Scheduling theory, part 1
Aperiodic jobs

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Real-time systems?

Not necessarily fast...

...but always predictable
Real-time systems: theory & practice

Practice

Building systems

Software, middleware, RTOS...

Hardware design

...
Real-time systems: theory & practice

Theory

Verification

Scheduling theory

Control theory

Practice

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Real-time systems: theory & practice

Theory
- Verification
- Scheduling theory
- Control theory
- ...

Practice
- Building systems
- Software, middleware, RTOS...
- Hardware design
- ...


Classical scheduling theory (e.g., in operations research) generally deals with *finite* processes (job-shop, flow-shop &c.) to *optimize* some metric.

Real-time scheduling theory generally deals with *infinite* processes (control loops &c.) to *guarantee* a safety specification.
The components of real-time scheduling theory

Task models:
Formalisms to specify workload and timing constraints

Scheduling algorithms:
Run-time strategies for scheduling workload

Analysis:
Offline methods for proving timing safety
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Modeling: the art of abstraction

The best material model of a cat is another, or preferably the same, cat.
— Norbert Wiener, 1945

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The job: a unit of work

A job $j_i$ is given by a triple $(A_i, C_i, D_i) \in \mathbb{N}^3$, where

- $A_i$ is the arrival time (or release time).
- $C_i$ is the worst-case execution time (WCET), and
- $D_i$ is the deadline.
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$$j_i = (A_i, C_i, D_i) = (3, 6, 21)$$
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Scheduling a collection of independent jobs

The problem

Given a (multi-)set \( \mathcal{J} = \{j_1, \ldots, j_n\} \) of \( n \) jobs, find a schedule where all jobs meet their deadlines.
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Given a (multi-)set $\mathcal{J} = \{j_1, \ldots, j_n\}$ of $n$ jobs, find a schedule where all jobs meet their deadlines.

Assumptions

- All jobs are independent
- A single processor
- Fully preemptive -or- non-preemptive scheduling
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Preemptive:  

Non-preemptive:
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**Challenge**

Schedule these jobs

\[ J = \left\{ (0, 2, 6), (0, 2, 14), (0, 2, 3), (0, 7, 13), (0, 1, 15), (0, 1, 2) \right\} \]

Note: All jobs in \( J \) have the same arrival time. Such jobs are called *synchronous*.
A solution

Earliest Deadline First (EDF)

Scheduling rule: Choose among the ready jobs to execute the job with the earliest deadline (ties broken arbitrarily).
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Earliest Deadline First (EDF)

Scheduling rule: *Choose among the ready jobs to execute the job with the earliest deadline (ties broken arbitrarily).*

Note 1: We didn’t need to use preemption!
A solution

Earliest Deadline First (EDF)

Scheduling rule: Choose among the ready jobs to execute the job with the earliest deadline (ties broken arbitrarily).

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Note 2: In this setting, EDF is also called *Earliest Due Date (EDD)* or *Jackson’s algorithm.*
Is this a good general strategy?

Question

Is EDF a good strategy for all sets of synchronous jobs?
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Theorem (Jackson, 1955)
If it is possible to schedule a set \( J \) of synchronous jobs, then \( J \) can also be scheduled by EDF.

Proof on black board!
**Some important definitions**

**Schedulability**

\( J \) is \( A \)-schedulable iff scheduling algorithm \( A \) always generates a schedule without deadline misses for \( J \).

**Feasibility**

\( J \) is feasible iff there exists a scheduling algorithm \( A \) such that \( J \) is \( A \)-schedulable.

**Optimality**

\( A \) is optimal iff all feasible \( J \) are also \( A \)-schedulable.
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How do we know if a set $\mathcal{J}$ of synchronous jobs is EDF-schedulable?

Schedulability test (Jackson, 1955)

Without loss of generality, let the indices of the jobs in $\mathcal{J} = \{j_1, \ldots, j_n\}$ be ordered by non-decreasing deadlines, and let all the arrival times be zero. Then, $\mathcal{J}$ is EDF-schedulable iff

$$\forall i, 1 \leq i \leq n : \sum_{k=1}^{i} C_k \leq D_i.$$
How do we know if a set $\mathcal{J}$ of synchronous jobs is EDF-schedulable?

