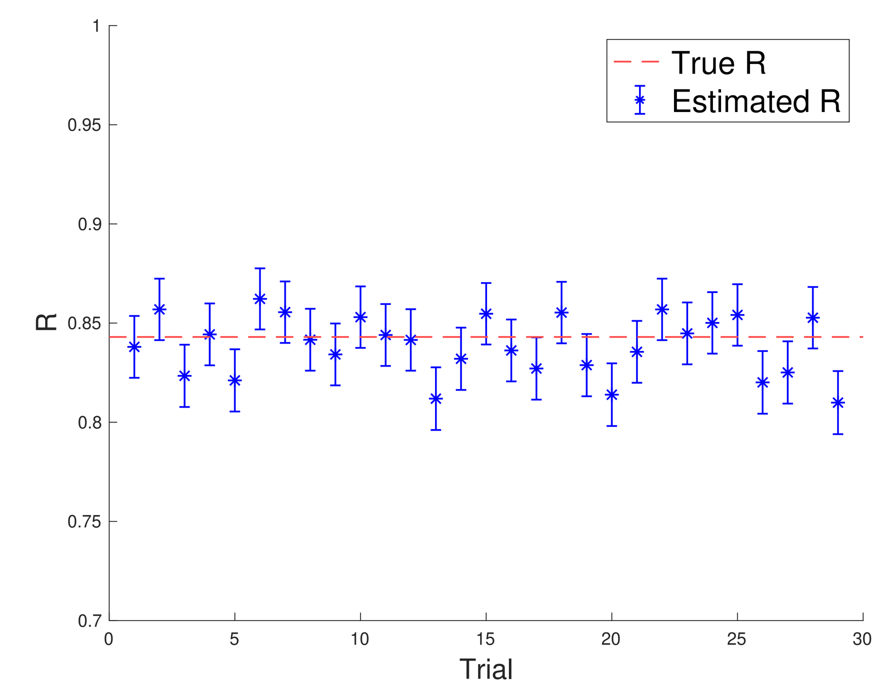


Figure 2. Estimated values of R using the method of moments on data generated using hit-and-iss Monte-Carlo. Error bars show one standard deviation. Trials consist of $N = 10^5$ events.

We use a density heatmap to visualise samples and confirm that they match experimental data. Figure 3 illustrates generated set using the Metropolis method with parameters $[\eta, \Delta\Phi, \alpha_1, \alpha_2] = [-0.460, \frac{\pi}{4}, 0.75, -0.75]$ as in [2]. $N_{steps} = 4\times10^6$.

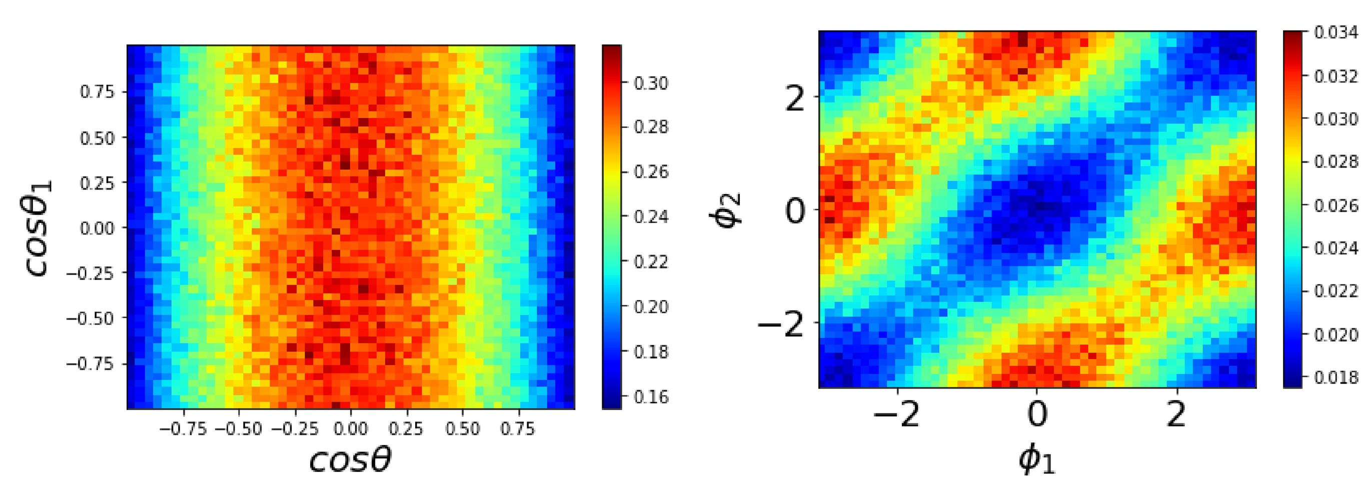


Figure 3. Example of generated data using Metropolis method

Future Work

- Use a number of parameter estimation methods to reconfirm that our generation methods function as intended.
- Explore whether a third Monte-Carlo method, the *inverse transformation* method, could be applied to $\mathcal{W}$.
- Compare the efficiency and computational cost of the different Monte-Carlo generation methods used in the study.

References

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