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Now let’s do arbitrary arrival times

For a set $J$ of jobs with asynchronous arrival times, does it matter if we allow preemptions?

Question
Now let’s do arbitrary arrival times

For a set $\mathcal{J}$ of jobs with *asynchronous* arrival times, does it matter if we allow preemptions?

**YES!**
The preemptive case

Question

Is EDF still optimal for preemptive scheduling of job sets with asynchronous arrival times?
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Theorem (Dertouzos, 1973)

EDF is optimal for scheduling any set of independent jobs on a single preemptive processor.
Proof of the optimality of EDF

First, let’s define an important function.

**The demand bound function**

For a job $j_i$ and time instants $t_1$ and $t_2$, where $0 \leq t_1 \leq t_2$, let the *demand bound function* $\text{dbf}(j_i, t_1, t_2)$ be defined as

$$\text{dbf}(j_i, t_1, t_2) = \begin{cases} C_i, & \text{if } t_1 \leq A_i \text{ and } D_i \leq t_2 \\ 0, & \text{otherwise}. \end{cases}$$

For a job set $\mathcal{J}$, let $\text{dbf}(\mathcal{J}, t_1, t_2)$ be defined as

$$\text{dbf}(\mathcal{J}, t_1, t_2) = \sum_{j_i \in \mathcal{J}} \text{dbf}(j_i, t_1, t_2).$$
Proof of the optimality of EDF

Feasibility test / EDF-schedulability test

A job set $\mathcal{J}$ is feasible on a single preemptive processor iff

$$\forall t_1, t_2 \text{ such that } 0 \leq t_1 \leq t_2 : \quad \text{dbf}(\mathcal{J}, t_1, t_2) \leq t_2 - t_1.$$
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Let’s prove the validity of this test and the optimality of EDF in one go!

Step 1: Prove that the condition is necessary.

Step 2: Prove that it is sufficient for EDF.

Conclusion: The test is valid and EDF is optimal. (Why?)

Proofs on the blackboard!
Proof of the optimality of EDF

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\[ \forall t_1, t_2 \text{ such that } 0 \leq t_1 \leq t_2 : \text{ dbf}(J, t_1, t_2) \leq t_2 - t_1. \]

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**Step 1**: Prove that the condition is *necessary*.
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**Conclusion**: The test is valid *and* EDF is optimal. (Why?)

Proofs on the blackboard!
The preemptive case: conclusions

Theorem (Dertouzos, 1973)

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A job set $\mathcal{J}$ is feasible on a single preemptive processor iff

$$\forall t_1, t_2 \text{ such that } 0 \leq t_1 \leq t_2 : \quad \text{dbf}(\mathcal{J}, t_1, t_2) \leq t_2 - t_1.$$  

(It is enough to consider values of $t_1$ picked from the arrival times and values of $t_2$ picked from the deadlines.)
The non-preemptive case

Is EDF still optimal for non-preemptive scheduling of job sets with asynchronous arrival times?

Question

NO! (But it is still optimal if idling is forbidden! Proof omitted.)
The non-preemptive case

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Is EDF still optimal for non-preemptive scheduling of job sets with asynchronous arrival times?

NO!
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How can we then find the best schedule for a job set $\mathcal{J}$ of non-preemptive jobs?
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How can we then find the best schedule for a job set $J$ of non-preemptive jobs?

One possible approach

Step 1: Assume the execution time of all jobs is the WCET.
Step 2: Try all possible orderings of executing the jobs.
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**Question**

How can we then find the best schedule for a job set $\mathcal{J}$ of non-preemptive jobs?

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Good news: This works!
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How can we then find the best schedule for a job set $\mathcal{J}$ of non-preemptive jobs?

**One possible approach**

Step 1: Assume the execution time of all jobs is the WCET.
Step 2: Try all possible orderings of executing the jobs.

**Good news: This works!**

**Bad news: There are $n!$ orderings of $n$ jobs.**
The non-preemptive case

Question

Is there an *efficient* way to find a valid schedule for a set of non-preemptive jobs?
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Is there an efficient way to find a valid schedule for a set of non-preemptive jobs?

Probably not: This problem is strongly NP-hard. (There is a simple reduction from 3-PARTITION.)
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In practice, various heuristic search techniques could work well.